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ONSET OF CONVECTION IN A COUPLE STRESS FLUID SATURATED ANISOTROPIC POROUS LAYER WHEN BOTH FLUID AND SOLID PHASES ARE HAVING DIFFERENT TEMPERATURES

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ABSTRACT
Anisotropy and local thermal non-equilibrium are discussed in connection to the onset of convection in a horizontal couple stress fluid saturated inhomogeneous porous layer. The flow is described using the Darcy model, and the energy equation is solved using a two-field model that separates the solid and fluid phases differently. The critical Rayleigh number and corresponding wave number for the thermal convection are determined using linear stability theory. This article discusses how anisotropy, thermal non-equilibrium, and the couple-stress parameter affect the start of convection. It has been done to do asymptotic analysis for both very small and high values of the interphase heat transfer coefficient. It is demonstrated that in limiting instances, the results that are consistent with a thermally non-equilibrium anisotropic porous material can be recovered.

***Keywords**: Convection, Thermal Non-Equilibrium, Couple Stress Fluid*

NOMENCLATURE

a	horizontal wave number
c	specific heat
d	height of the porous layer
g	gravitational acceleration
h	inter phase heat transfer coefficient
H	non-dimensional inter phase heat transfer coefficient, $\dfrac{hd^2}{\varepsilon k_f}$
k	horizontal wave number
k_f, k_s	thermal conductivities of fluid phase and solid phase respectively
K	permeability of the porous medium
K_x	permeability parameter along x direction
K_y	permeability parameter along y direction
$\mathbf{k}$	unit vector in the vertical direction,
p	pressure
$\vec{q}$	velocity vector, (u,v)
Ra	Rayleigh number, $\dfrac{\rho_0 g\beta\,(T_l - T_u)Kd}{\varepsilon\mu_f\,\kappa_f}$
T	temperature
t	time

(x,y) space co-ordinates

Greek symbols

α diffusivity ratio
β co-efficient of thermal expansion
γ porosity-modified conductivity ratio, $\dfrac{\varepsilon k_f}{(1-\varepsilon)k_s}$
ε porosity
κ thermal diffusivity
μ_e effective viscosity
μ_f fluid viscosity
ρ_f fluid density
θ non-dimensional temperature of the fluid phase
ϕ non-dimensional temperature of the solid phase
∇^2 $\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$
μ dynamic viscosity
κ thermal diffusivity, $k_f/(\rho_0 c)_f$
ϕ^{-1} anisotropic permeability parameter, $\dfrac{K_z}{K_h}$

Subscripts

b basic state
f fluid
l lower
s solid
u upper
* non-dimensional
0 reference
/ perturbed quantity

INTRODUCTION

Many geophysical and technological issues are very interested in thermal convection in fluid-saturated porous media. Geothermal power use, oil reservoir modeling, thermal insulation of constructions, and nuclear waste disposal are all significant applications. Numerous writers have already conducted in-depth research on the issue of convective instability of a horizontal fluid-saturated porous layer confronted to an

unfavorable temperature gradient. The most recent reviews by Ingham and Pop [1998], Nield and Bejan [2006], and Vafai [2000] provide excellent documentation of the expanding body of work in this field.
Isotropic materials have been a major focus of theoretical and experimental research on convective flow in porous media. However, the mechanical and thermal properties of porous materials are anisotropic in many real-world circumstances. Anisotropy typically results from the asymmetric shape or preferential orientation of porous fibres or matrix [Mckibbin, 1986].
Particulate-containing fluids are a common working medium in industrial settings. The majority of fluids do not conform to a Newtonian description, and the development of micro-momentum field theories in such a situation opened up new application areas, including polymeric suspensions, animal blood, and liquid crystals, which have been reported in recent years to contain very small suspended particles of various shapes. These particles have the ability to rotate independently of the fluid's rotation and flow, as well as to alter shape, contract and expand. The majority of real-world issues involving these working fluids are non-isothermal, and the development of thermally responsive fluids has opened up new application fields. In many of these applications, convection is a dominant and significant mechanism of heat transfer.
Researchers have paid a lot of attention in recent years to the study of Rayleigh-Benard convection in porous layers when the fluid and solid phases are not in a local thermal equilibrium because the assumption of local thermal equilibrium is insufficient for many practical applications involving high-speed flows or significant temperature differences between the fluid and solid phases, and it is crucial to take thermal non-equilibrium effects into account. It is anticipated that local thermal non-equilibrium theory will play a significant role in future developments due to the applications of porous media theory in drying, freezing, and other commonplace materials as well as applications in everyday technology like microwave heating and rapid heat transfer from computer chips via use of porous metal foams and their use in heat pipes.
A two field model for the energy equation has been studied by Nield and Bejan [2006]. Two equations are utilized for the fluid and solid phases, as opposed to a single energy equation that describes the common temperature of the saturated porous media. In a two-field model, the terms that take into consideration heat loss or gain from one phase are connected to the energy equations. In a number of investigations, Rees and colleagues [1999, 2000, 2002] have examined the impact of thermal non-equilibrium (LTNE) on free convective fluxes in porous media. Detailed information on the works on thermal non-equilibrium effects is provided in the review of Kuznetsov [1998].

