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Research Article

A COMPARISON BETWEEN MEAN-RISK MODEL AND PORTFOLIO SELECTION MODELS WITH FUZZY APPROACH IN COMPANIES LISTED IN TEHRAN STOCK EXCHANGE

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ABSTRACT
This study is conducted to compare between Mean-risk Model and Portfolio Selection Models of with fuzzy approaches. To do so, 8 portfolios are reviewed. Results of hypothesis testing showed that the value at risk model with fuzzy approach is not capable of estimating market risk and selecting optimum portfolio on its own. The proposed portfolio leads to an expected return approaching the return from Markowitz method showing the model power in estimating portfolio. In this model, portfolio selection is significantly affected by confidence curve. In all calculations, confidence curve values are applied to estimate investment ratios.

Keywords: *Mean-risk Model, Fuzzy Approach, Portfolio Selection, Value at Risk*

INTRODUCTION

Developing investment, on one hand, leads to the attraction of inefficient capitals and directing them to economic productive sectors and on the other hand the investments will be led to industries having higher or lower risk regarding investors' orientation (based on risk and return). This will result in the optimum allocation of limited resources and this is the main objective of the emergence of capital markets. Concerning the evolutions happened in today world, especially developing countries facing with serious threats need suitable solutions to better exploit their God's endowments and gifts for solving their economic problems. And, one of the best solutions is to expand and develop investments (Tehrani and Nourbakhsh, 2009). On the other hand, in countries with high cash flow in people's hand, it is possible to attract these capitals by developing and expanding capital markets, prevent from economic losses, and direct economy toward sublimity (Yari, 2008).

Since organizing global economy and removing the imposed shocks are feasible due to the flexibility and strength of the advanced markets, the markets (including centralized and decentralized) have monopolized a great part of human's productive, commercial, and financial activities (Castelo *et al.*, 2008). Namely, it can be said that today a wide amount of capitals is traded via stock market across the world. And, national economy of any country is related to the performance of its stock market and gets influenced by the market in a wide range. In addition, such market has recently turned into investment tools; not only for professional investors but also for novice ones. Hence, the issue not only is related to macroeconomic parameters but also affects human's daily life. As a result, they form a mechanism with direct and major social effects (Woo and Shi, 2007).

Return from investment is of particular significance to investors. This is because all investment activities are done to gain return. Return assessment is the only rational way (before risk estimation) which the investors can do in order for comparing between alternative and different investments (Tehrani and Nourbakhsh, 2009).

In researches on determining the optimum portfolio, "risk" is mentioned as one of the main indices of determining portfolio. This is also crystal clear in Markowitz and classic economists' early theories. To measure the risk of a portfolio, indices like delta, gamma, vega, teta, and methods such as variance-covariance, delta-gamma simulation, linear VaR, and historical data VaR can be applied. Each of the indices and methods evaluate different aspects of a portfolio risk. Nevertheless, the main objective of the calculation of these indices is to measure the risk of portfolios the components of which include one or more derived tools exchangeable in bourse. As well as constraining the application of the above indices, the issue makes their application in bourses with only cash trade-offs impossible. In the meantime, it seems that using median risk method – as compared to other conventional methods – provides the possibility for more accurate estimation of return for companies listed in Tehran stock exchange.

Hence, research hypothesis is formulated as follow:

- Fuzzy Mean-risk Model is more efficient compared to the portfolio selection methods.

Literature Review and Background of the Study

In 1959, Markowitz and Tobin presented their investment in uncertainty conditions and based on profit mean and standard deviation. In late 1960s, many financial-economic issues were reviewed using operational research techniques. And, optimization methods were developed for solving such problems. Namely, today, operational research tools including decision analysis, statistical estimations, simulation, random processes, optimization, systems supporting decision making, and artificial intelligence have become an indispensable part of some financial operation aspects (You, 2009).

In most models proposed for portfolio management, a simplified method replaces the actual model. Considering ordinary investment rather than special investment and whatever exists is itself a proof for the claim. The assumptions on describing ordinary investment are mostly inadequate and misleading. For example, applying Markowitz's tow-dimensional models (Mean-Variance [M-V] measure) is probably difficult in the real world. The measure takes many unreal assumptions on investors' preferences and (or) in mentioning investment appositions. Hence, we must act so that the complexities of the real world can be presented in the simplest models regarding application.

