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# SUBSPACE-BASED 2 DIMENSIONAL LOCALIZATION IN A MULTISTATIC PASSIVE RADAR USING 2 RECIEVER ARRAY

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### ABSTRACT

A novel subspace-based 2-D localization using a multistatic passive radar has proposed in this paper because of its importance in military affairs. We know that a passive radar is a radar which no need to emit signal to detect targets, then it has so benefits as compared with active ones and also multistatic passive radar have higher resolution than biostatics. At the end, MATLAB simulation result related to this study will propose in different states and situation to illustrate the goals of this paper.

Keywords: Passive Radar, Transfer Matrix, Localization, Green Function

## **INTRODUCTION**

Now a day, it is paid attention to use passive radars in place of active radars especially in military affairs. Some of advantages of passive radar are as: lower cost, no dedicated transmitter no need for frequency locations, difficulty of jamming virtually immune to anti-radiation missiles, fast updates and potential ability to detect stealth targets.

Thus many efforts are done in array processing and detection and target location in order to increase radar imaging resolution.

The goal of this paper is array processing in passive radar with tow receiver array. By exploiting common RF energy such as commercial FM broadcast as an illuminators of opportunity, scattered by a target, and receiving it in receiver arrays. With proper processing the distance, direction of arrival and the scatterers location could be found. Location in passive radars usually done for far distances (Suberviola *et al.*, 2012).

In this paper every array is consist of N receiver antenna with distance of at least  $\lambda/2$  to minimize the mutual coupling effects between the antenna ( $\lambda$  is the wave length). However the mutual coupling effect is not considered for this case. The two array are considered in (x,z) plane. Green function is employed to modeling the scattered wave between every target and every antenna in fact green function is the propagated wave equation between 2 point. In the other hand to detect *M* target, a *N*×*M* matrix is achieved in every receiver array (Antoniou *et al.*, 2012).

That is the some received data matrix. Then with multiplying the received data matrix of one array in the transpose of the other one, a  $N \times N$  matrix is obtained called the transfer matrix. To determine the position of targets first need to perform Singular Value Decomposition (SVD) on this matrix. SVD decomposes the transfer matrix to a mapping from the targets space to the first array space and also a mapping from the targets space to the 2<sup>nd</sup> array space.

Then with implementation of sub space based direction finding and to location matrix, position of targets could be found. A kind of MUSIC algorithm is use as a subspace based Method that is combined with time reversed methods (Jamil *et al.*, 2012).

On radar imaging the sensitivity of the MUSIC algorithm to the coherence of the wave scattered from targets, causes (the estimated auto correlation  $N \times N$  dimensional matrix) to have a rank less than *N*. i.e. the matrix in place of autocorrelation matrix cause the matrix not to be singular.

In section 2 a model for the received scattered radar signal is defined and the problem formulated. In section 3 the processing technique is discussed and the super resolution MUSIC algorithm combined with maximum likelihood estimation and time reversal method for tow dimensional radar imaging is proposed. The numerical experiments and simulation results are shown in section 4. And section 5 contains the conclusions of this study (Odendaal *et al.*, 1994), (Oristaglio and Blok, 2004).

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Figure 1: Configuration of the passive radar using two receivers located apart

#### Signal Modeling and Transfer Matrix Formulation

As mentioned in this paper two receiver array are applied for 2D location. It is necessary to determine the propagated wave between every target and every receiver antenna (Alam *et al.*, 2012). For this purpose homogeneous Green Function is applied. Green function as the propagated wave equation between 2 point r and r in angle frequency  $\omega$  is defined as follow:

$$G(r, r', \omega) = \sqrt{1/8\pi k} \frac{e^{(-ik|r-r'|)}}{|r-r'|} = \frac{i}{4} H_0^{(1)}(\frac{\omega}{c_0}|r-r'|)$$
(1)

Where k is the wave number of the background (Devaney, 2000) and H is the zeroth order Hankel function of the first kind in the 2D medium (Jamil *et al.*, 2012). The constant Co is the known background speed (Ammari *et al.*, 2011). |r - r'| shows that there is no difference between the propagated wave from r to r and r to r then:

$$G(r, r', \omega) = G(r', r, \omega)$$
<sup>(2)</sup>

All of the calculations are done in narrow band so because of constant  $\mathcal{O}$  this parameter will not be shown in the equations. Assume there is M detectable target and N receiver antenna in every array (Jamil *et al.*, 2012). Every target is located at position  $x_m$ , that m=1,2,...,M and every antenna of the first array are located at point  $\alpha_k$  where k=1,2,...N, and also every antenna at the second array is at position  $\beta_j$  that j=1,2,...,N, then the wave scattered from the target in  $x_m$  to the antenna at  $\alpha_k$  or  $\beta_j$  is as follow:

$$G(\alpha_k, x_m) = \sqrt{1/8\pi k} \frac{e^{\left(-ik\left|\alpha_k - x_m\right|\right)}}{\left|\alpha_k - x_m\right|}$$

$$G(\beta_j, x_m) = \sqrt{1/8\pi k} \frac{e^{\left(-ik\left|\beta_j - x_m\right|\right)}}{\left|\beta_j - x_m\right|}$$
(3)

