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## MEASUREMENT OF SUPER EFFICIENCY BASED ON EUCLIDEAN DISTANCE IN DATA ENVELOPMENT ANALYSIS

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#### ABSTRACT

Amirteimoori and Kordrostami (2010) presented a method for determining the super efficiency measure based on the Euclidean distance between the evaluated making unit (DMU) and Pareto frontier. Then (Aparicio and Pastor, 2011) gave the drawbacks of the model of Amirteimoori and Kordrostami (2010) according to their main goals, took the action to correct the problems and proposed a new model. The model (Aparicio and Pastor, 2011) may be infeasible. In this paper, the example provided by Aparicio and Pastor (2011) reviewed, then the researchers presented a quadratic programming problem that is firstly feasible and secondly ,they determined an unit in new technology that would be minimum Euclidean distance to the unit under evaluation and the measure of super efficiency determined based the Euclidean distance.

**Keywords:** Data Envelopment Analysis, Measure of efficiency, Euclidean Distance, Super Efficiency Measurement, Ranking

#### INTRODUCTION

Data Envelopment Analysis (DEA) was originally proposed by Charnes *et al.*, (1978) as a method evaluating the relative efficiency of Decision Making Units (DMUs) performing essentially the same task. Units use similar multiple inputs to produce similar multiple outputs. The basic models of DEA similar to CCR (Charnes *et al.*, 1978) and BCC Banker *et al.*, (1984) can recognize the performance of efficient and inefficient units and the will not be able to distinguish between DMUs. The efficient DMUs will be ranked using super efficiency models. The super efficiency model was presented for the first time by Andersen and Petersen (1993). The infeasibility of the super efficiency models lead to different analysis on efficient units and consequently a limited application of super efficiency models in DEA. Amirteimoori and Kordrostami (2010) determined the measure of Euclidean distance based (EDB) super efficiency based on the minimum Euclidean distance of the DMU under evaluation and projection point on Pareto frontier of new technology obtained from the omission of under evaluation efficient unit and then (Aparicio and Pastor, 2011) gave the drawbacks of their model and corrected the model. In this paper, reviewing one of the examples presented in (Aparicio and Pastor, 2011), we will present a model for ranking including the shortest Euclidean distance of under evaluation unit from the frontier of new technology.

In the second section, some basic definitions, models and concepts used in other sections, will be presented. In the third section, the model of Amirteimoori and Kordrostami (2010) and the drawbacks given by Aparicio and Pastor and then (Aparicio and Pastor, 2011) corrective model will be presented. In the fourth section, examining the example presented in (Aparicio and Pastor, 2011) and pointing out the drawbacks in the methodology of Aparicio and Pastor (2011), we will present a quadratic programming problem by which we can achieve the coordinates of the point at which there is the shortest Euclidean distance as super efficiency measure and criterion for ranking the extreme efficient units and show that the proposed quadratic programming problem is always feasible. The proposed model will be described in fifth section using numerical example and the results will be compared to Andersen and Petersen super efficiency model (Andersen and Petersen, 1993). The results are stated in the sixth section.

## DEA Preliminary and Concepts and Basic Models

In this section we describe some definitions, basic models and concepts used in other sections. Consider n as decision making units (DMUs) which use m inputs to produce s outputs. We will show them as

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 $(x_j, y_j), j = 1, 2, ..., n$ . assume that  $x \in R^m_+, y \in R^s_+$  that all data are non-negative and at least one of the components of each input and output vector is positive. The Production Possibility Set (PPS) will be represented as

 $T = \{(x, y) | y \ge 0 \text{ can be produced by } x \ge 0\}$ 

The set of production possibility with constant return to scale technology presented by Charnes *et al.*, 1978) is as follows:

$$T_C = \{(x, y) \mid \sum_{j=1}^n \lambda_j x_j \le x, \sum_{j=1}^n \lambda_j y_j \ge y, \ \lambda_j \ge 0\}$$

CCR model in input oriented is as follows:

$$\begin{array}{ll} (\text{CCR- I}) \\ \min \theta_{o} \\ S.t. \sum_{j=1}^{n} \lambda_{j} x_{ij} \leq \theta_{0} x_{io}, & i = 1, 2, ..., m, \\ \\ \sum_{j=1}^{n} \lambda_{j} y_{rj} \geq y_{ro}, & r = 1, 2, ..., s, \\ \lambda_{i} \geq 0, & j = 1, 2, ..., n. \end{array}$$
(1)

