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Research Article

A METHOD FOR DEFUZZIFICATION BASED 0N PROBABILITY DENSITY FUNCTION (II)

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ABSTRACT
The concept of (fuzzy) probability density function of fuzzy random variable is proposed in this paper. Due to the "resolution identity", we can construct a closed fuzzy number from a family of closed intervals. Using the same technique, we can construct the (fuzzy) probability density function of fuzzy random variable from the known probability density function. The basic idea of the new method is to obtain a method to rank fuzzy number which a fuzzy quantity is related to. The paper must have abstract.

Keywords: *Ranking, Defuzzification, PFD, Fuzzy Number*

INTRODUCTION
The concept of fuzzy sets as introduced is widely used in many different fields today, ranging from control applications, robotics, image and speech processing, biological and medical sciences to applied operation research and expert systems.
Most of these applications can be regarded as systems with numerical input (e.g. sensor data) and numerical output (e.g. voltages). Internally these systems work with fuzzy values, which have to be mapped to non-fuzzy (crisp) values after processing. This conversion is called defuzzification. Various defuzzification methods have been proposed in (Broekhoven *et al.,* 2006; Filev *et al.,* 1991; Kandel *et al.,* 1998; Kosko, 1992; Leekwijck *et al.,* 1999; Roychowdhury, 1996; Roychowdhury *et al.,* 2001). The most popular methods are the center of gravity method and the mean of maxima method, which are computationally inexpensive and easy to implement within fuzzy hardware chips although a full scientific reasoning has not been established. Many researchers attempted to understand the logic of the defuzzification process. Although so many defuzzification methods have been proposed so far, no one method gives a right effective defuzzified output. The computational results of these methods often conflict, and they don't have a uniform framework in theoretical view. We often face difficulty in selecting appropriate defuzzification methods for some specific application problems. Most of the existing defuzzification methods tried to make the estimation of a fuzzy set in an objective way. However, an important aspect of the fuzzy set application is that it can represent the subjective knowledge of the decision maker; different decision makers may have different perception for the defuzzification results. This article proposes here a method to use the concept probability density function of a fuzzy number, so as to find the order of fuzzy numbers. This method can distinguish the alternatives clearly. The main purpose of this article is that, this defuzzification of a fuzzy number can be used as a crisp approximation of a fuzzy number. Therefore, by the means of this difuzzification, this article aims to present a new method for ranking of fuzzy numbers. In addition to its ranking features, this method removes the ambiguous results and overcome the shortcomings from the comparison of previous ranking. Text of section 1.

Preliminary Notes
The basic definition of a fuzzy number given in (Filev *et al.,* 1993; Genther *et al.,* 1994; Heilpem, 1992; Kauffman *et al.,* 1991) as follows:

Definition 2.1 Let U be an universe set. A fuzzy set A of U is defined by a membership function μ_A (x) → [0, 1], where; μ_A(x) indicates the degree of x in A.

Definition 2.2. A fuzzy subset A of universe set U is normal iff $sup_{x\in U}\mu_A(x) = 1.$

Definition 2.3. A fuzzy set A is a fuzzy number iff A is normal and convex on U. The set of all fuzzy numbers is denoted by F.

Definition 2.4. The membership function μ_A of extended fuzzy number A is expressed by

$$\mu_A = \begin{cases} \mu_A^L(x), & when \quad a_1 \leq x \leq a_2, \\ \omega, & when \quad a_2 \leq x \leq a_3, \\ \mu_A^R(x), & when \quad a_3 \leq x \leq a_4, \\ 0, & otherwise \end{cases} \tag{1}$$

Where $\mu_A^L(x)$: $[a_1\ ,a_2] \rightarrow [0, \omega]$ and μ_A^R: $[a_3\ ,a_4] \rightarrow [0, \omega]$.

Based on the basic theories of fuzzy numbers, A is a normal fuzzy number if ω = 1, whereas A is a non-normal fuzzy number if 0 < $\omega \leq$ 1. Therefore, the extended fuzzy number A in Definition (2.4) can be denoted as (a1, a2, a3, a4, ω). The image –A of A can be expressed by (−a1, −a2, −a3, −a4, ω) (Kauffman *et al.*, 1991).

With Zadeh's extension principle, the arithmetic operation of fuzzy sets especially the fuzzy numbers can be defined.

Here, this article recalls the two simplest cases of scalar addition and scalar multiplication. For the fuzzy set with membership function μ_A(x) , the membership function of scalar addition A + c and scalar multiplication kA(k ≠ O) are [μ_{A+c}(x) = μ_A(x- c) and μ_{kA} = $\mu_A\left(\frac{x}{k}\right)$, respectively.

