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Research Article

RANKING DMUS WITH FUZZY DATA BY L_2-NORM

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ABSTRACT
This paper develops DEA models using imprecise data represented by fuzzy sets. By a metric distance on interval and fuzzy numbers, we introduce the approach to solve the fuzzy CCR model (FCCR) and ranking of DMUs with fuzzy data.

Keywords: *Fuzzy DEA, DMU, Fuzzy Input-output*

INTRODUCTION
The concept of decision making in fuzzy environment was first proposed by Bellman and Zadeh (1970). Some researchers have proposed several fuzzy models to evaluate DMUs with fuzzy data (see [3,4,5,6,7,8]). The DEA models with fuzzy data (FDEA models) can more realistically represent real-world problems than the conventional DEA models. The fuzzy set theory also allows linguistic data to be used directly within the DEA models.

The fuzzy DEA models take the form of fuzzy linear programming models. A typical approach to fuzzy linear programming requires a method to rank fuzzy sets and different fuzzy ranking methods may lead to different results. The problem of ranking fuzzy sets has been addressed by many researchers.

This paper develops DEA models using imprecise data represented by fuzzy sets. By a new metric distance on interval and fuzzy numbers we introduce the approach to solving the fuzzy CCR model (FCCR) and ranking of DMUs with fuzzy data.

The paper is organized as follows: In Section 2 fuzzy notation and the equivalence of Fuzzy Production Possibility Set and Ranking with Fuzzy Data in DEA are considered. In Section 3 the method is illustrated by solving an applied numerical example and conclusions are drawn in section 4.

Fuzzy DEA
Fuzzy Production Possibility Set
Suppose that we have a sample of n, DMUs to be evaluated. Each DMU consumes varying amounts of m different inputs to produce s different outputs; we assume that all the input and output data cannot be exactly obtained due to the existence of uncertainty.
They are approximately known and can be represented by positive triangular fuzzy numbers. All inputs and outputs are assumed to be nonnegative, especially, DMU_j consume amounts $\tilde{X}_j\ (j=1,...,n)$ of fuzzy input (vector) and produces amounts $\tilde{Y}_j\ (j=1,...,n)$ of fuzzy output (vector). Now we define the fuzzy Production Possibility Set:

$$FT_c=\left\{\left(\tilde{X},\tilde{Y}\right)\middle| d(\tilde{X}_j)\geq d(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}), d(\tilde{Y}_j)\leq d(\sum_{j=1}^{n}\lambda_j\tilde{y}_{rj}),\lambda\geq 0\right\}$$

Assumption1. All actually observed fuzzy input-fuzzy output combinations are feasible. An fuzzy input-fuzzy output bundle $\left(\tilde{X},\tilde{Y}\right)$ is feasible when the fuzzy output bundle $\tilde{y}$ can be produced from the fuzzy input bundle $\tilde{x}$, by Assumption (1) each $\left(\tilde{x}_j,\tilde{y}_j\right)$ $j=1,...,n$ is a feasible bundle fuzzy input-fuzzy output.

Assumption2. Consider two feasible fuzzy input-fuzzy output bundles $\left(\tilde{X}^{'},\tilde{Y}^{'}\right),\left(\tilde{X}^{''},\tilde{Y}^{''}\right)$. Then, the (weighted) average fuzzy input-fuzzy output bundle $\left(\tilde{\tilde{X}},\tilde{\tilde{Y}}\right)$, where

$\tilde{\tilde{X}} = \lambda\tilde{X}^{'} + (1-\lambda)\tilde{X}^{''}$, $\tilde{\tilde{Y}} = \lambda\tilde{Y}^{'} + (1-\lambda)\tilde{Y}^{''}$ for some λ satisfying $0 \le \lambda \le 1$:

$$d(\tilde{\tilde{X}}) = d(\lambda\tilde{X}^{'} + (1-\lambda)\tilde{X}^{''}) = \lambda d(\tilde{X}^{'}) + (1-\lambda)d(\tilde{X}^{''}) \ge \lambda d(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}) + (1-\lambda)d(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij})$$

$$d(\tilde{\tilde{Y}}) = d(\lambda\tilde{Y}^{'} + (1-\lambda)\tilde{Y}^{''}) = \lambda d(\tilde{Y}^{'}) + (1-\lambda)d(\tilde{Y}^{''}) \ge \lambda d(\sum_{j=1}^{n}\lambda_j\tilde{y}_{rj}) + (1-\lambda)d(\sum_{j=1}^{n}\lambda_j\tilde{y}_{rj})$$

