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Research Article

THE COMPARISON OF DIFFERENT METHODS OF CHRIP SIGNAL PROCESSING AND PRESENTATION OF A NEW METHOD FOR PROCESSING IT

***Zahra Shourangiz and Masoud Mahzoon**
Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Fars, Iran
**Author for Correspondence*

ABSTRACT
In this paper we introduce and compare Chrip Fourier transforms and fractional Fourier transforms. At first we review different ways to model Chrip signals. We know that the Chrip signal is a signal whose momentary frequency change with time and it in fact doesn't have the short comings of classic frequency transforms like Fourier. At the end, a new and effective way to model a hypothetical Chrip signal will be presented.

Keywords: *Chrip, Fractional Fourier, Wigner Chrip, Fourier Chrip*

INTRODUCTION
Classical methods of signal analysis take place in two time or frequency states, as these methods provide no information about the frequency changes over time or time dispersion of a particular frequency in a signal (Yali *et al.,* 2009), they are not suited to analyze non-static signals like Chrip signals. In a Chrip signal there is information about how frequency changes over time, so in this article we are looking for ways to improve Chrip signal processing by using this information.
Chrip signal was first introduced by Mr. Klader *et al*., in the Bell Telephone Laboratory for radar use in 1960. The Chrip signals have extensively applications in areas such as radar and sonarray ultrasound systems (Engen and Yngvar, 2011).
The main aim: to optimize previous methods based on Chrip signal and to introduce a new transform on the basis of Chrip signal which can be used to process the Chrip signal in the optimization area. The concept of Pulse compression is used in order to have a simultaneous long range and good resolution.

Introduction and Characterization of the Chrip Signal

A. Simple Chrip Model

The ability of the radars for perfect differentiation is due e wide bandwidth frequency of these systems. From the topic of Fourier transform we know that a pulse with a constant range and carrier frequency has a narrow bandwidth and low differentiation limits.
However, in the spectrum of these signals, a signal with wide bandwidth and a long time interval can be obtained using the Frequency Modulation (FM).

$$fi = \frac{1}{2\pi}\frac{d\phi}{dt} = f_0 + kt \qquad (1)$$

$$\phi = 2\pi\left[F_0 t + k\frac{t}{2}\right] \qquad (2)$$

$$K = \frac{F_{\max} - F_{\min}}{T_{\max} - T_{\min}} \quad if \rightarrow k > 0 \rightarrow upchirp \qquad (3)$$
$$if \rightarrow k < 0 \rightarrow downchirp$$

Where k represents Chriprate and f, the momentary frequency.

B. Ambiguity Function

This function is a very useful and significant tool in radar and sonarray signal processing. This function can be very useful in comparing different signals by their resolutions. Ambiguity function can be expressed as a time response filter for the signal that has time lag and frequency displacement.

Individual ambiguity function:

$$|X(\tau,\nu)| = \left|\int_{-\infty}^{\infty} u(t)u^*(t+\tau)\exp(j2\pi\nu t)\,dt\right| \quad (4)$$

Reciprocal ambiguity function:

$$|AF(\tau,\nu)| = \left|\int_{-\infty}^{\infty} s(t)r^*(t+\tau)\exp(j2\pi\nu t)\,dt\right| \quad (5)$$

1. Introduction of the important features of the ambiguity function:

Maximum value:

$$|X(\tau,\nu)| \le |X(0,0)| = 1 \quad (6)$$

Constant value:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |X(\tau,\nu)|^2\, d\tau\, d\nu = 1 \quad (7)$$

Symmetry with respect to the origin:

$$|X(-\tau,-\nu)| = |X(\tau,\nu)| \quad (8)$$

Effect of linear modulation frequency:

$$u(t)\exp(j\pi k t^2) \Leftrightarrow |X(\tau,\nu-k\tau)| \quad (9)$$

C. Analysis of the Signal Using the Ambiguity Function

In radar and sonarray systems, the choice of waveform and its parameters has a large impact on the system performance, including the determination range and resolution of the system, and one of the most important and most basic design stages is the selection of the sending waveform. In this section, we study different waveforms using the ambiguity function and calculate the range with the help of the time axis and the speed with the help of Doppler shift.