In this paper we are intended to study the effect of anisotropy and local thermal non-equilibrium on the onset of convection in a couple stress fluid saturated anisotropic porous layer heated from below. Our objective in this paper is to study how the onset criterion is affected by the combined effect of anisotropy and thermal non-equilibrium in steady cases. The effect of couple stress parameter on the stability is also presented. We have also carried out the asymptotic analysis for very small and very large values of the inter phase heat transfer coefficient.

Mathematical Formulation

We consider a horizontal couple stress fluid saturated anisotropic porous layer of depth d, which is heated from below and cooled from above. The lower surface is held at a temperature T_1, while the upper surface is at T_u. We assume that the solid and fluid phases of the medium are not in local thermal equilibrium and use a two-field model for temperatures. It is assumed that at the bounding surfaces the solid and fluid phases have identical temperatures. The Darcy model is employed for the momentum equation. The basic governing equations are

$$\nabla.\mathbf{q} = 0 , \qquad (1)$$

$$\rho_f\left[\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\varepsilon^2}\mathbf{q}.\nabla\mathbf{q}\right]=-\nabla p-\frac{1}{K}(\mu_f-\mu_c\nabla^2)\mathbf{q}+\rho_f\mathbf{g} \tag{2}$$

$$\varepsilon(\rho c)_f\frac{\partial T_f}{\partial t}+(\rho c)_f\,\boldsymbol{q}.\nabla T_f=\varepsilon k_{fh}\nabla^2 T_f+h(T_s-T_f), \tag{3}$$

$$(1-\varepsilon)(\rho c)\frac{\partial T}{\partial t}=(1-\varepsilon)k\nabla T-h(T-T), \tag{4}$$

$$\rho_f=\rho_o[1-\beta(T_f-T_u)]. \tag{5}$$

We eliminate the pressure from the momentum equation and render the resulting equation and the energy equations for fluid phase and solid phase dimensionless by using the following transformations.

$$(x,y)=d(x^*,y^*),\qquad (u,v,w)=\frac{\varepsilon k_f}{(\rho c)_f d}(u^*,v^*,w^*),\quad p=\frac{k_f\mu}{(\rho c)_f K}p^* \tag{6}$$

$$T_f=(T_1-T_u)\theta+T_u,\qquad T_s=(T_1-T_u)\phi+T_u,\qquad t=\frac{(\rho c)_f d^2}{k_f}t^*$$

to obtain

$$\frac{1}{\xi}\frac{\partial^2 w}{\partial z^2}+\nabla_1^2 w=Ra\nabla_1^2 w, \tag{7}$$

$$\frac{\partial T_f}{\partial t}+(\boldsymbol{q}.\nabla)T_f=\eta_f\nabla_1^2 T_f+\frac{\partial^2 T_f}{\partial z^2}+H(T_s-T_f), \tag{8}$$

$$\alpha\frac{\partial T_s}{\partial t}=\eta_s\nabla^2 T_s+\frac{\partial^2 T_s}{\partial z^2}-\gamma H(T_s-T_f) \tag{9}$$

where,

$$Ra=\frac{\rho_f g\beta(T_1-T_u)K_z d}{\varepsilon\mu_f\kappa_f},\qquad \gamma=\frac{\varepsilon k_f}{(1-\varepsilon)k_s},\qquad H=\frac{hd^2}{\varepsilon k_{fz}}, \tag{10}$$

$$\alpha=\frac{(\rho c)_s}{(\rho c)_f}\frac{k_f}{k_s}=\frac{\kappa_f}{\kappa_s},\quad \eta_s=\frac{k_{sh}}{k_{sz}},\quad \eta_f=\frac{k_{fh}}{k_{fz}},\quad \xi=\frac{\kappa_x}{\kappa_z},C=\frac{\mu_e}{\mu_f d^2}.$$

