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Research Article

RELATIONSHIP BETWEEN THE SATURATED HYDRAULIC CONDUCTIVITY AND THE PARTICLE SIZE DISTRIBUTION

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ABSTRACT
Hydraulic conductivity is one important soil parameter in hydrological studdies. The hydraulic conductivity is measured on soil samples in the lab and sometimes tests carried out in the field. In this study, 50 set of soil samples with sand texture selected. The results show approximately success in predicting hydraulic conductivity from particle diameters data. The following relationship obtained from regression on data *(R^2 = 0.40)*: $K_s = 8.4477 + 52.911(d_{10})$. Where d_{10} is the soil particle diameter (mm) that 10% of all soil particles are finer (smaller) by weight and K_s, saturated hydraulic conductivity is expressed in m/day. Comparison between linear and quadratic single parameter equations showed K_s, saturated hydraulic conductivity predicted from linear equation based on d_{10}, estimated better than quadratic single parameter equation based on d_{10} but saturated hydraulic conductivity predicted from quadratic equations based on d_{50} and d_{60}, estimated better than linear single parameter equations based on d_{50} and d_{60}. Also the results showed that among seven empirical formulae (Hazen, Kozeny-Carman, Breyer, Slitcher, Terzaghi, USBR and Alyamani&Sen), the Slitcher formula predicted Ks, saturated hydraulic conductivity, better than other formulae with 0.629 R; 6.78 RMSE; 5.73 MAE; 26.71% RE & 1.46 DT and the Breyer formula estimated Ks, with largest prediction error with 0.634 R; 58.11 RMSE; 48.80 MAE; 227.54% RE & 3.21 DT.

Keywords: *Empirical Equation, Hydraulic Conductivity, Particle Diameter, Particle-size Distribution and Soil*

INTRODUCTION
Saturated hydraulic conductivity is one of the most important of soil physical characteristics that are very important in ground water studies. Various studies indicate that the hydraulic conductivity is dependent on the particle size distribution.
Curve of particle size distribution (PSD) is one of the basic physical properties of soils that usually expressed the cumulative probability distribution of the diameter of the soil particles (Hwang *et al.*, 2002). Since, direct measurement of hydraulic conductivity is time consuming and costly, indirect methods such as predicting from readily available soil properties e.g. particle-size distribution have been developed (Alyamani and Sen, 1993; Chakraborty *et al.*, 2006; Mualem, 1976; Shao and Robert, 1998; Van and Leji, 1989). In recent years, many studies have been to estimate soil hydraulic functions based on soil particle size distribution (Hwang and Powers, 2003b; Hwang and Hong, 2006).
Although in hydromechanics, it would be more useful to characterize the diameters of pores rather than those of the grains, the pore size distribution is very difficult to determine, so that approximation of hydraulic properties are mostly based on the easy-to-measure grain size distribution as a substitute. There have been attempts to estimate saturated hydraulic conductivity based on particle-size distribution *(PSD)* (Boadu, 2000; Mualem, 1976; Tyler and Wheatcraft, 1989; Van *et al.*, 1992; Van, 1980; Van Genuchten and Leji, 1989).
Many different relationships have been developed from readily available soil properties e.g. particle-size distribution. Research findings showed that USBR and Slitcher equation calculated the amounts of hydraulic conductivity less than the other equations (Cheng and Chen, 2007). The research results showed