Rectangular Pulse wave form

Rectangular Pulse form is one of the simplest wave forms to be used is known as the single-frequency wave form. The smaller the width of the pulse, the greater there solution. In the following figures, the figure resulted from a cut on the time lag axis of the ambiguity function is the pulse autocorrelation function, the width of which represents the timing precision or resolution of the range; and the width of the pulse figure resulted from a cut on the Doppler axis represents the resolution for the Doppler.

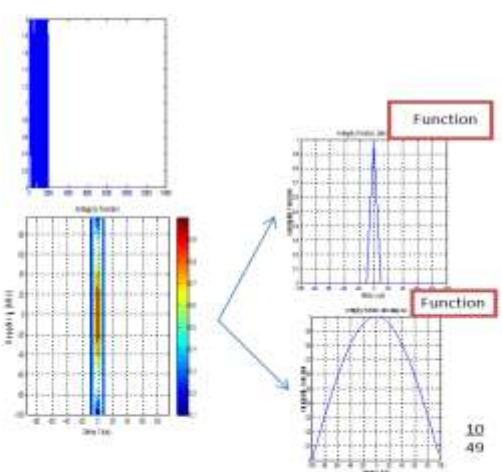

Figure1: The ambiguity function of the rectangular pulse wave form

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Chrip Wave Form

As shown in the figures, we come to the conclusion that the time resolution of the Chrip pulse has improved in comparison with the single-frequency pulse.

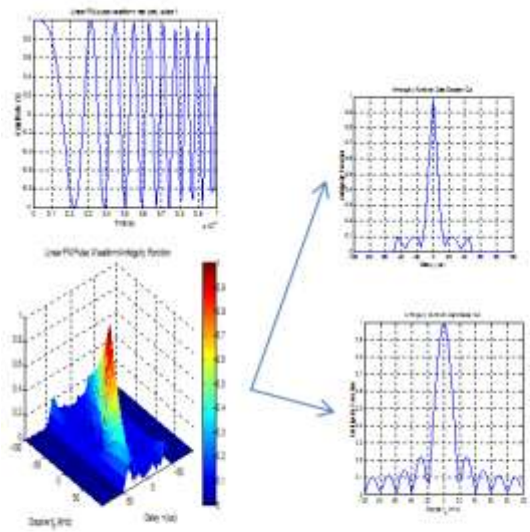

Figure 2: LFM wave form ambiguity function

The display of the linear Chrip signal in the time domain is as follows:

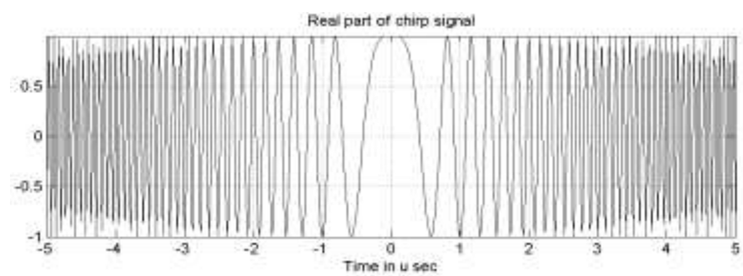

Figure 3: The display of the linear Chrip signal in the time domain

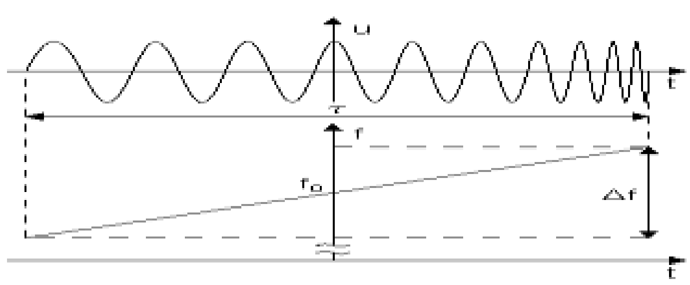

Figure 4: The display of the Chrip frequency time

Chrip signal Fourier transform is shown below:

$$\tilde{\epsilon}_1(f) = \int_{-\infty}^{\infty} \epsilon_1(t) e^{-j2\pi ft}\, dt = \int_{-T/2}^{T/2} e^{-j2\pi\left[(f_0-f)t+\frac{1}{2}kt^2\right]}\, dt \qquad (10)$$