(The asterisks have been dropped for simplicity)

Basic State

The basic state is assumed to be quiescent and is given by

$$u=v=w=0,\qquad T_f=T_{fb}(z),\qquad T_s=T_{sb}(z). \tag{11}$$

The basic state temperatures of fluid phase and solid phase satisfy the equations

$$\frac{d^2 T_{fb}}{dz^2}+H(T_{sb-}T_{fb})=0, \tag{12}$$

$$\frac{d^2 T_{sb}}{dz^2}-\gamma H(T_{sb-}T_{fb})=0 \tag{13}$$

with boundary conditions

$$T_{fb} = T_{sb} = 1 \quad at\, z = 0,$$
$$T_{fb} = T_{sb} = 0 \quad at\, z = 1 \tag{14}$$

so that the conduction state solutions are given by

$$T_{fb} = T_{sb} = (1-z). \tag{15}$$

The perturbed state

The basic state is perturbed and the quantities in the perturbed state are given by

$$(u,v,w) = (u',v',w'), \quad T_f = T_{fb} + \theta, \quad T_s = T_{sb} + \phi. \tag{16}$$

Substituting the Equations (16) into Equations (7)-(9) and using the basic state solutions, we obtain the following equations for the perturbed quantities (after neglecting the primes)

$$\frac{\partial^2 w}{\partial z^2} + \xi \nabla_1^2 w - \xi C \nabla^4 w = Ra \xi \nabla_1^2 \theta, \tag{17}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{q}.\nabla \theta + w = \eta_f \nabla_1^2 \theta + \frac{\partial^2 \theta}{\partial z^2} + H(\phi - \theta), \tag{18}$$

$$\frac{\partial \phi}{\partial t} = \eta_s \nabla_1^2 \phi + \frac{\partial^2 \phi}{\partial z^2} - \gamma H(\phi - \theta). \tag{19}$$

Since the fluid and solid phases are not in local thermal equilibrium, the use of appropriate thermal boundary conditions may pose a difficulty. However, the assumption that the solid and fluid phases have equal temperatures at the bounding surfaces made at the beginning of this section helps in overcoming this difficulty. Accordingly, Equations (17) to (19) are solved for impermeable isothermal boundaries. Hence the boundary conditions are

$$w = 0 \quad \text{at} \quad z = 0,\ 1, \tag{20a}$$
$$\theta = \phi = 0 \quad \text{at} \quad z = 0,\ 1. \tag{20b}$$

Linear Stability Theory

To study the linear stability theory, we use the linearized version of equations (17)-(19). The principle of exchange of stabilities holds in the presence of anisotropy and non-LTE effects (there is only one destabilizing agency) so that the onset of convection is stationary.

We seek the solutions to the linearized equations in the form

$$(w\ ,\theta,\ \phi) = [A_1,\ A_2,\ A_3]\exp(ilx + imy)\sin \pi z, \tag{21}$$

where A's are constants. Substituting the equations (21) in equations (17) – (19) we obtain the following matrix equation

$$\begin{pmatrix} \pi^2 + a^2\xi + C\xi(a^2+\pi^2)^2 & -Ra\xi a^2 & 0 \\ 1 & -(\pi^2 + a^2\eta_f + H) & H \\ 0 & \gamma H & -(\pi^2 + \eta_s a^2 + \gamma H) \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{22}$$

By setting the determinant of the above matrix to zero we get

$$Ra = \frac{(\pi^2 + \xi a^2 + C\xi(a^2+\pi^2)^2)\left(\left(\pi^2 + \eta_f a^2\right)\left(\pi^2 + \eta_s a^2\right) + H\left(\gamma\left(\pi^2 + \eta_f a^2\right) + \pi^2 + \eta_s a^2\right)\right)}{\xi a^2(\pi^2 + \eta_s a^2 + H\gamma)} \tag{23}$$

For given values of H, γ, ξ, C, η_f and η_s Equation (23) describes the neutral curves for the onset of thermal convection. The value of Ra given in Equation (23) may be minimized with respect to a, and, although condition for extreme value may be written down, it is difficult to obtain a closed-form explicit expression for the minimizing value of a. Therefore, we used the Newton–Raphson iteration scheme to obtain the critical values of Ra and a as functions of H, γ, ξ, η_f and η_s.