that the Breyer equation is very suitable for soils with low uniformity coefficient (Pinder and Celia, 2006).
Rawls and Brakensiek (1989) used field data across the U.S. to develop a regression equation that relates porosity, and the percentages of sand and clay-sized particles in the sample to K_s (Rawls and Brakensiek, (1989). Ahuja *et al.,* (1989) estimated Ks using the generalized form of the Kozeny-Carmen equation (Ahuja *et al.*, 1989).
Cronican and Gribb (2004) developed multiple linear regressions for southeastern U.S. sandy soils based on regional soil data (Cronican and Gribb, 2004).
Odong (2007) have been evaluated several empirical equations to calculate hydraulic conductivity using grain size distribution of unconsolidated aquifer materials. Odong (2007) noted that all the seven empirical formulae reliably estimated hydraulic conductivities of the various soil samples well within the known ranges. Kozeny-Carman formula proved to be the best estimator of most samples analyzed, and may be, even for a wide range of other soil types (Odong, 2007).
Han *et al.,* (2008) developed a new model to estimate saturated hydraulic conductivity from soil structural properties derived from water retention curve (Han *et al.*, 2008).
Chakraborty *et al.,* (2006) reported that considerable success in predicting hydraulic conductivity from PSD data of soils (Chakraborty *et al.*, 2006).
Hazen (1982) proposed the relationship between saturated hydraulic conductivity and soil particle diameter, $K_S = c(d_{10})^2$, Where Ks is expressed in cm/sec, c is a constant that varies from 1.0 to 1.5, and d_{10} size gives the diameter for which 10% (by mass) of the particles in a soil sample are finer (Cronican and Gribb, 2004; Hazen, 1892). Also, Hazen equation is presented in the following form.

$$K = \frac{g}{v} \times 6 \times 10^{-4} [1 + 10(n - 0.26)] d_{10}^2 \tag{1}$$

Kozeny-Carman formula is one of the most applicable equations used for determine of hydraulic conductivity as a function of the soil properties

$$K = \frac{g}{v} \times 8.3 \times 10^{-3} \left[\frac{n^3}{(1-n)^2} \right] d_{10}^2 \tag{2}$$

Breyer presented the following formula (Odong, 2007):

$$K = \frac{g}{v} \times 6 \times 10^{-4} \log \frac{500}{U} d_{10}^2 \tag{3}$$

Where U is the grain uniformity coefficient.
Slitcher presented the following formula (Odong, 2007):

$$K = \frac{g}{v} \times 1 \times 10^{-2} n^{3.287} d_{10}^2 \tag{4}$$

Terzaghi presented the following formula:

$$K = \frac{g}{v} \cdot C_t \cdot \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 d_{10}^2 \tag{5}$$

Where the C_t = sorting coefficient and $6.1 \times 10^{-3} < C_t < 10.7 \times 10^{-3}$. In this study, an average value of C_t is used (Terzaghi and Peck, 1964).
The following formula presented by U.S. Bureau of Reclamation (USBR) (Odong, 2007):

$$K = \frac{g}{v} \times 4.8 \times 10^{-4} d_{20}^{0.3} \times d_{20}^2 \tag{6}$$

Alyamani and Sen (1993) proposed the relationship between saturated hydraulic conductivity and soil particle diameters (Alyamani and Sen, 1993):

$$K = 1300\left[I_o + 0.025(d_{50} - d_{10})\right]^2 \tag{7}$$

Where K_s is expressed in cm/sec, I_o is the x-intercept of the straight line formed by joining d_{50} and d_{10} of the grain-size distribution curve (mm). d_{50} is the mean grain-size for which 50% of the particles are finer by weight (mm).
The aim of the study was to determine relationship between saturated hydraulic conductivity and particle-size distribution. Also several empirical equations have been evaluated to calculate hydraulic conductivity using grain size distribution.

MATERIALS AND METHODS

In this study the 50 sets of soil samples were collected to estimate hydraulic conductivity based on particle-size distribution (PSD). Standard methods were applied to investigate particle size distribution (grain size curve), and finally determine of parameters of d_{10}, d_{50} and d_{60}, Where d_{10}, d_{50} and d_{60}, are the soil particle diameter (mm) where d_{10}, d_{50} and d_{60} size gives the diameter for which 10%, 50% and 60% (by mass) of the particles in a soil sample are finer.
Soil texture was classified according to the International Society of Soil Science (ISSS) classification system. The soil texture was sand. The values of parameters of d_{10}, d_{50}, d_{60} and saturated hydraulic conductivity are summarized in Table 1.