Assumption3. Fuzzy inputs are freely disposal, if $\left(\tilde{X}^{o},\tilde{Y}^{o}\right)$ is feasible, then for any $\left| d(\tilde{X}) \ge d(\tilde{X}^{o}) \right.$ is also feasible and Fuzzy outputs are freely disposal, if $\left(\tilde{X}^{o},\tilde{Y}^{o}\right)$ is feasible, then for Any $d(\tilde{Y}) \le d(\tilde{Y}^{o})$ is also feasible (that is easy to see).

Assumption4. If $\left(\tilde{X},\tilde{Y}\right)$ is feasible, then for any $\mathrm{k} \ge 0$, $\left(k\tilde{X},k\tilde{Y}\right)$ is also feasible(that is easy to see).

Assumption5. FTC is a smallest set that satisfying assumption (1-4) (that is easy to see).

Now consider the input-oriented technical efficiency of firm (o), we examine whether and to what to reduce its fuzzy inputs without reducing the fuzzy outputs.

The input-oriented technical efficiency of firm (o) is θ^{*}, where:

$$\theta^{*} = \min\theta \quad ,\left(\theta\tilde{X}^{o},\theta\tilde{Y}^{o}\right) \in FT_{c} \tag{1.1}$$

Definition 2.1. $\mathrm{DMU_o}$ $\left(\tilde{X}^{o},\tilde{Y}^{o}\right)$ is a technical efficient if and only if the input-oriented technical efficiency of firm (o) equal one ($\theta^{*} = 1$).

Fuzzy L_2-NORM Model in DEA

Suppose that we have used CCR or BCC models to obtain the efficiency score of observed DMUs and also assume that $\mathrm{DMU_b}$ is one of the observed DMUs.

Now we omit $\mathrm{DMU_b}$ from the reference set of all the other DMUs so, the original efficient frontier will change if and only if $\mathrm{DMU_b}$ is Extreme efficient (E).

The new efficient frontier (without $\mathrm{DMU_b}$) gets closer to the inefficient DMUs and it is possible that some of these inefficient DMUs change to efficient.

Obviously, among the extreme efficient DMUs, the one that affects the efficient frontier to get further to the remaining DMUs should be ranked as the best one.

In order to carry out our method, we re-evaluate all of the Inefficient and Non-extreme efficient (I,N) DMUs by the following model:

$$
\begin{aligned}
Min \quad & \varphi_a^b=\theta\text{-}\varepsilon\left(\sum_{i=1}^{m}s_i^- + \sum_{r=1}^{s}s_r^+\right) \\
s.t. \quad & \sum_{\substack{j=1\\ j\neq b}}^{n}\lambda_j x_{ij} + s_i^- = \theta x_{ia} && i=1,\dots,m \\
& \sum_{\substack{j=1\\ j\neq b}}^{n}\lambda_j y_{rj} - s_r^+ = y_{ra}, && r=1,\dots,s \\
& \lambda_j \ge 0 && \text{j=1,....,n ,j} \neq \text{b} \\
& s_i^- \ge 0, && i=1,\dots,m \\
& s_r^+ \ge 0, && r=1,\dots,s
\end{aligned} \tag{1.2}
$$

Ranking with Fuzzy Data in DEA

In this section, we suppose that inputs and outputs of DMUs (decision making units) are fuzzy numbers. Consider fuzzy input-output levels, we assume that the DMU_b is extreme efficient. By omitting $(\tilde{X}_b,\tilde{Y}_b)$ from *FTC*, we define the production possibility set $FT_c^{'}$ as:

$$
FT_c^{'} = \left\{ (\tilde{X},\tilde{Y}) \middle| d(\tilde{X}_j) \ge d(\sum_{\substack{j=1\\ j\neq b}}^{n}\lambda_j\tilde{x}_{ij}), d(\tilde{Y}_j) \le d(\sum_{\substack{j=1\\ j\neq b}}^{n}\lambda_j\tilde{y}_{rj}), \lambda \ge 0 \right\} \tag{1.3}
$$