Defuzzitication with PDF from Membership Function

Let $A = (a_1, a_2, a_3, a_4, 1)$ is an arbitrary fuzzy number. The function f_1 defined by $f_1(x) = c_1 . \mu_A(x)$, where $C_1 = \frac{2}{a_4 + a_3 - a_1 - a_2}$ is a probability density function associated with A.

Remark 3.1. Note that we obtained c_1 by the property that $\int_{-\infty}^{\infty} f_1(x)dx = 1.$

Definition 3.2. The Mellin transform $M_x(s)$ of a probability density function $f\ (x)$, where x is positive, is defined as:

$$M_x(s) = \int_0^{+\infty} x^{s-1} f(x)dx.$$

Whenever the integral exist. Now it is possible to think of the Mellin transform in terms of expected values.

Recall that the expected value of any function g(x) of the random variable X, whose distribution is $f(x)$, is given by $\mathrm{E}[\mathrm{g(x)}] = \int_{-\infty}^{+\infty} g(x)f(x)dx.$ Therefore, it follows that $M_x(s) = E[X^{s-1}] = \int_0^{+\infty} x^{s-1}f(x)dx.$ Hence $[X^s] = M_x(s + 1)$. Thus, the expectation of random variable X is $E[X] = M_x(2)$.

Remark 3.3. Let $A = (a_1, a_2, a_3, a_4, 1)$ is an arbitrary triangular fuzzy number, the density function $f\ (x)$ corresponding to A is s follows:

$$f_{xA}(x) = \begin{cases} \frac{2(x-a_1)}{(a_4+a_3-a_1-a_2)(a_2-a_1),} & a_1 \leq x < a_{2,} \\ \frac{2}{(a_4+a_3-a_1-a_2)} & a_2 \leq x < a_{3,} \\ \frac{2(a_4-x)}{(a_4+a_3-a_1-a_2)(a_4-a_3),} & a_3 \leq x < a_{4,} \\ 0, & otherwise \end{cases} \quad (2)$$

The Mellin transform is then obtained by:

$$M_{XA}(s) = \int_0^{+\infty} x^{s-1} f_{XA}(x)dx.$$

$$= \frac{2}{(a_4+a_3-a_1-a_2)(s^2+s)}\left[\frac{(a_4^{s+1}-a_3^{s+1})}{a_4-a_3} - \frac{(a_2^{s+1}-a_1^{s+1})}{a_2-a_1}\right], \quad (3)$$

And

$$E[X_A] = M_{XA}(2) = \frac{1}{3}\left[(a_1+a_2+a_3+a_4) + \frac{a_1a_2-a_3a_4}{a_4+a_3-a_1-a_2}\right], \quad (4)$$

In the following, we present a new approach for ranking fuzzy numbers based on the distance method. The method not only considers the PDF of a fuzzy number, but also considers the minimum crisp value of fuzzy numbers. For ranking fuzzy numbers, this study firstly defines a minimum crisp value τ_{min} to be the benchmark and its characteristic function $\mu_{\tau min}(x)$ is as follows:

$$\mu_{\tau min}(x) = \begin{cases} 1, & x = \tau_{min} \\ 0, & x \neq \tau_{min} \end{cases} \quad (5)$$

When ranking n fuzzy numbers $A_1, A_2, \dots, A_n$ the minimum crisp value τ_{min} is defined as:

$$\tau_{min} = min\{x \,|\, x \in Domain(A_1, A_2, \dots, A_n)\}. \quad (6)$$

Assume that there are n fuzzy numbers $A_1, A_2, \dots, A_n$ the proposed method for ranking fuzzy numbers $A_1, A_2, \dots, A_n$ is now presented as follows:

Use the point $(E\,[X_{Aj}]\,,0)$ to calculate the ranking value $M_x(A_j) = dist(E[X_{Aj}], \tau_{min})$ of the fuzzy numbers Aj, where $1 \leq j \leq n$, as follows:

$$dist(E[X_{Aj}], \tau_{min}) = \| E[X_{Aj}] - \tau_{min} \| \quad (7)$$

From formula (7), we can see that $M_x(A_j) = dist(E[X_{Aj}], \tau_{min})$, can be considered as the Euclidean distance between the point $(E[X_{Aj}], 0)$ and the point $\tau_{min}, 0$). We can see that the larger the value of $M_x(A_j)$, the better the ranking of Aj, where $1 \leq j \leq n$.

Ranking Fuzzy Numbers by the MX (.)

ln this section, the researchers will propose the ranking of fuzzy numbers associated with the PDF approximation. Ever, the probability function can be used as a crisp approximation of a fuzzy number; therefore the resulting approximation is used to rank the fuzzy numbers. Thus, MX (.) is used to rank fuzzy numbers.

Definition 4.1. Let A and $B \in F$ be two fuzzy numbers, and $M_x(A)$ and $M_x(B)$ be the PDF approximation of their. Define the ranking of A and B by MX(.) on F, i.e.