Table 1: Values of statistics of d_{10}, d_{50}, d_{60} and saturated hydraulic conductivity

Statistics	Parameters			
	d_{10}[a]	d_{50}	d_{60}	K_s
Mean	**0.246**	**0.641**	**0.840**	**21.45**
Minimum	**0.13**	**0.35**	**0.51**	**12.5**
Maximum	**0.38**	**1.05**	**1.31**	**33.6**

a. d_{10}, d_{50} and d_{60}, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and Ks, saturated hydraulic conductivity is expressed in m/day.

The mean values of parameters of d_{10}, d_{50} and d_{60} were 0.246, 0.641and 0.840 [mm], respectively, also the mean values of saturated hydraulic conductivity was 21.45 (m/day). In this study saturated hydraulic conductivity was measured by the falling head method. The samples were first wetted by capillarity for 24 hours. The water is then allowed to flow through the soil with maintaining a constant pressure head and saturated hydraulic conductivity was measured when outflow rate becomes constant.
The results were analyzed with SPSS 21.0 and EXCEL software with statistics such as, Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Relative Error (RE) and Deviation Time (DT) that calculated using equation (8), (9) and (10) respectively, where n represents the number of instances presented to the model and O_i and P_i represents measured and predicted, and O_{ave} and P_{ave} represents mean values of measured and predicted respectively.

$$RMSE = \left[\frac{\sum_{i=1}^{n}(P_i - O_i)^2}{n}\right]^{0.5} \tag{8}$$

$$MAE = \sum_{i=1}^{n}\left[\left|P_i - O_i\right|/n\right] \tag{9}$$

$$RE = (MAE / O_{ave}) \times 100$$

$$Log_{10}^{DT} = \left[n^{-1} \sum_{i=1}^{n} [Log_{10}(P_i / O_i)]^2 \right]^{0.5} \quad (10)$$

Empirical equations including Hazen, Kozeny-Carmen, Breyer, Slitcher, Terzaghi, USBR and Alyamani and Sen Equation have been evaluated.

RESULT AND DISCUSSION

The following equations for *Ks*, saturated hydraulic conductivity (m/day), were obtained from multiple regressions on data.

$$K_S = 8.4477 + 52.911(d_{10}) \quad (11)$$

$$K_S = 13.792 + 11.935(d_{50}) \quad (12)$$

$$K_S = 13.641 + 9.2877(d_{60}) \quad (13)$$

$$K_S = 14.683 + 105.17(d_{10})^2 \quad (14)$$

$$K_S = 17.836 + 8.2169(d_{50})^2 \quad (15)$$

$$K_S = 17.803 + 4.8243(d_{60})^2 \quad (16)$$

$$K_S = 17.987 + 8.7434(d_{50} - d_{10}) \quad (17)$$

$$K_S = 16.524 + 8.2761(d_{60} - d_{10}) \quad (16)$$

$$K_S = 36.019 - 4.2149(d_{60} / d_{10}) \quad (19)$$

$$K_S = 37.225 - 5.9843(d_{50} / d_{10}) \quad (20)$$

Whered_{10}, d_{50} and d_{60}, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and K_s, saturated hydraulic conductivity is expressed in m/day.Table 2 was indicated the various statistics of equations mentioned above.

Table 2: Values of statistics of multiple regressions equations estimate to hydraulic conductivity

Equation	Statistics			
	RMSE	***MAE***	***RE***	***DT***
11	**4.11**	**3.28**	**15.31**	**1.22**
12	6.77	5.46	25.46	1.37
13	7.41	6.07	28.28	1.42
14	19.79	19.07	88.94	1.98
15	5.23	4.22	19.69	1.27
16	5.67	4.51	21.02	1.30
17	5.13	4.16	19.41	1.27
18	5.76	4.55	21.20	1.30
19	14.60	13.54	63.12	1.79
20	15.34	14.31	66.72	1.82

a. RMSE is the Root Mean Square Error; MAE, Mean Absolute Error; RE, is the Relative Error and DT is Deviation Time.