To obtain the ranking score of DMU_b,we consider the following model:

$$
\begin{aligned}
Min \quad & \tilde{\varphi}_a^b=\theta\text{-}\varepsilon\left(\sum_{i=1}^{m}s_i^- + \sum_{r=1}^{s}s_r^+\right) \\
s.t. \quad & \sum_{\substack{j=1\\ j\neq b}}^{n}d(\lambda_j \tilde{x}_{ij}) + s_i^- = \theta d(\tilde{x}_{ia}) && i=1,\dots,m \\
& \sum_{\substack{j=1\\ j\neq b}}^{n}d(\lambda_j \tilde{y}_{rj}) - s_r^+ = d(\tilde{y}_{ra}), && r=1,\dots,s \\
& \lambda_j \ge 0 && \text{j=1,....,n ,j} \neq \text{b} \\
& s_i^- \ge 0, && i=1,\dots,m \\
& s_r^+ \ge 0, && r=1,\dots,s
\end{aligned} \tag{1.4}
$$

where $a \in \tilde{\Gamma}_{I,N}$ *and* $b \in \tilde{\Gamma}_E$. Note that $\tilde{\Gamma}_{I,N}$ is the set of inefficient and non-extreme efficient DMUs with fuzzy data and $\tilde{\Gamma}_E$ is the set of extreme efficient DMUs with fuzzy data. Now we consider vector $1=(1,1,\dots,1)^{'} \in \mathrm{R}^{card(\tilde{\Gamma}_{I,N})}$ and call it *ideal vector*. After obtaining the measure of efficiency $\tilde{\varphi}_a^b$ for each $a \in \tilde{\Gamma}_{I,N}$ by model (3), we define vector $\tilde{X}^{(b)} \in \mathrm{R}^{card(\tilde{\Gamma}_{I,N})}$ for each $b \in \tilde{\Gamma}_E$ as follows:

$$\tilde{X}^{(b)} = (\tilde{\varphi}_a^b)^t, \quad \forall\ a \in \tilde{\Gamma}_{I,N}$$

$$\tilde{\omega}^b = \left\| 1 - \tilde{X}^{(b)} \right\|_2 = \left(\sum_{a \in \Gamma_{I,N}} \left| 1 - \tilde{\varphi}_a^b \right|^2 \right)^{\frac{1}{2}}, \forall b \in \tilde{\Gamma}_E \qquad (1.5)$$

$$\tilde{\omega}^b \ (\forall \text{b: b} \in \tilde{\Gamma}_E)$$

After calculating $\tilde{\omega}^b$ $(\forall \text{b:b} \in \tilde{\Gamma}_E)$, we classify DMU_b s (the extreme efficient DMUs with fuzzy data) based on comparing as follows:

At first, we choose the smallest of $\tilde{\omega}^b$ s and then let its corresponding DMU_b as the first extreme efficient fuzzy DMU. Now, among the rest of $\tilde{\omega}^b$ s, choose the smallest of them, and then let its corresponding DMU_b with fuzzy data as the second extreme efficient fuzzy DMU. Similarly, we can classify all of the extreme efficient DMU with fuzzy data in this method. Obviously, the biggest of $\tilde{\omega}^b$ s is corresponding with the last of extreme efficient DMU with fuzzy data.

MATERIALS AND METHODS

Methodology and Examples

In this example, the data of 20 branch banks of Iran is evaluated by the proposed method. Each branch uses four inputs in order to produce tow outputs. The labels of inputs and outputs are presented in the table below.

Table 1: The labels of inputs and outputs

	Input	Output
1	Staff	Loans
2	Computer terminals	Charge
3	Space (m2)	
4		
	Deposits	

Table 2: The ranking of technically efficient branch banks of Iran

DMU	E	L_2	Rank
15	1	1.110	1
4	1	1.322	2
7	1	1.317	3
20	1	1.367	4
17	1	1.382	5
12	1	1.378	6
1	1	1.385	7

Conclusion

In this paper, we presented a criterion for ranking Fuzzy data. Using the fuzzy distance, the L_2-Norm ranking method of DMUs with crisp data was extended to a ranking method of DMUs with Fuzzy data. A numerical example was also provided for ranking different branch banks of Iran. Our proposed method can be utilized for ranking any set of DMUs with Fuzzy data.

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