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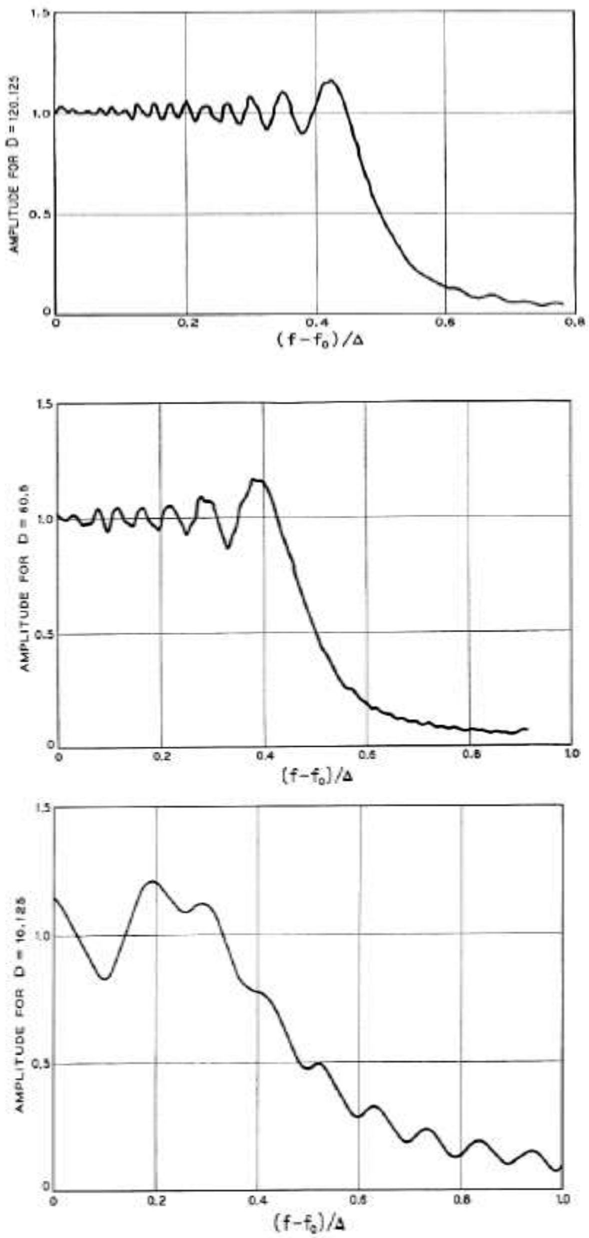

Figure 5: The display in different frequency domains with varying Ds; increasing D gets us closer to the desired range

The Introduction of New Transforms in Chrip Signal Processing
Here we introduce three new transforms in Chrip signal processing: FRFT, Fourier Chrip transform, CFT Wigner Chrip transform WFT. These three transforms are powerful tools that are used in the recent years' signal processing. The Fourier fractional transform and the Fourier Chrip transform are generalized conventional Fourier transforms and Wigner Chrip transform is the generalized Wigner transform.
A. Fractional Fourier Transform
Fractional Fourier transform is more flexible in processing non-constant signals in comparison with the ordinary Fourier transform, due to having a degree of freedom and therefore is widely used in filtering, revealing and estimating the parameters of the linear Chrip signal. The transform is unitary and is very efficient in Chrip transform signal processing. The transform appears with a low rank and little abrupt

changes and has relatively low computational load. Chrip's constant rate is needed to calculate the transform. Fractional Fourier transform is independent from the frequency information and has high accuracy in estimating the Chrip signal range and very low error as to zero. It can be considered linear. It can be considered as linear. The original formulation of the fractional Fourier transform is as follows (Diego *et al.*, 2012):

$$X_\alpha(u) = \mathrm{F}^p[x(t)] = \int_{-\infty}^{\infty} x(t)\,K_\alpha(t,u)\,dt \quad (11)$$

$$x_1(t) = rect\left(\frac{t}{T}\right)e^{j2\pi\left(f_0 t + \frac{1}{2}kt^2\right)} \quad (12)$$

$$K_\alpha(t,u)\begin{cases}\sqrt{\dfrac{1-j\cot\alpha}{2\pi}}\exp\left(j\dfrac{t^2+u^2}{2}\cot\alpha - tu\csc\alpha\right), & \alpha \neq n\pi \\ \delta(t-u), & \alpha = 2n\pi \\ \delta(t+u), & \alpha = (2n+1)\pi\end{cases} \quad (13)$$

$$\alpha = p\times(\pi/2) \quad (14)$$

Where p is the transform rate, α is the transform angle, K is the core of FRFT from a kernel function. Some of the properties of FRFT are as follows:

1. The transformation is linear
2. The turns are zero, $F^0 = 1$

3. Compatibility with conventional Fourier transform,
$F^1 = F$

4. The addition in turns, $F^P.F^Q = F^{p+q}$

5. Alternation with period 4, $F^{P+4} = F^P$

6. Parseval's theorem for any given α:

$$\int_{-\infty}^{\infty} x(t)\,y^*(t)\,dt = \int_{-\infty}^{\infty} X_\alpha(u)\,Y_\alpha(u)\,du \quad (15)$$

7. Reverse $x(t) = \int_{-\infty}^{\infty} X_\varphi(u)k_{-\varphi}(u,t)\,du$

The relationship between FRFT and Wigner distribution and ambiguity function:

Wigner distribution and ambiguity function are of the most important tools in the analysis of time and frequency such that they are frequently used in radar applications and quantum mechanics. Below after the introduction of Wigner distribution and ambiguity function, its relationship with the fractional Fourier transform is presented. Wigner distribution signal $X(t)$ is defined as follows:

$$W_x(t,f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-jft}d\tau \quad (16)$$

Ambiguity function:

$$A_x(\eta,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-jt\eta}dt \quad (17)$$

Wigner distribution is described as the signal power distribution in the frequency time plate and it is shown that the image of this distribution on the time and frequency axes respectively represents the signal's square size of frequency distribution and the square size of time distribution:

$$\int_{-\infty}^{\infty} w_x(t,f)\,df = |x(t)|^2 \qquad (18)$$

and

$$\int_{-\infty}^{\infty} w_x(t,f)\,df = |x(f)|^2 \qquad (19)$$

The relationship between the ambiguity function and the fractional Wigner Fourier transform distribution:

$$W_x(u,v) = W_x(u\cos\alpha - v\sin\alpha, u\sin\alpha + v\cos\alpha)\ A_x(\eta,v) = W_x(\eta\cos\alpha + r\sin\alpha, \eta\sin\alpha - r\cos\alpha) \qquad (20)$$

For example, if the frequency signal is displayed as figure a, the fractional Fourier transform with the proper angle can display the frequency time as a figure.
Schematic display of the FRFT effect on the Wigner distribution

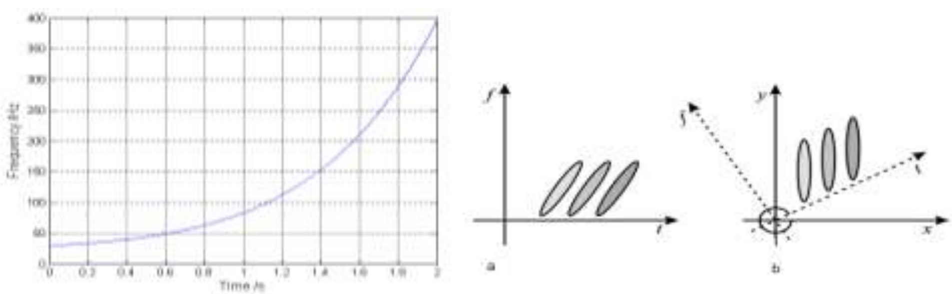

Figure 6: The display of the momentary frequency of the Chrip signal by time

B. Wigner Chrip Transform

Wigner Chrip transform is more complete than the previous transforms and can be used for non-linear Chrip signal processing whose momentarily frequency coefficients of a polynomial time changes by time (Osama *et al.*, 2012). To process signals whose momentary frequencies can change by time like a polynomial, but its implementation is heavier and is defined as follows (Liu *et al.*, 2013):

$$w_c(t,f) = \frac{1}{T}\int_{-\infty}^{\infty} x^*\left(t-\frac{\tau}{2}\right).x^*\left(t+\frac{\tau}{2}\right)\times \exp(-j2\pi(f+C(t)\tau))d\tau \qquad (21)$$