The results showed as per the table the equation (11) was the best model for predicting K_s, saturated hydraulic conductivity (m/day), with 4.11 RMSE, 3.28 MAE, 15.31 RE and 1.22 DT. Comparison of observed vs. predicted values of saturated hydraulic conductivity obtained from the equation (11) as a 1:1 scale has been depicted in figure (1) that indicates good match.

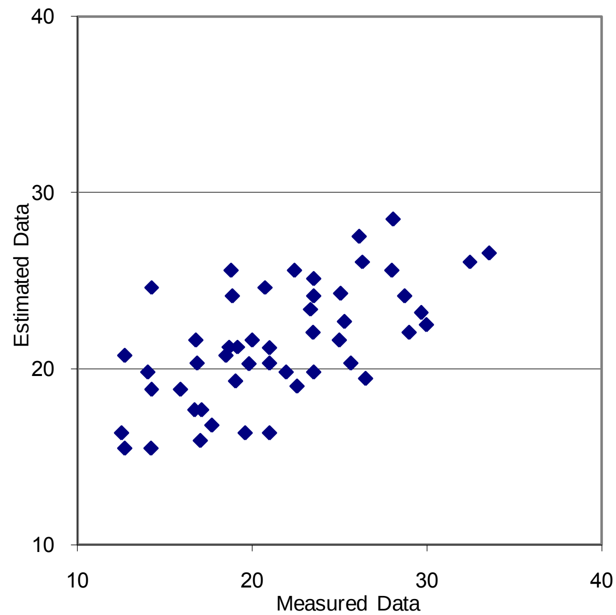

Figure 1 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by equation (11)

The results showed that among single parameter linear equations (equation 11, 12 and 13) in this study, the equation that predicted K_s, saturated hydraulic conductivity (m/day), from d_{10} estimated better (less prediction error) than d_{50} and d_{60} with 4.11 RMSE; 3.28 MAE; 15.31 RE, and 1.22 DT and the equation that predicted K_s, from d_{50} and d_{60} estimated with larger prediction error and the higher trend is evident between *Ks* and d_{10}. The results of single parameter regression analysis showed that when d_{10}, d_{50} and d_{60} increase, *Ks*, saturated hydraulic conductivity (m/day), increases.

Table 3: Values of statistics of empirical equations estimate to hydraulic conductivity

Equation	**Statistics**			
	RMSE	**MAE**	**RE**	**DT**
Hazen	54.45	47.25	220.32	3.17
Kozeny-Carmen	43.92	38.45	179.27	2.81
Breyer	58.11	48.80	227.54	3.21
Slitcher	**6.78**	**5.73**	**26.71**	**1.46**
Terzaghi	19.63	16.03	74.74	1.82
USBR	27.04	20.99	97.87	2.08
Alyamani and Sen	22.76	16.60	77.39	1.88

a. RMSE is the Root Mean Square Error; MAE, Mean Absolute Error; RE, is the Relative Error and DT is Deviation Time.