When $\xi = \eta_f = \eta_s$ =1 and C=0 (i.e. for the isotropic porous medium with non-LTE effects), Equation (23) reduces to

$$Ra = \frac{(\pi^2 + a^2)^2}{a^2}\left[\frac{(\pi^2 + a^2) + H(1+\gamma)}{\pi^2 + a^2 + \gamma H}\right]. \tag{24}$$

This expression for the Rayleigh number is same as the one given by Banu and Rees (2002) for the non-LTE isotropic case.

Further when $H \to \infty$ (i.e in the LTE limit) the above expression (24) reduces to

$$Ra = Ra(\frac{\gamma}{(1+\gamma)}) = \left(\frac{\pi^2 + a^2}{a}\right)^2. \tag{25}$$

Note that the expression on the LHS simplifies to

$$Ra\left(\frac{\gamma}{1+\gamma}\right) = \frac{\rho_f\, g\,\beta(T_1 - T_u)Kd}{[\varepsilon\kappa_f + (1-\varepsilon)\kappa_s]\mu_f} \tag{26}$$

which is the expression for the Rayleigh number now based on the mean properties of the porous medium. In fact it is this value, which is used in the local thermal equilibrium case. Equation (25) gives the critical values for the Rayleigh number and the wave number for the onset of convection as $4\pi^2$ and π, respectively which are the classical Darcy-Benard results. In the limit of $H \to \infty$ and in the presence of anisotropic effects, equation (23) reduces to

$$Ra = \frac{(\pi^2 + \xi a^2 + C\xi(a^2 + \pi^2)^2)\left(\pi^2 + \eta a^2\right)}{\xi a^2} \tag{27}$$

where $\eta_f = \eta_s = \eta$ for the LTE case. The Rayleigh number given by Eq. (27) attains the critical value for the wave number $a_c = \sqrt{x}$, which satisfies the equation

$$-\pi^4 - C\pi^6\xi + 2x^3 C\eta\xi + x^2(C\pi^2\xi + \eta\xi + 2C\pi^2\eta\xi) = 0$$

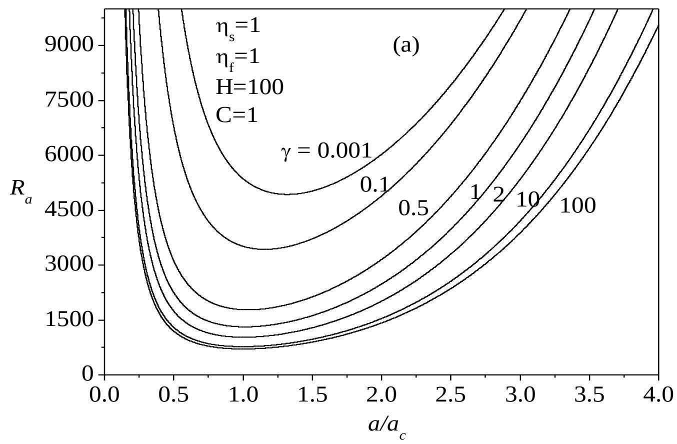
The expression for the critical Rayleigh number Ra_c and the critical wave number a_c for both small H and large H are evaluated and comparison of these values with the exact values obtained from Equation (23) are given in Table 1 & 2. It is important to note that an excellent agreement is found between these two results.