Fuzzy sets theory seeks for creating more proximity between classic math accuracy and also general ambiguity existing in the real world. In real issues, many decisions are made in environments where the objectives and constraints and results gained are not fully known. And, decision making for taking optimum portfolio is based on information from financial-economic environment (the annual reports of companies, inflation rate, growth rate, and monetary and fiscal policies of government and …) which accompany with a degree of ambiguity. Expressing these inaccuracies by means of classic and definite mathematical concepts is impossible in terms of portfolio planning and optimizing models. Hence, employing fuzzy logic to overcome the environmental-information ambiguity will contribute to the financial decision makers (Raii and Telangi, 2004).

The first systematic approach for portfolio selection issue has been M-V approach indicated by Markowitz (Jang *et al.*, 2007). In Markowitz's portfolio selection, return on bonds is taken to be random variable. In the model, return and risk are respectively set by mean and standard deviation of the historical return on stock. The main assumption of Markowitz's M-V is that the future situation of bonds can be reflected by past historical data. In the model, in fact, only one aspect of uncertainty (i.e. randomness of return on stock) is considered. On a contrary, in the real world, there are other uncertainty factors based on which randomness cannot be assured for ever-changing return on companies' stocks per se. The uncertainty can be regarded as the fuzziness of the return on stock. And, assuming that the return on stock is in terms of fuzzy variable, we can take action to select portfolio (Lee, 2007). Fuzzy variable is a commensurable function of validation space for real number set. On the other hand, the factors of random and fuzzy uncertainty can be seen in the real world simultaneously. In fact, it is possible to achieve a new integrative variable by integrating random and fuzzy uncertainty factors (Lio, 2008). The integrative variable introduced by Lio (2006) is a commensurable function of chance space for real number set. The variable itself can be presented as fuzzy random variable and random fuzzy variable (Lio, 2009). Fuzzy random variable was presented by Kavakrank (1978). The variable is a commensurable function of probability space for a number of fuzzy sets. Yet, random fuzzy variable was presented by Lio (2002) which is a function of possibility space for a number of random variables (Lee, 2007).

Results from Garkaz *et al.*, (2010) study showed that there is no significant difference in employing two models (Mean-Variance model and Mean-Semi-Variance model). On a contrary, Amiri and Khalouzadeh (2006) have demonstrated that the efficiency of market risk modeling method based on value at risk theory and genetic algorithm optimization method is in gaining the optimum weights of portfolio with respect to the constraints on risk. So far, various risk measures have been presented such as variance (Markowitz, 2000), value at risk VaR, conditional value at risk CVaR (Rakcefeler and Jovarsef, 2000) and the like. During recent years, researchers have directly included the uncertainty from estimation errors in portfolio optimization process. In the method, model inputs are not the same conventional inputs like the expected value of return rates and covariance rather they are uncertainty sets.

MATERIALS AND METHODS

Research Method

This applied-descriptive study was done in terms of survey to compare between Mean-risk Model and the Portfolio Selection Models with fuzzy approach. Sample of the study consisted of companies listed in Tehran Stock Exchange. Sampling procedure was carried out using systematic omission method and based on the following criteria:

1- Company shall be listed in the exchange between 2006 and 2012.
2- Its fiscal year shall be ended in March.
3- The company shall not make any financial changes between 2006 and 2012.
4- The company shall not have operational stoppage between 2006 and 2012.
5- Broker companies shall be omitted due to the special nature of the activity.
6- The company shall not be bad granted during respective years.

After practicing the sampling constraints, 112 companies (out of 486 companies) remained and were studied.

In this study, Mean-risk Model was applied to set optimum portfolio. Before presenting the method, uncertainty function is introduced for normal fuzzy variables and then risk curve and confidence curve are presented.