For the efficient units (frontier),  $\theta_o^* = 1$ . Thus the model (1) is not able to distinguish between efficient units. Andersen and Petersen (1993) introduced the super efficient model for ranking the efficient units. For this order, let  $E \subset \{1, 2, ..., n\}$  is set of efficient units and consider  $DMU_o$  at which  $o \in E$  and  $T'_c$  is defined as follows:

$$T'_{C} = \{(x, y) \mid \sum_{\substack{j=1\\j\neq o}}^{n} \lambda_{j} x_{j} \le x, \sum_{\substack{j=1\\j\neq o}}^{n} \lambda_{j} y_{j} \ge y, \lambda_{j} \ge 0, j = 1, 2, \dots, n, j \neq o\}$$

In fact,  $T'_{c}$  is the production possibility set by all observation except  $DMU_{o}$ . The supper efficient model of Andersen and Petersen (1993) for ranking  $DMU_{o}$  is as follows:

$$(\text{SE- CCR- I})$$

$$\min \theta_o^{AP}$$

$$S.t. \sum_{\substack{j=1\\j\neq o}}^n \lambda_j x_{ij} \le \theta_o^{AP} x_{io}, \quad i = 1, 2, ..., m,$$

$$\sum_{\substack{j=1\\j\neq o}}^n \lambda_j y_{rj} \ge y_{ro}, \quad r = 1, 2, ..., s, \quad (2)$$

$$\lambda_j \ge 0, \qquad j = 1, 2, ..., n, j \ne o.$$

**Theorem1.**  $DMU_o$  is extreme efficient iff  $\theta_o^{*AP} > 1$  or model (2) is infeasible. **Proof.** [3].

# The Method of Amirteimoori and Kordrostami for Measuring Super Efficiency and the Drawbacks on their Model given by Aparicio and Pastor and Corrective Model

In this section, we will present the model presented by Amirteimoori and Kordrostami (2010) for determining the measure of EDB based on the minimum Euclidean distance between the unit under

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evaluation and projection point on new Pareto frontier. Then we will point out briefly the drawbacks of their model presented by Aparicio and Pastor (2011) and discuss about the mixed- integer quadratic program presented by Aparicio and Pastor (2011) and suggested for removing the drawbacks of EDB measure and main goals of Amirteimoori and Kordrostami (2010).

Amirteimoori and Kordrostami (2010) measure of EDB has been programming based on the following mathematical programming problem:

$$\max -\alpha^{t} x_{o} + \beta^{t} y_{o}$$

$$S.t. -\alpha^{t} x_{j} + \beta^{t} y_{j} + s_{j} = 0, \quad j \in E,$$

$$1\alpha + 1\beta = 1,$$

$$s_{j} \leq (1 - \gamma_{j})M, \qquad j \in E,$$

$$\sum_{j \in E} \gamma_{j} \geq m + s - 1,$$

$$\alpha \geq 0, \quad \beta \geq 0,$$

$$\gamma_{i} \in \{0,1\}, \quad s_{j} \geq 0, \qquad j \in E.$$
(3)

Where 1 = (1,1,...,1), *M* is a large positive constant and  $E \subset \{1,2,...,n\}$  is set of extreme efficient DMUs (Amirteimoori and Kordrostami, 2010) search the full- dimensional reference supporting surface corresponding to  $DMU_o$ .

Let  $(\alpha^*, \beta^*, \gamma^*, s^*)$  is an optimal solution of (3), (Amirteimoori and Kordrostami, 2010) EDB measure is defined as follows:

$$\pi_o = \frac{1 + \frac{1}{card(A_o)} \sum_{i \in A_o} \frac{\rho \alpha_i^*}{x_{io}}}{1 - \frac{1}{card(B_o)} \sum_{r \in B_o} \frac{\rho \beta_r^*}{y_{ro}}}$$

Where:

$$A_{o} = \{i \mid x_{io} > 0\}, B_{o} = \{r \mid y_{ro} > 0\}, \rho = \frac{-\alpha^{*t}x_{o} + \beta^{*t}y_{o}}{(\|(\alpha^{*}, \beta^{*})\|_{2})^{2}}.$$

Generally. In this method, a hyperplane is determined by the problem (3) as  $-\alpha^{*t}x_o + \beta^{*t}y_o = 0$  that  $\rho, \pi_o$  could be determined and  $(\hat{x}_o, \hat{y}_o)$  projection point corresponding to  $DMU_o$  will be defined that the point has minimum Euclidean distance from  $DMU_o$  to hyperplane  $-\alpha^{*t}x_o + \beta^{*t}y_o = 0$ .