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Where  $G(\alpha_k, x_m)$  and  $G(\beta_j, x_m)$  are the scattered wave equation from m<sup>th</sup> target location to the k<sup>th</sup> and j<sup>th</sup> antenna position of the first and the 2nd array respectively. Then with putting the calculated wave equations between every target cmd every antenna in a matrix as follow the received signal model in an array could be defined:

$$A_{1} = \begin{bmatrix} G(\alpha_{1}, x_{1}) & G(\alpha_{1}, x_{2}) & \cdots & G(\alpha_{1}, x_{M}) \\ G(\alpha_{2}, x_{1}) & G(\alpha_{2}, x_{2}) & \cdots & G(\alpha_{2}, x_{M}) \\ \vdots & \vdots & \ddots & \vdots \\ G(\alpha_{N}, x_{1}) & G(\alpha_{N}, x_{2}) & \cdots & G(\alpha_{N}, x_{M}) \end{bmatrix}_{N \times M}$$

$$A_{2} = \begin{bmatrix} G(\beta_{1}, x_{1}) & G(\beta_{1}, x_{2}) & \cdots & G(\beta_{1}, x_{M}) \\ G(\beta_{2}, x_{1}) & G(\beta_{2}, x_{2}) & \cdots & G(\beta_{2}, x_{M}) \\ \vdots & \vdots & \ddots & \vdots \\ G(\beta_{N}, x_{1}) & G(\beta_{N}, x_{2}) & \cdots & G(\beta_{N}, x_{M}) \end{bmatrix}_{N \times M}$$
(4)

Where  $A_1$  and  $A_2$  are the received signal modeled with the Green function in the first and second receiver array respectively. After modeling the received signal,  $N \times N$  dimensional transfer matrix is formed as follow:

$$K = A_2 \times \sigma \times A_1^t \tag{5}$$

Where denote t is the transpose and  $\sigma$  is a  $M \times M$  dimensional diagonal matrix that its diagonal elements are the scattering potential of the targets:

$$\sigma = diag[\sigma_1, \sigma_2, ..., \sigma_M]$$

Where  $\sigma_m$  as the  $m^{,th}$  target scattering potential is obtained from this equation:

$$\sigma_m = \rho_j \left(\sum_{k=1}^N \left| G(\alpha_k, x_m) \right|^2 \right)^{\frac{1}{2}} \left(\sum_{j=1}^N \left| G(\beta_j, x_m) \right|^2 \right)^{\frac{1}{2}}$$
(6)

Where  $\rho_i$  is the reflection coefficient defined by:





Figure 2: Receivers arrays and the target location

Where  $\eta_j$  is the dielectric contrast (or the reflector strength at coordinate  $x_m$ ) and  $l_j^2$  is the physical volume of the m'th scatterer (Ammari *et al.*, 2011).

Then transfer matrix elements could be denoting as follow:

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$$K_{k,j} = \sum_{m=1}^{M} G(\alpha_k, x_m) \sigma_m G(\beta_j, x_m)$$
(8)

#### Signal Processing and Imaging using Multi Spectrum Data

The goal is to estimate the location of the targets from the scattered field data. One method is described in this section that is MUSIC to be used in conjunction with time-reversal processing. Thus the response matrix k still will be required the MUSIC algorithm is based on the signaler value decomposition (SVD) of the transfer matrix which allows an arbitrary complex phase. Let  $K = U \sum V^H$  be the singular value decomposition of the transfer matrix k (Suberviola *et al.*, 2012)