$x(t)$ is a signal on which the transformation is going to be done, f is the variable for frequency, t the variable for time, τ time lag and N Chrip signal level. $C(t)$ is the extraction sequence (Durak and Aldirmaz, 2010):

$$C(t) = \frac{c_2(t)}{2}.\tau^2 + \frac{c_3(t)}{2}.\tau^3 + \frac{c_4(t)}{2}.\tau^4 + \ldots + \frac{c_N(t)}{N}.\tau^N \qquad (22)$$

This transform similar to the fractional Fourier transform has high accuracy for estimating the Chrip signal range and is a double linear transform and in case of the signals being multi-components, we we'll have conflict terms (Tao *et al.*, 2007) and that is why we should improve the transform in a way to minimize the effect of these conflicts (Osama *et al.*, 2012). Wigner Chrip transform contains a large set of signals and can be even suited for signals whose Chrip rate and parameters change over time and it has little error, but heavier implementation in comparison with the fractional Fourier (Diego *et al.*, 2012).

C. A New Approach: Fourier Chrip Transform

Signal representation using the Fourier Chrip transform is as follows:

$$x(t) = A.\exp(j2\pi[f_0 t + \frac{c_{20}}{2}t^2 + \frac{c_{30}}{3}t^3 + \frac{c_{40}}{4}t^4 + \frac{c_{50}}{5}t^5 + \frac{c_{60}}{6}t^6]) \quad (23)$$

Where we have:

$$f_0 = 30Hz \,;\, c_{20} = 15Hz/s \,;\, c_{30} = 20Hz/s^2$$
$$c_{40} = 10Hz/s^3 \,;\, c_{50} = 5Hz/s^4 \,;\, c_{60} = 3Hz/s^5 \quad (24)$$

Also the following equation calculates the error in reconstructing the signal, where $x_i(t)$ is the original signal, $x_r(t)$ the reconstructed signal and Nrestored and the rate of the signal (Pei and Ding, 2010).

$$Error = \frac{1}{N}(x_i(t) - x_r(t)) \quad (25)$$

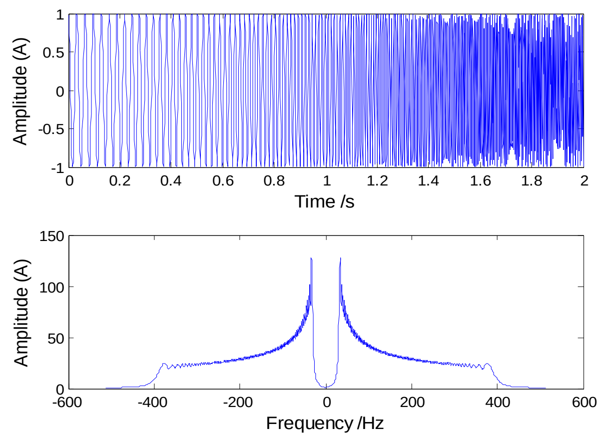

Figure 7: The display of time frequency Chrip

IV. *Computer Simultion*

Here we present the results of computer simulations by the MATLAB software. First we reconstruct the signal using the Fourier Chrip method.