 $(\hat{x}_{o}, \hat{y}_{o}) = (x_{o} + \rho \alpha^{*}, y_{o} - \rho \beta^{*})$ 

Aparicio and Pastor (2011), presenting numerical examples, pointed the drawbacks of model (3) including:

1. Model (3) determines a full- dimensional supporting surface that really minimizes the Chebyshev distance to  $DMU_o$  instead of Euclidean distance. As a direct consequence, it is possible to find a closer projection point to the assessed unit than the produced by their method. In fact the method of Amirteimoori and Kordrostami fined a super plate that has minimum distance of Chebyshev to  $DMU_o$  and obtains the projection point corresponding to  $DMU_o$  based on Euclidean norm. Thus, measure of EDB defined by Amirteimoori and Kordrostami (2010) applied two different norms for final conclusion.

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2. The projection point presented by Amirteimoori and Kordrostami (2010) can be outside of the DEA technology. In addition, the projection point can belong to the weak efficient frontier instead of desired strongly efficient frontier.

Aparicio and Pastor (2011) presented the mixed- integer quadratic program for removing above mention drawbacks with emphasizing on the main goals of (Amirteimoori and Kordrostami, 2010) as follows:

$$\min \sum_{i=1}^{m} (s_i^{-})^2 + \sum_{r=1}^{s} (s_r^{+})^2$$

$$S.t. \sum_{j \in E} \lambda_j x_{ij} = x_{io} - s_i^{-}, \qquad i = 1, 2, ..., m,$$

$$\sum_{j \in E} \lambda_j y_{rj} = y_{ro} + s_r^{+}, \qquad r = 1, 2, ..., s,$$

$$- \sum_{i=1}^{m} \alpha_i x_{ij} + \sum_{r=1}^{s} \beta_r y_{rj} + d_j = 0, \qquad j \in E,$$

$$\alpha_i \ge 1, \qquad i = 1, 2, ..., m,$$

$$\beta_r \ge 1, \qquad r = 1, 2, ..., s,$$

$$d_j \le M b_j, \qquad j \in E,$$

$$\lambda_j \le M (1 - b_j), \qquad j \in E,$$

$$d_j \ge 0, \lambda_j \ge 0, \qquad j \in E,$$

$$s_i^{-} \ge 0, \qquad i = 1, 2, ..., m,$$

$$s_r^{+} \ge 0, \qquad r = 1, 2, ..., s.$$

In order to determine full- dimensional reference supporting surfaces, the following constrain was added to model (4):

$$\sum_{j \in E} (1 - b_j) \ge m + s - 1 \Leftrightarrow \sum_{j \in E} b_j \le card(E) - m - s + 1$$
(5)

The projection point is:

$$(\hat{x}_o, \hat{y}_o) = (\sum_{j \in E} \lambda_j^* x_j, \sum_{j \in E} \lambda_j^* y_j)$$
(6)

Where "\*" denotes an optimal solution.

The DB measure of Amirteimoori and Kordrostami (2010) was corrected by Aparicio and Pastor (2011) as follows:

$$\pi_{o} = \frac{\frac{1}{card(A_{o})} \sum_{i \in A_{o}} \frac{\hat{x}_{io}}{x_{io}}}{\frac{1}{card(B_{o})} \sum_{r \in B_{o}} \frac{\hat{y}_{ro}}{y_{ro}}} = \frac{\frac{1}{card(A_{o})} \sum_{i \in A_{o}} \frac{\sum_{i \in E} \lambda_{j}^{*} x_{ij}}{x_{io}}}{\frac{1}{card(B_{o})} \sum_{r \in B_{o}} \frac{\sum_{i \in E} \lambda_{j}^{*} y_{rj}}{y_{ro}}}$$
(7)

Where  $\pi_o$  is introduced as the measure of super efficiency in DEA.

## 1. The Drawbacks of Aparicio and Pastor Method and the Suggested Method

In this section, we discuss the drawback in the method of Aparicio and Pastor (2011) using the example (Aparicio and Pastor, 2011) and then the proposed model will be presented to find a criterion for ranking extreme efficient units based on Euclidean distance.

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#### 1.1. The Drawback of the Method of Aparicio and Pastor

Consider example 2 of [4]. The data of four DMUs are in table 1.