1. MX (A) < MX (B) if only if A $\prec$ B,

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2. MX (A) > MX (B) if only if A $\succ$ B,
3. MX (A) = MX (B) if only if A ~ B.

Then, this article formulates the order $\succcurlyeq$ and $\succcurlyeq$ as A $\succcurlyeq$ B ifand only if A $\succ$ B or A~B, A$\preccurlyeq$ B ifandonlyifA$\prec$ B or A~B.

Remark 4.2. If A $\prec$ B, then –A $\succ$ –B.
Hence, this article can infer ranking order of the images of the fuzzy numbers.

Using the Proposed Ranking Method in Selecting Army Equip System

From experimental results, the proposed method with some advantages: (a) without normalizing process, (b) tit all kind of ranking fuzzy number, (c) correct Kerre's concept. Therefore we can apply PDF value of fuzzy ranking method in practical examples. In the following, the algorithm of selecting equip systems is proposed, and then adopted to ranking an army example.

4.1.1 An Algorithm for Selecting Equip System

We summarize the algorithm for evaluating equips system as below:

Step 1: Construct a hierarchical structure model for equips system

Step 2: Build a fuzzy performance matrix$\tilde{A}$. We compute the performance score of the sub factor, which is represented by triangular fuzzy numbers based on expert's ratings, average all the scores corresponding to its criteria. Then, build a fuzzy performance matrix$\tilde{A}$.

Step 3: Build a fuzzy weighting matrix $\widetilde{W}$. According to the attributes of the equip systems, experts give the weight for each criterion by fuzzy numbers, and then form a fuzzy weighting matrix$\widetilde{W}$.

Step 4: Aggregate evaluation. To multiple fuzzy performance matrix and fuzzy weighting matrix $\widetilde{W}$, then get fuzzy aggregative evaluation matrix $\tilde{R}$. (i.e. $\tilde{R} = \tilde{A} \otimes \widetilde{W}^t$).

Step 5: Determinate the best alternative. After step 4, we can get the fuzzy aggregative performance for each alternative, and then rank fuzzy numbers by PDF value of fuzzy numbers.

4.1.2 The Selecting of Best Main Battle Tank

In (Cheng *et al.*, 2002), the authors have constructed a practical example for evaluating the best main battle tank, and they selected $x_1 = M_1A_1\ (USA), x_2$ = Challenger 2 (UK), x33 = Leopard2 (Germany) as alternatives.

In (Cheng *et al.*, 2002), the expert's opinion was described by linguistic terms, which can be repressed in triangular fuzzy numbers .The fuzzy Delphi method is adopted to adjust the fuzzy rating of each expert to achieve the consensus condition. The evaluating criteria of main battle tank are a1: attack capability, a2: mobility capability, a3: self-defense capability and, a4: communication and control capability.

Table 1: Linguistic values for the ratings.

Linguistic value	**TFNs**
Very Poor(VP)	(0,0,0.16)
Poor	(0,0.16,0.33)
Slightly (SP)	(0.16,0.33,0.5)
Fair (F)	(0.33,0.5,0.66)
Slightly good (SG)	(0.5,0.66.0.83)
Good (G)	(066,0.83,1)
Very good (VG)	(0.83,1,1)

Table 2: Basic performance data for five types of main battle Tanks.

Item	**Type**				
	Tank A	**Tank B**	**Tank C**	**Tank D**	**Tank F**
Armament	120 mm gun	120 mm gun	120 mm gun	105 mm gun	120 mm gun
	15.2 mm MG	15.2 mm MG	15.2 mm MG	15.2 mm MG	7.62 mm MG
	12.7 mm MG				12.7 mm MG
Ammunition	40 1000 11400	Up to 50 4000	42 4750	40 4	44 1500 10000
Smoke grenade discharges	2 × 6	2 × 5	2 × 8	None	2 × 9
Power to weight ratio (hp/t)	26.2	19.2	27.2	19.0	27.5
Max. road Speed (km/h)	67	56	72	60	71
Max. range(km)	480	450	550	300	550
Fording(m)	1.21	1.07	1.0	1.2	1.23
Gradient	60	60	60	55	60
Trench	2.74	2.43	3.00	2.51	2.92
Armor protection	Good	Excellent	Good	Fair	Excellent
Acclimatization	Good	Fair	Good	Fair	Good
Communication	Fair	Fair	Fair	Poor	Fair
Scout	Medium	Medium	Medium	Medium	Good

Table 3: Linguistic values importance weights

Linguistic value	**TFNs**
Very Low (VL)	(0.00,0.00,0.167)
Low	(0.0,0.167,0.333)
Slightly	(0.167,0.333,0.5)
Medium (M)	(0.333,0.5,0.667)
Slightly High (SH)	(0.5,0.667,0.833)
High (H)	(O.667,0.833,1.0)
Very High (VH)	(0.833,1.00,1.00)