The results showed that among single parameter quadratic equations (equation 14, 15 and 16), the equation that predicted K_s, saturated hydraulic conductivity (m/day), from d_{50} estimated better (less prediction error) than d_{10} and d_{60} with 5.23 RMSE; 4.22 MAE; 19.69 RE, and 1.27 DT.
Also the results showed that among equation 17, 18, 19 and 20, the equation 17 that predicted K_s, from d_{50}-d_{10} (without d_{60}) estimated better than other tow parameter equations with 5.13 RMSE; 4.16 MAE; 19.41 RE, and 1.27 DT and K_s predicted based on d_{50}/d_{10} estimated with largest prediction error. Overall vriations between predicted and observed K_s are reported in the literature (Alyamani and Sen, 1993; Chakraborty *et al.*, 2006; Jabro, 1992; Mualem, 1976; Rawls and Brakensiek, 1989; Tyler and Wheatcraft, 1989; Uma *et al.*, 1989; Van *et al.*, 1992; Van, 1980; Van and Leji, 1989). The values of various statistics of empirical equations were indicated in Table 3.
The results showed as per the table the slitcher equation was the best model for predicting Ks, saturated hydraulic conductivity, with 6.78 RMSE, 5.73 MAE, 26.71% RE and 1.46 DT. Also the results showed that among studied equations, the Breyer equation predicted Ks, saturated hydraulic conductivity with high prediction error with 58.11 RMSE, 48.80 MAE, 227.54% RE and 3.21 DT. Meanwhile, the high value of DT statistic indicates that the accuracy and efficiency of the model in estimating the saturated hydraulic conductivity is low.
Comparison of observed vs. predicted values of saturated hydraulic conductivity of slitcher equation as a 1:1 scale has been depicted in figure (2) that indicates good match.

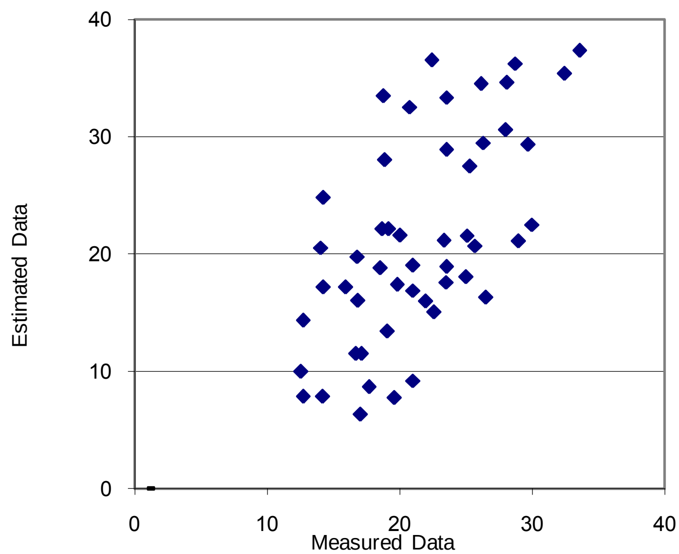

Figure 2 Comparison of measured saturated hydraulic conductivity, K_s (m/day) and K_s estimated by slitcher equation

Odong (2007) have been evaluated several empirical equations to calculate hydraulic conductivity using grain size distribution and Results showed that Kozeny-Carman formula proved to be the best estimator of most samples analyzed, and may be, even for a wide range of other soil types. However, some of the formulae underestimated or overestimated hydraulic conductivity; even of the same soils (Jabro, 1992). The difference between measured values of saturated hydraulic conductivity with estimated values has been reported by variousresearchers (Jabro, 1992; Van *et al.*, 1992; Van, 1980).
It is concluded that the values of hydraulic conductivity calculated by the Slitcher equation is lower than the other equations, which is consistent with the conclusions by Vukovic and Soro (1992), Cheng and Chen (2007) and Odong (2007) (Hwang and Powers, 2003b; Jabro, 1992; Shao and Robert, 1998). Alyamani and Sen, Terzaghi and USBR equations calculated saturated hydraulic conductivity lower than Breyer, Hazen and Kozeny-Carmen equations.

Conclusion

In this study several multiple regression and empirical equations to calculate hydraulic conductivity based on grain size distribution have been evaluated. The results showed approximately success in predicting hydraulic conductivity from particle diameters data. The following relationship obtained from regression on data *($R^2 = 0.40$)*:

$$K_s = 8.4477 + 52.911(d_{10})$$

Where d_{10} is the soil particle diameter (mm) that 10% of all soil particles are finer (smaller) by weight and K_s, saturated hydraulic conductivity is expressed in m/day. The results showed that the slitcher equation was the best formula for predicting saturated hydraulic conductivity among studied equations. The results showed that the Breyer equation predicted saturated hydraulic conductivity with high prediction error.

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