DMUj	I <sub>1</sub>	01	<b>O</b> <sub>2</sub>	03
1	1	1	11	1
2	1	1	1	11
3	1	10	6	6
4	1	4	6	6

Table	1:	Data of four	DMUs

Using Theorem 1, it can be seen that  $DMU_1$ ,  $DMU_2$  and  $DMU_3$  are extreme efficient and  $DMU_4$  not extreme efficient. Thus  $E = \{DMU_1, DMU_2, DMU_3\}$ . Consider  $DMU_4$ . Considering relation (5), the problem (4) for this example will be as follows:

min $(s_1^-)^2 + (s_1^+)^2 + (s_2^+)^2 + (s_3^+)^2$	
S.t. $\lambda_1 + \lambda_2 + \lambda_3 = 1 - s_1^-$ ,	(8.1)
$\lambda_1 + \lambda_2 + 10\lambda_3 = 4 + s_1^+,$	(8.2)
$11\lambda_1 + \lambda_2 + 6\lambda_3 = 6 + s_2^+,$	(8.3)
$\lambda_1 + 11\lambda_2 + 6\lambda_3 = 6 + s_3^+,$	(8.4)
$-\alpha_1 + \beta_1 + 11\beta_2 + \beta_3 + d_1 = 0,$	(8.5)
$-\alpha_1 + \beta_1 + \beta_2 + 11\beta_3 + d_2 = 0,$	(8.6)
$-\alpha_1 + 10\beta_1 + 6\beta_2 + 6\beta_3 + d_3 = 0,$	(8.7)
$\alpha_1 \ge 1, \ \beta_1 \ge 1, \ \beta_2 \ge 1, \ \beta_2 \ge 1,$	(8.8)
$d_1 \leq Mb_1, d_2 \leq Mb_2, d_3 \leq Mb_3,$	(8.9)
$\lambda_1 \le M(1-b_1), \ \lambda_2 \le M(1-b_2), \ \lambda_3 \le M(1-$	$-b_3$ ), (8.10)
$b_1 \in \{0,1\}, \ b_2 \in \{0,1\}, \ b_3 \in \{0,1\},$	(8.11)
$d_1 \ge 0, d_2 \ge 0, d_3 \ge 0,$	(8.12)
$\lambda_1 \geq 0, \; \lambda_2 \geq 0, \; \lambda_3 \geq 0,$	(8.13)
$b_1 + b_2 + b_3 \le 3 - 1 - 3 + 1$ ,	(8.14)
$s_1^- \ge 0,$	(8.15)
$s_1^+ \ge 0, \ s_2^+ \ge 0, \ s_3^+ \ge 0,$	(8.16)

From (8.14) and (8.11) we conclude that  $b_1 = b_2 = b_3 = 0$ , thus from (8.9) we have  $d_1 = d_2 = d_3 = 0$  (note that *M* is a very large positive constant). So the constraints (8.5), (8.6), (8.7) and (8.8) will be as follows:

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$$-\alpha_{1} + \beta_{1} + 11\beta_{2} + \beta_{3} = 0,$$
  

$$-\alpha_{1} + \beta_{1} + \beta_{2} + 11\beta_{3} = 0,$$
  

$$-\alpha_{1} + 10\beta_{1} + 6\beta_{2} + 6\beta_{3} = 0,$$
  

$$\alpha_{1} \ge 1, \ \beta_{1} \ge 1, \ \beta_{2} \ge 1, \ \beta_{2} \ge 1,$$

The first and second constraints for  $(\alpha_1, \beta_1, \beta_2, \beta_3) = (13,1,1,1)$  are true, but in third constraints is not true. Thus we conclude that the aforementioned system for  $DMU_4$  is infeasible.

According to what was said, we conclude that there is no guarantee for feasibility of the set including constraints (8.5) through (8.8) in the model of Aparicio and Pastor (2011); that is, their model may be infeasible.