Uncertainty Function (Risk Curve)

Suppose that ε is a fuzzy random variable from uncertain distribution $N(\mu,\sigma)$. Then, its uncertainty value can be calculated from the equation below which is uniquely reversible, as well:

$$\Phi(r) = R(r) = M\left(r_f - \varepsilon \geq r\right) = \left(1 + exp\left(\frac{\pi(\mu - r_f + r)}{\sqrt{3}\sigma}\right)\right)^{-1} \quad , \quad \forall r \geq 0$$

So that r_f is defined the amount of risk expected by investors and can be desirably determined with respect to the investor's sensitivity and risk taking. In this study, the amount of r_f risk is taken to be 0.05.

Mutually, the reverse of the function is also defined as follow:

$$\Phi^{-1}(z) = \mu - \frac{\sqrt{3}\sigma}{\pi} \ln\left(\frac{1-z}{z}\right)$$

Confidence Curve

Since median risk and fuzzy sets methods are developed based on data uncertainty, the amount of risk estimation will also be with uncertainty. Investors cannot absolutely determine their own amount of critical risk. And, it is better to set an interval for accepting investment risk. The interval which directs the investor with more confidence toward decision making is known as confidence curve. In this curve, the amount of risk is estimated not in terms of point rather in terms of interval. And, the estimated risks create safe and non-safe intervals. The curve is determined using the expected risk of capital and defined as follow:

$$\alpha(r) = \begin{cases} a_1 - b_1 r & 0 \leq r \leq r_1 \\ a_2 - b_2 r & r_1 \leq r \leq r_2 \\ \quad \vdots & \\ a_k - b_k r & r_{k-1} \leq r \end{cases}$$

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So that $a_k - b_k r$ is regarded to be the same r_f and, to ensure consistency of the function, a_i and b_i values must be set in a way that for each r value, we have:

$$a_i - b_i r = a_{i+1} - b_{i+1} r$$

In this study, since the amount of r_f risk is taken to be 0.05, confidence curve is defined as follow:

$$\alpha(r) = \begin{cases} 0.6 - 2.25r & 0 \le r \le 0.2 \\ 0.25 - 0.5r & 0.2 \le r \le 0.4 \\ 0.05 & 0.4 \le r \end{cases}$$

Hence, the issue of optimizing portfolio section for an investor who intends to divide his capital between n companies in ratios of x_1 ،x_2 to x_n will be as follow:

$$\max_{\{x\}} E(\varepsilon_1 x_1 + \varepsilon_2 x_2 + \cdots + \varepsilon_n x_n)$$

$$s.t. \quad R(x_1, x_2, \dots, x_n; r) \le \alpha(r), \sum_{i=1}^{n} x_i = 1, \;\; x_i \ge 0 \quad i = 1،2، \dots ،n$$

Regarding the definition of uncertainty function, the value of median risk for an uncertain variable is defined as follow:

$$\max_{\{x\}} [x_1 E(\varepsilon_1) + x_2 E(\varepsilon_2) + \cdots + x_n E(\varepsilon_n)]$$

$$s.t. \quad x_1 \Phi_1^{-1}(\alpha(r)) + x_2 \Phi_2^{-1}(\alpha(r)) + \cdots + x_n \Phi_n^{-1}(\alpha(r)) \ge r_f - r,$$

$$\sum_{i=1}^{n} x_i = 1, \;\; x_i \ge 0 \quad i = 1،2، \dots ،n$$

Which if the uncertain variable under study (stock rate of return) has normal distribution, then:

$$\max_{\{x\}} [x_1 E(\varepsilon_1) + x_2 E(\varepsilon_2) + \cdots + x_n E(\varepsilon_n)]$$

$$s.t. \quad \sum_{i=1}^{n} (\mu_i - \frac{\sqrt{3}\sigma_i}{\pi} ln \frac{1 - \alpha(r)}{\alpha(r)}) x_i \ge r_f - r, \quad \forall r \ge 0$$

$$\sum_{i=1}^{n} x_i = 1, \;\; x_i \ge 0 \quad i = 1،2، \dots ،n$$

And, if the uncertain variable under study (stock rate of return) has consistent uncertain distribution (a_i , b_i) for each ε_i, then median risk estimation is done as follow:

$$\max_{\{x\}} [x_1 E(\varepsilon_1) + x_2 E(\varepsilon_2) + \cdots + x_n E(\varepsilon_n)]$$

$$s.t. \quad \sum_{i=1}^{n} \alpha(r)(b_i - a_i) x_i + \sum_{i=1}^{n} a_i x_i \ge r_f - r, \quad \forall r \ge 0$$

$$\sum_{i=1}^{n} x_i = 1, \;\; x_i \ge 0 \quad i = 1،2، \dots ،n$$

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Estimating median risk is aimed to estimate x_i weights which belong to each of the companies. Accordingly, optimum portfolio is set based on these weights.

Portfolio Constitution

Portfolios are constituted based on two B/M and E/P ratios during the years under study. To constitute portfolio in year t, respective ratios in year t-1 were calculated and then companies were separately arranged based on two respective ratios. Stocks of the listed companies %30 above B/M and E/P ratios were regarded as value portfolio (portfolios 3, 4, 7, and 8) and stock of companies listed %30 below B/M and E/P ratios as growth portfolio (portfolios 1, 2, 5, and 6). Then, monthly return of value and growth portfolios was calculated for year t. The calculation of monthly portfolios were done based on two approaches including the weight of stock market value existing in portfolio and the equal weight of stock existing in portfolio. Hence, four growth portfolios and four value portfolios were gained. In this method, B/M is the same book value to market value of stockholders and E/P is earnings per share at the stock price.

RESULTS AND DISCUSSION

Results

As shown in Table 1, return rate mean in portfolios 1 to 8 were respectively 0.016, 0.012, 0.035, 0.036, 0.013, 0.018, 0.034, and 0.032. Considering the amplitude of estimated Standard Deviation (SD) for each of the return rates of portfolios, it is seen that maximum SD belongs to portfolio 5 and minimum SD to portfolio 6. This indicates that rate of return in this portfolio is further centralized around its mean. And, its variations cannot be generally different from its mean value. Maximum rate of return is 0.462 for portfolio 5 and minimum rate is 0.183 for portfolio 7.

Table 1: Descriptive indices of portfolios return rate

Portfol 8 io	Portfoli 7 o	Portfoli 6 o	Portfoli 5 o	Portfoli 4 o	Portfoli 3 o	Portfoli 2 o	Portfoli 1 o	Index
0/032	0/034	0/018	0/013	0/036	0/035	0/012	0/016	**Mean**
0/017	0/035	0/011	0/008	0/022	0/019	0/002	0/011	**Median**
0/247	0/183	0/129	0/462	0/281	0/286	0/238	0/345	**Max**
-/060	-0/095	-0/068	-0/145	-0/093	-0/048	-0/094	-0/173	**Min**
0/059	0/056	0/041	0/072	0/066	0/058	0/044	0/063	**SD**
1/137	0/389	0/403	2/293	1/427	1/133	1/626	1/535	**Skewness**
4/470	2/711	2/922	15/27	5/734	5/029	8/813	10/41	**Stretch**

Figure 1 shows risk curve of portfolio 8 with confidence curve together. And, based on the diagram, the risk curves of different portfolios partially overlap.

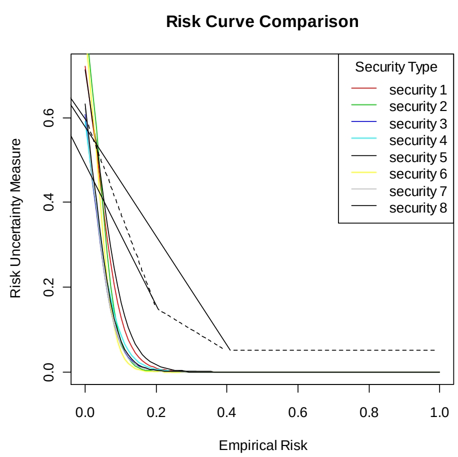

Diagram 1: Risk curve and confidence curve for comparing portfolios risk

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As also implied in earlier pages, to determine optimum portfolio, we have solved the following nonlinear system:

$$\underset{\{x\}}{\mathrm{Max}}[x_1E(\varepsilon_1) + x_2E(\varepsilon_2) + \cdots + x_nE(\varepsilon_n)]$$

$$s.t. \quad \sum_{i=1}^{n}(\mu_i - \frac{\sqrt{3}\sigma_i}{\pi} ln \frac{1-\alpha(r)}{\alpha(r)}) x_i \geq r_f - r, \quad \forall r \geq 0$$

$$\sum_{i=1}^{n} x_i = 1, \ \ x_i \geq 0 \ \ i = 1, 2, \ldots, n$$

In this method, for the sake of default data consistency from portfolios and the past investment behaviors in them, their effect coefficient which confirms the amount of the ratio of proposed investment in that portfolio was taken to be the same and as 0.125. Problem solving is continued till all employed constraints and the amount of statement $x_1E(\varepsilon_1) + x_2E(\varepsilon_2) + \cdots + x_nE(\varepsilon_n)$ reach their maximum amounts. Results of the method based on investment ratio in each of the portfolios are as Table 2.

Table 2: Estimating investment ratios in portfolios using Mean-risk Model

Portfoli 8 o	**Portfoli 7 o**	**Portfoli 6 o**	**Portfoli 5 o**	**Portfoli 4 o**	**Portfoli 3 o**	**Portfoli 2 o**	**Portfoli 1 o**	**Index**
0/125	0/125	0/125	0/125	0/125	0/125	0/125	0/125	**Basic share**
0	0	0	0	0/8351	0	0	0/1649	**Estimated share**
%0	%0	%0	%0	%83/51	%0	%0	%16/49	**Investmen t percentage**

$\boldsymbol{basic}$: $[\boldsymbol{x_1E(\varepsilon_1) + x_2E(\varepsilon_2) + \cdots + x_nE(\varepsilon_n)}]$**= 0/02498615**

$\boldsymbol{max}$: $[\boldsymbol{x_1E(\varepsilon_1) + x_2E(\varepsilon_2) + \cdots + x_nE(\varepsilon_n)}]$**= 0/033023041**

As seen in the table, only two portfolios (1 and 4) were selected in this method. The amount of proposed investment in these portfolios is %16.49 in portfolio 1 and %83.51 in portfolio 4. Regarding mean rate of return in each of respective portfolios, mean rate of return for investing in these portfolios is 0.033.

Selecting Portfolio by Markowitz Risk Management Method

To determine optimum portfolio, we have solved following nonlinear system:

$$\delta_p^2 = \sum_{i=1}^{N}\sum_{i=1}^{N} x_i x_j cov_{ij}$$

$$Min \ \delta_p^2$$

$$S.T: \ \bar{r}_p = \sum_{j=1}^{M} x_j \bar{r}_j$$

$$\sum_{j=1}^{M} x_j = 1$$

In this method, for the sake of default data consistency from portfolios and the past investment behaviors in them, their effect coefficient which confirms the amount of the ratio of proposed investment in that portfolio was taken to be the same and as 0.125. Problem solving and estimating x_i will be continued till all employed constraints and the amount of statement $Z = \delta_p^2$ reach their minimum amounts. Results of the method based on investment ratio in each of the portfolios are as Table 3.

Table 3: Estimating investment ratios in portfolios using Markowitz risk method

Portfolio 8	Portfolio 7	Portfolio 6	Portfolio 5	Portfolio 4	Portfolio 3	Portfolio 2	Portfolio 1	Index
0/125	0/125	0/125	0/125	0/125	0/125	0/125	0/125	**Basic share**
0	0	0	0	0/2929	0/7071	0	0	**Estimated share**
%0	%0	%0	%0	%29/29	%70/71	%0	%0	**Investment percentage**

$\boldsymbol{basic}$: $\boldsymbol{\delta_p^2}$**= 0/00509129**

$\boldsymbol{min}$: $\boldsymbol{\delta_p^2}$**= 0/003678763**

As seen in the table, only two portfolios (3 and 4) were selected in this method. The amount of proposed investment in these portfolios is %70.71 in portfolio 3 and %29.29 in portfolio 4. Regarding mean rate of return in each of respective portfolios, mean rate of return for investing in these portfolios is 0.0359.