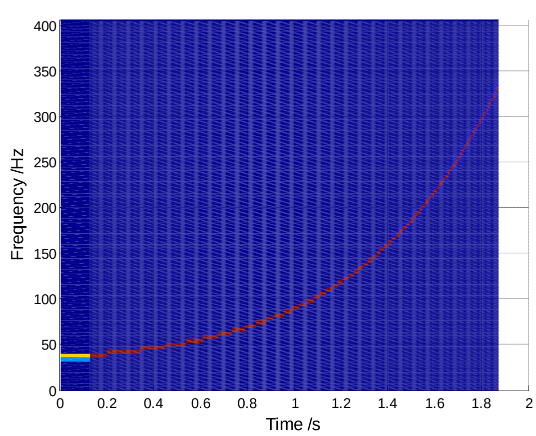

Figure 8: The display of the frequency time signal by Fourier Chrip method

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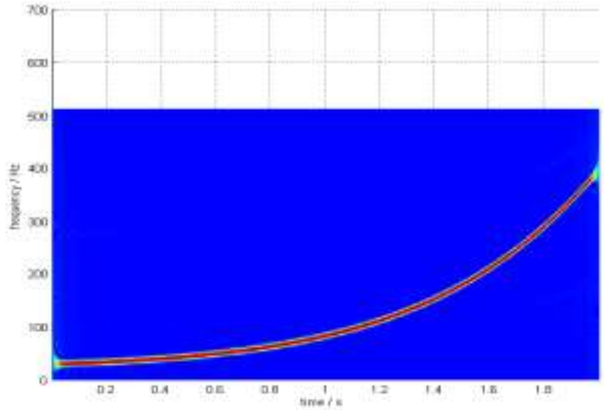

Figure 9: The display of the frequency time signal by Wigner Chrip method

As can be seen, the Fourier Chrip signal has easily and with no errors tracked and displayed the time related changes. The Wigner Chrip transform has some errors in comparison with the Fourier Chrip transform.

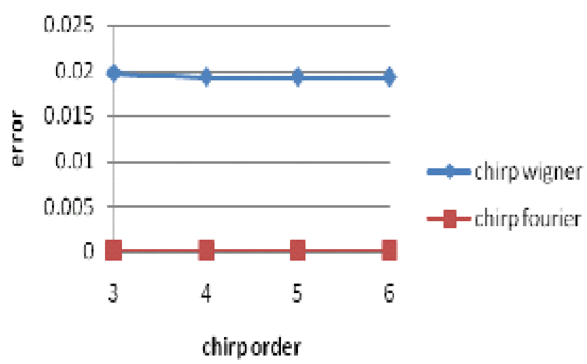

Figure 10: The comparison of error in two methods of Chrip Fourier and Chrip Wigner at different Chrip signal rates

We can see that in this case the errors are small compared to the Fourier Chrip. Similarly, we see that in the Fourier Chrip we have zero error which is highly desirable but little error in Wigner Chrip.

CONCLUSIONS

According to the simulation results in MATLAB software we can see that the Fourier Chrip methods reconstruct the Chrip signal with higher resolution than the Chrip Wigner method and track its frequency path. Error calculating curves for the Chrip signal rate also indicative better performance of Chrip Fourier method due to its low rate of error.

REFERENCES

Yali Qin, Lin Wenyao, Zhou Shouli and Hu Hairong (2009). Detection of Chirp Signal by Combination of Kurtosis Detection and Filtering in Fractional Fourier Domain" Open Research Fund of State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, 2009

Geir Engen and Yngvar Larsen (2011). Efficient Full Aperture Processing of TOPS Mode Data Using the Moving Band Chirp Z-Transform. *IEEE Transactions on Geosciences and Remote Sensing* **49**(10).

Diego Cristallini, Debora Pastina, Fabiola Colon and Pierfrancesco Lombardo (2012). Efficient Detection and Imaging of Moving Targets in SAR Images Based on Chirp Scaling. *IEEE Transactions On Geosciences and Remote Sensing*.

Osama Alkishriwo A and Luis Chaparro F (2012). Signal Compression Using The Discrete Linear Chirp Transform (DLCT). *20th European Signal Processing Conference*.

Liu Gao-gao, Zhang Lin-rang, Liu Nan, Chen Guang-feng and Zhang Yan (2013). Focusing Highly Squinted Using the Extended Nonlinear Chirp Scaling Algorithm. *IEEE Geosciences and Remote Sensing Letters* **10**(2).

Durak L and Aldirmaz S (2010). Adaptive Fractional Fourier Domain Filtering. *Signal Processing* **90** 1188–119.

Tao R, Zhan F and Wang Y (2008). Fractional Power Spectrum. *IEEE Transaction Signal Processing* **56**(9) 4199–4206.

Pei SC and Ding JJ (2010). Fractional Fourier Transform, Wigner Distribution, and Filter Design for Stationary and Non stationary Random Processes. *IEEE Transactions on Signal Processing* **58**(8).