Now, consider the examples conclusions obtained by Aparicio and Pastor (2011) for the aforementioned example that is follows:

$$(\hat{x}_4, \hat{y}_4) = (\sum_{j \in E} \lambda_j^* x_j, \sum_{j \in E} \lambda_j^* y_j) = (0.7940; 5.5038, 3.4106, 6.1174)$$
(9)

(10.1)

We have from the relation (9):  $2^* + 2^* + 2^* = 0.7040$ 

$$\lambda_{1} + \lambda_{2} + \lambda_{3} = 0.7940, \quad (10.1)$$
  

$$\lambda_{1}^{*} + \lambda_{2}^{*} + 10\lambda_{3}^{*} = 5.5038, \quad (10.2)$$
  

$$11\lambda_{1}^{*} + \lambda_{2}^{*} + 6\lambda_{3}^{*} = 3.4106, \quad (10.3)$$

 $\lambda_1^* + 11\lambda_2^* + 6\lambda_3^* = 6.1174$ , (10.4) From (9), relations (10.1) through (10.4) and relations (8.1) through (8.4) it can be concluded that

 $s_2^{-*} = -2.5894$  that is in contradiction with (8.16). In addition, we have substituting the values of (9) and (7):  $\pi_4 = 0.8037$  that is in contradiction to the value determined in [4] that is  $\pi_4 = 0.2679$ .

## 1.2. The Proposed Model

The proposed model will be for achieving a criterion to rank the phrase from the shortest Euclidean distance  $DMU_o(x_o, y_o)$  from frontier  $T'_c$  that is proposed for this.

$$\pi_o = \min \| (\bar{x}, \bar{y}) - (x_o, y_o) \|_2^2$$
  
S.t.  $(\bar{x}, \bar{y}) \in T'_C$ . (11)

According to the definition of Euclidean distance and structure of  $T'_{c}$ , the programming problem (11) will be as follows:

$$\eta_{o} = \min \sum_{i=1}^{m} (\bar{x}_{i} - x_{io})^{2} + \sum_{r=1}^{s} (y_{ro} - \bar{y}_{r})^{2}$$

$$S.t. \sum_{\substack{j=1 \ j\neq o}}^{n} \lambda_{j} x_{ij} \leq \bar{x}_{i}, \qquad i = 1, 2, ..., m,$$

$$\sum_{\substack{j=1 \ j\neq o}}^{n} \lambda_{j} y_{rj} \geq \bar{y}_{r}, \qquad r = 1, 2, ..., s, \qquad (12)$$

$$\bar{x}_{i} \geq 0, \qquad i = 1, 2, ..., m,$$

$$\bar{y}_{r} \geq 0, \qquad r = 1, 2, ..., s,$$

$$\lambda_{j} \geq 0, \qquad j = 1, 2, ..., n, \quad j \neq o.$$

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Problem (12) is a quadratic programming that can be solved using the L. C. P. method [7] or other methods due to convexity of the objective function and feasible region.

**Theorem 2.** The programming problem (12) is always feasible.

**Proof.** For  $k \neq o, j = 1, 2, ..., n, j \neq k$  Let  $\lambda_k = 1, \lambda_j = 0$  and define:

 $\bar{x}_i = \max_{1 \le i \le m} \{x_{ik}, x_{io}\}, \bar{y}_r = \min_{1 \le r \le s} \{y_{rk}, y_{ro}\}, \text{ It is easy to see that } (\bar{x}, \bar{y}, \lambda) \text{ is a feasible solution for (12).}$ 

#### Numerical Example

Example 1. Consider ten DMUs with two inputs and two outputs. The data of ten DMUs are in table 2.

$DMU_{j}$	$I_1$	$I_2$	$O_1$	$O_2$	
1	15	27	45	78	
2	24	32	44	54	
3	31	44	75	24	
4	20	57	37	65	
5	18	83	86	76	
6	16	82	24	35	
7	14	56	24	65	
8	75	35	98	76	
9	25	68	73	46	
10	72	91	24	53	

Table 2: Data of the ten DMUs

The results of application of model (12) in compared to the model (Amirteimoori and Kordrostami, 2010) are shown in table 3.

$DMU_{j}$	$ heta_{\scriptscriptstyle AP}^*$	Rank (AP)	$\eta_{_o}$	Rank (12)
1	1.95	1	494.52	1
2	0.76	inefficient		
3	0.96	inefficient		
4	0.63	inefficient		
5	1.59	3	102.4	3
6	0.44	inefficient		
7	0.98	inefficient		
8	1.65	2	361.63	2
9	0.82	inefficient		
10	0.2	inefficient,		

 Table 3: The results of the models (2) and (12)

According to theorem 1,  $DMU_1$ ,  $DMU_5$  and  $DMU_8$  are extreme efficient and the remaining DMUs are inefficient. The proposed model (12) will be applied for extreme efficient units. The results are shown in

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fourth and fifth columns of table 3. The rank of extreme efficient units is the same by both model (4) and (12). We have  $DMU_1 \succ DMU_8 \succ DMU_5$  (the symbol  $\succ$  shows the higher rank.