Figure 3: Eigen value chart results (SVD) of the transfer matrix K

Where  $\sum_{s} = diag(\gamma_{1}, \gamma_{2}, ..., \gamma_{M})$  and  $\sum_{n} = diag(\gamma_{M+1}, \gamma_{M+2}, ..., \gamma_{N})$  are the signal and the noise subspace eigenvalues and  $U_{s}$  and  $U_{n}$  are the signal and the noise subspace respectively. So  $U = \begin{bmatrix} U_{s} & U_{n} \end{bmatrix}$ and  $V = \begin{bmatrix} U_{s}^{H} & U_{n}^{H} \end{bmatrix}^{t}$  (Devaney, 2000). Where  $U_{N \times N} = \begin{bmatrix} u_{1} \dots u_{M} & u_{M+1} \dots & u_{N} \end{bmatrix}$  and  $V_{N \times N} = \begin{bmatrix} v_{1} \dots v_{M} & v_{M+1} \dots & v_{N} \end{bmatrix}$  are orthogonal matrices.  $u_{n}$  and  $v_{n}$  for  $n = \begin{bmatrix} 1, 2, \dots, N \end{bmatrix}$  the n<sup>°th</sup> column of U and V are orthogonal unitary eigenvectors of matrix k containing the signal and the noise subspaces between the targets and the first array and also the second array respectively. In other word the MUSIC algorithm is based on this fact that the transfer matrix K is a projection operator on to the subspace of  $C^{N}$ spanned by the complex conjugate of the Green function vector (the signal subspace) and that the noise subspace N is spanned by the eigenvectors of K that have zero eigenvalue. The complex conjugate of each Green function vector is orthogonal to the noise subspace and especially to the eigenvectors of the transfer matrix with zero eigenvalues.

$$\langle u_{m_0}, g_m^* \rangle = \langle u_{m_0}^*, g_m \rangle = 0 \tag{10}$$

For m=1,2,...,M and  $m_0=M+1,...,N$  Where  $\langle . \rangle$  is the inner product and  $u_{m_0}$  are the eigenvectors of *K* with zero eigenvalue. Then a pseudo-spectrum is form according to the MUSIC algorithm:

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$$I(x_{p}) = \frac{1}{\sum_{m_{0}}^{N} \left( \left| \langle u_{m_{0}}^{*} \cdot g_{p_{1}} \rangle \right|^{2} + \left| \langle v_{m_{0}}^{*} \cdot g_{p_{2}} \rangle \right|^{2} \right)}$$
(11)

Where  $u_{m_0}$  is the  $m_0^{,th}$  column of the matrix U and  $v_{m_0}$  is the  $m_0^{'th}$  column of the matrix V and also:

$$g_{p_1} = \left[ G\left(\alpha_1, X_p\right), G\left(\alpha_2, X_p\right), \dots, G\left(\alpha_N, X_p\right) \right]^t$$

$$g_{p_2} = \left[ G\left(\beta_1, X_p\right), G\left(\beta_2, X_p\right), \dots, G\left(\beta_N, X_p\right) \right]^t$$
(12)

Are the steering vectors. The steering vector is the Green function vector for a target located at a test location like  $X_p$  Because the signal subspace is orthogonal to the noise subspace the scalar product  $\langle u_{m_0}^*, g_{p_1} \rangle$  and  $\langle v_{m_0}^*, g_{p_2} \rangle$  that  $m_0$ =M+1,...,N will vanish whenever the test location  $X_p$  is equal to the real location of one of the targets  $X_m$ . Then the pseudo-spectrum I will peak at each target location

the real location of one of the targets  $X_m$ . Then the pseudo-spectrum *I* will peak at each target location When  $X_p = X_m$ . Equation (10) is the MUSIC algorithm combined with time reversal method (Aubry and Derode, 2010).

#### Numerical Experiments

As seen in figure 4 two receiver array containing 8 antenna for each one is used in this paper to increase passive localization for proposed method. The distance between each 2 antenna work at frequency 1 GHZ assumed  $2\lambda$  ( $\lambda$ =0.3m is the wavelength). The first and the 2<sup>nd</sup> array antennas are located in (*x*,*z*) plane at coordinate (50,0) to (64,0) and (-50,0) to (-64,0) respectively. Each unit in this coordinate assumed to be equal to 1 wave length. Also point target is located at position (40,19810). Numerical experiment results of computer MATLAB simulation from detected targets and 2D MUSIC pseudo-spectrum are demonstrated in figures that show the high resolution of this method in 2-D target localization.





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In another experiment the number of receivers increases in each array to 12 sensors. Locations of arrays are (50,0) to (72,0), (-50,0) to (-72,0). Figure 5 shows 2-D localization using TRMUSIC. Comparing tomography images of two simulations shows that the resolution will be better with increasing in number of sensors.



Figure 5: Tomography image results from 2D music with (a) 12 sensors (b) 8 sensors

Also figure 6 shows an experiment for tow targets located at (73,-10) and (73,10). Clearly this simulation assumed to be in near field state. We can see the sensor number effect on the 2-D localization resolution.



Figure 6: Figure 6: tomography image results from 2D music with 2 target located at (73,-10) and (73,10) in the state of (a) 12 sensors (b) 8 sensors

## Conclusion

In this paper we can see that using 2 receiver arrays and multiplying the received signal matrix of one array in transpose of the other one's as transfer matrix, results a high resolution 2D localization in multistatic passive radars. MUSIC algorithm in conjunction with time reversal processing is applied to detect the targets position. Also localization resolution will go better with increasing number of sensors. The computer simulation performed for this work shows the desired results of proposed method in obscured targets localization for multistatic passive radars.

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