Table 4: Estimating ratios of investment in portfolios using value at risk method

Portfoli 8 o	Portfoli 7 o	Portfoli 6 o	Portfoli 5 o	Portfoli 4 o	Portfoli 3 o	Portfoli 2 o	Portfoli 1 o	Index
0/125	0/125	0/125	0/125	0/125	0/125	0/125	0/125	**Basic share**
0/1204	0/1304	0/1316	0/1292	0/1296	0/1303	0/1265	0/1019	**Estimated share**
%12/04	%13/04	%13/16	%12/92	%12/96	%13/03	%12/65	%10/19	**Investmen t percentage**

$\boldsymbol{basic}$: $\boldsymbol{VaR}$**= 0/000835918**

$\boldsymbol{min}$: $\boldsymbol{VaR}$**= 0/0000000600482**

Selecting Portfolio by Value at Risk Method

To determine optimum portfolio, we have solved following nonlinear system:

$$Min\ VaR = M.Z_\alpha.\delta\sqrt{T} - \mu.x_i = M.Z_\alpha.\delta\sqrt{T} - r_p$$

$$S.T:\ \bar{r}_p = \sum_{j=1}^{M} x_j \bar{r}_j$$

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$$\sum_{j=1}^{M} x_j = 1$$

$$x_j \geq 0$$

In this method, for the sake of default data consistency from portfolios and the past investment behaviors in them, their effect coefficient which confirms the amount of the ratio of proposed investment in that portfolio was taken to be the same and as 0.125. Problem solving and estimating x_i will be continued till all employed constraints and the amount of statement $Z = VaR$ reach their minimum amounts. VaR criterion indicates the probability that investment risk goes beyond the standard risk. Results of the method based on investment ratio in each of the portfolios are listed in Table 4. As seen in the table, investing in all portfolios is proposed in this method. Value at risk method has estimated maximum share of investment as related to portfolio 6 with capital %13.16 and minimum to portfolio 1 with capital %10.19. The expected rate of return on this investment is 0.0252 regarding mean rate of return.

Results

According to risk curves estimation, it is concluded that portfolios 3, 4, 7, and 8 are the most secure portfolios in investment with respect to the amount of return rate risk. This is because risk curve in these portfolios were located in safe investment area for any amount of empirical risk. Results from investment ratios in these portfolios showed that Markowitz method has worked better in finding the best portfolios. This is because, in this method, the third and fourth portfolios which have desirable status regarding investment risk uncertainty are proposed. In comparison, in Mean-risk Model and value at risk method, the portfolios are also suggested that investing in them is at high risk. Hence, based on the indices, the hypothesis (the efficiency of Mean-risk to estimate the investment risk) is rejected. And, we claim that Markowitz method has had more accurate performance in identifying portfolios compared to Mean-risk Model.

Besides, estimating the expected return rate of the investment in optimum portfolios also indicated that Markowitz method works better compared to Mean-risk Model. And, the amount of expected rate of return from this method is to some extent larger than the same amount from Mean-risk Model. Yet, value at risk method has worked better compared to both above methods. In general, it can be claimed that the hypothesis (the better efficiency of Mean-risk to estimate the investment return compared to other methods) is also rejected.

This is because Markowitz showed better results regarding portfolio selection in comparison with Mean-risk Model. Here, Mean-risk model has been considered as a merely fuzzy model and results show that the model cannot have significant and considerable power in estimating market risk and selecting optimum portfolio on its own. It must be noted that although the method could not appear as the best method among the models under study, its desirable power in selecting suitable portfolio must not be ignored. The portfolio proposed by this method leads to an expected return approximate the return resulted from Markowitz method indicating the model power for portfolio estimation. However, it must be noted that selecting portfolio in the model is significantly affected by confidence curve. And, in all calculations for estimating investment ratios, the values of confidence curve are used. Since the selection of coefficients in the analytical shape of the function can be done in a variety of ways, it cannot be absolutely declared that this method has worked worse than Markowitz method. This is because the optimum selection of coefficients in this function can directly improve the estimation of the investment ratios.

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