**Example 2.** Consider fifteen DMUs with for inputs and three outputs. The data of fifteen DMUs are in table 4.

$DMU_{j}$	$I_1$	I <sub>2</sub>	$I_3$	$I_4$	$O_1$	<i>O</i> <sub>2</sub>	<i>O</i> <sub>3</sub>
1	63.46	21.39	0	0	42.23	0	4063.68
2	0	0	0	229.24	13.89	0	11250.82
3	0	22.86	15.54	21.59	55.56	0	0
4	199.62	5.47	0	52.29	56.17	0	9829.4
5	302.82	15.52	0	0	92.59	0	0
6	109.1	0	20.47	0	36.29	0	2410.96
7	0	0	56.21	14.03	0	34.13	0
8	6.38	11.52	43.03	0	15.14	13.04	10574.7
9	33.61	28.34	0	75.24	69.46	0	4971.83
10	167.28	0	55.79	0	0	29.87	12458.02
11	99.64	9.77	6.76	0	0	19.89	10099.37
12	0	23.71	0	0	0	27.78	0
13	194.02	8.24	0	0	46.17	0	6506.39
14	0	6.42	119.09	0	0	74.07	0
15	0	10.83	7.31	10.23	26.32	0	0

Table 4: Data of Inteen DNUS	Table 4	: Data	of fifteen	<b>DMUs</b>
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The results of application of model (12) in compared to the model (2) are shown in table 5.

	${\cal A}^*$	Rank	n	Rank	
	$O_{AP}$	( <b>AP</b> )	$\eta_o$	(12)	
1	2.60	4	238.84	6	
2	infeasible		2200	2	
3	1	10			
4	1.64	7	32.61	12	
5	1.22	8	69.03	9	
6	Infeasible		55.16	11	
7	Infeasible		8.68	13	
8	17.67	1	7900	1	
9	2.58	5	131.04	7	
10	Infeasible		124.24	8	
11	2.22	6	4891.19	5	
12	Infeasible		546.70	3	
13	4.16	3	57.78	10	
14	9.85	2	532.13	4	
15	1.01	9	0.002	14	

 Table 5: The results of the models (2) and (12)

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According to theorem 1, all DMUs except  $DMU_3$  (non- extreme efficient point) are extreme efficient. Model (4) and proposed model (12) will be applied for extreme efficient DMUs. The rank of extreme efficient DMUs by models (4) and (12) are follows as:

The Ranking Results using Andersen and Petersen Model (2)

 $DMU_8 \succ DMU_{14} \succ DMU_{13} \succ DMU_1 \succ DMU_9 \succ DMU_{11} \succ DMU_4 \succ DMU_5 \succ DMU_{15} \succ DMU_3.$ 

The Ranking Results using the Proposed Model (12)

 $DMU_8 \succ DMU_2 \succ DMU_{12} \succ DMU_{14} \succ DMU_{11} \succ DMU_1 \succ DMU_9 \succ DMU_{10} \succ DMU_5 \succ DMU_{13} \succ DMU_6$ 

 $\succ DMU_{4} \succ DMU_{7} \succ DMU_{15}.$ 

The proposed model is feasible that is an advantage of model compared to Andersen and Petrsen model and (Aparicio and Pastor, 2011) model.

## Conclusion

(Amirteimoori and Kordrostami, 2010) presented a method for determining the super efficiency measure based on the Euclidean distance between the evaluated decision making unit and Pareto frontier. Then (Aparicio and Pastor, 2011) gave the drawbacks of the model of Amirteimoori and Kordrostami, according to their main goals, took the action to correct the problems and proposed a new model. With regard to the issue of programming problem of (Amirteimoori and Kordrostami, 2010) and its correction model by (Aparicio and Pastor, 2011) may be infeasible, in this paper, the example provided by (Aparicio and Pastor, 2011) would be reviewed, that their problem be infeasible. In the model presented by them, the envelopment and multiplier forms constrains are simultaneously used that is not justification and in addition, they are mixed integer quadratic programming that is a complex problem and is time consuming in terms of computation. In this paper, we presented a quadratic programming problem that was always feasible and we can determine projection point in new technology that has minimum Euclidean distance. Moreover the results of ranking by AP model and proposed model (12) are equal, if AP model is feasible (see example 1).

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