Table 4: The importance weights of linguistic criteria and its mean

Criteria	**Experts**			**Mean of TFNs**
	D1	**D2**	**D3**	
Attack ($\widetilde{W}_1$)	VH	H	H	(0.72,0.89,1)
Mobility ($\widetilde{W}_2$)	VH	H	VH	(078,0.94,1)
Self – defense ($\widetilde{W}_3$)	M	VH	SH	(0.56,0.72,0.83)
Communication – command ($\widetilde{W}_4$)	M	M	M	(0.33,0.5,0.67)

Table 5: Basic performance data for live types of main battle Tanks.

Criteria	**Type** **Tank A**	**Tank B**	**Tank C**	**Tank D**	**Tank E**
Attack					
Armament	G	SG	SG	F	SG
Ammunition	VG	SG	SG	F	G
Smoke grenade	G	SP	VG	VP	VG
Mean	(.7,.8,1)	(.3,.5,.7)	(.6,.7,.8)	(.2,.3,.5)	(.6,.8,.9)
Mobility					
Power to weight	G	F	G	F	G
Max. road speed	G	F	VG	SG	VG
Max. range	G	SG	Vg	P	VG
Fording/Gradient	G	SG	SG	F	G
Mean	(.6,.8,1)	(.4,.5,.7)	(.7,.8,.9)	(.2,.4,.6)	(.7,.9,1)
Self-defense					
Armor protection	SG	G	F	F	G
Acclimatization	SG	F	SG	F	G
Mean	(.5,.6,.8)	(.5,.6,.8)	(.4,.5,.7)	(.3,.5,.6)	(.5,.7,.9)
Communication					
Communication	G	G	G	F	G
Scout	SG	SG	SG	SG	G
Mean	(.5,.7,.9)	(.5,.7,.9)	(.5,.7,.9)	(.4,.5,.7)	(.6,.8,1)

In this example, we adopted the hierarchical structure constructed in (Cheng *et al.*, 2002) for selection of five main battle tanks, and the step-by-step illustrations based on Sec. 4.1.ls algorithm are described below :

Step 1: Construct a hierarchical structure model for equips system.

Step 2: Build a fuzzy performance matrix $\tilde{A}$. The basic performance data for five types of main battle tanks are summarized in Table 2. Then based on the linguistic values in Table 1, the fuzzy preference of five tanks toward four criteria are collected and shown in Table 5.

Step 3: Build a fuzzy weighting matrix $\widetilde{W}$. The aggregative fuzzy weights of four criteria, according to the linguistic values of importance in Table 3, are shown in Table 4.

Step 4: Aggregate evaluation. To multiple fuzzy performance matrix $\tilde{A}$ and fuzzy weighting matrix $\widetilde{W}$, then get fuzzy aggregative evaluation matrix $\tilde{R} = \tilde{A} \otimes \widetilde{W}^t$. therefore, from Table 4 and 5, we have

$$\tilde{R} = \begin{bmatrix} (0.7,0.9,1.0) & (0.7,0.8,1.0) & (0.5,0.7,0.8) & (0.6,0.8,0.9) \\ (0.4,0.6,07) & (0.4,0.6,0.8) & (0.5,0.7,0.8) & (0.6,0.8,0.9) \\ (0.6,0.8,0.9) & (0.7,0.9,0.96) & (0.4,0.6,0.8) & (0.6,0.8,0.9) \\ (0.2,0.3,0.5) & (0.3,0.5,0.6) & (0.3,0.5,0.7) & (0.4,0.6,0.8) \\ (0.7,0.8,0.9) & (0.8,0.9,1.0) & (0.6,0.8,0.9) & (0.7,0.8,1.0) \end{bmatrix} \otimes \begin{bmatrix} (0.7,0.9,1.0) \\ (0.8,0.9,1.0) \\ (0.6,0.7,0.8) \\ (0.3,0.5,0.7) \end{bmatrix}$$

Step 5: Determinate the best alternative. According to Eq. 7, we can get the PDF value of fuzzy numbers of Tanks A-E, which are equal to 0.234, 0.423, 0.236, 0.323 and 0.289, respectively. Therefore, we find that the ordering of PDF value is Tank A < Tank C < Tank F < Tank D < Tank B. So, the best type of main battle Tank is Tank F.

Conclusion

The modern approach to the evaluation of measurement data in metrology is based on the mathematical formulation of the simple idea that any kind of information that is relevant for inference the measurand generates a corresponding state of knowledge about the measurand. This paper briefly discusses the basic concept of probability density function (PDF), which is the mathematical description of the state of knowledge about the measurand corresponding to give information.

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