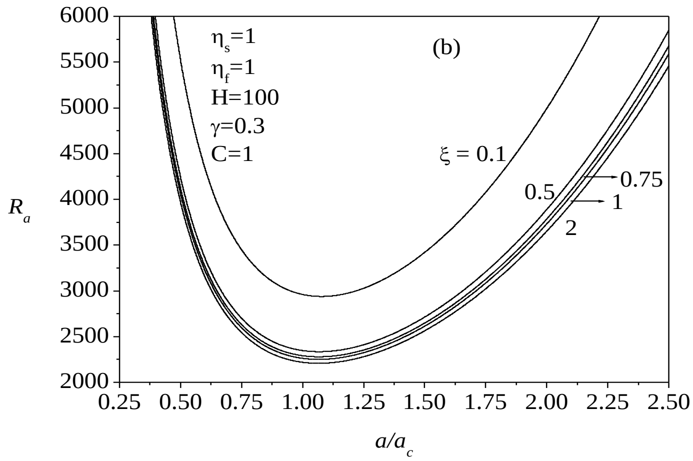
RESULTS AND DISCUSSION

In order to determine the impact of different values of physical factors on the commencement of convection, the expression for Rayleigh number provided by Equation (23) is numerically assessed. In Figures 1(a–d) the neutral curves for H=100 and for a range of values of the parameters γ, ξ, η_f, η_s and C are displayed. Figure 1(a) shows the neutral curves for the case of isotropic porous medium saturated with couple stress fluid (i.e. when $\xi = \eta_f = \eta_s$ =1) for different values of γ. It may be noted that, in general Ra_c and the minimum wave number decrease as γ increases, indicating that the effect of increasing γ is to destabilize the system. The effects of anisotropy on the onset of thermal convection are evident from Figures 1(b, d, e). In Figure 1(b) we show the effect of mechanical anisotropy parameter ξ ($= K_x/K_z$) on Rayleigh number *Ra* for a fixed values of γ =0.3 and $\eta_f = \eta_s$ =1.0. From this figure it is evident that increase in the value of ξ decreases Ra_c and thus augments the onset of convection. This may be understood as follows: let us keep the vertical permeability K_z fixed (or the horizontal permeability K_x fixed), and vary the horizontal permeability K_x (or the vertical permeability K_z). Then an increased horizontal permeability reduces the Rayleigh number, indicating that the system becomes unstable. The effect of the anisotropic parameter is more significant for ξ =0.1. In Figure1 (c) we display the effect of couple stress parameter on Rayleigh number. It is evident that with increase in the couple stress parameter decreases the Rayleigh number which shows that the couple stress parameter has stabilizing effect. The effect of the thermal anisotropy parameters η_f , and η_s for fluid and solid phases are shown in Figure 1(d) and 1(e), respectively for γ =0.3 and ξ =1.0 and C=1. The effect of the thermal anisotropy parameter for the fluid phase η_f is shown Figure 1(d). The effect of increasing η_f for fixed value of the othe, increases the critical Rayleigh number as well the minimizing wave number and thus delays the onset of convection. This is because increase in horizontal thermal conductivity of the fluid phase increases the stability of the system. Figure 1(d) shows the effect of thermal anisotropy parameter η_s for the solid phase. Similar effect is found as in the case of fluid phase.

Figures 2(a–d) shows the variation of critical Rayleigh number with interphase heat transfer coefficient *H* for a range of values of the parameters γ, ξ, η_f , η_s and C = 1. The variation of Ra_c with *H* for different values of γ for the case of isotropic porous medium is displayed in Figure 2 (a). We observe that the critical Rayleigh number is independent of γ for very small values of H while for large H critical Rayleigh number decreases with increase in γ. The physical reason for this is that there is almost no transfer of heat between the phases and therefore the condition for the onset of convection is not affected by the properties of the solid phase. The effect of mechanical anisotropy parameter ξ on the critical Rayleigh number is depicted in Fig 2(b) for γ =0.3, $\eta_f = \eta_s = 1$ and C=1. We found that the critical Rayleigh number decreases with increase in ξ .Further the value of R_c increases slowly with H reaches a maximum value and for large H R_c ultimately approaches to an asymptotic value depending on the value of ξ .The effect of thermal anisotropy parameter η_f of the fluid phase on R_c is shown in fig2(c) for $\gamma = 0.3,\ and\ \xi = \eta_s = 1\ and$ C=1. We find that an increase in the value of η_f increases the value of Ra_c indicating that the effect of increasing the thermal anisotropy parameter is to delay the onset of convection. Figure 2 (d) shows the effect of thermal anisotropy parameter η_s of the solid phase on Ra_c for γ =0.3 and $\eta_f = \xi$ =1.0. Its effect is found to be similar to that of η_f . However, for small values of *H*, Ra_c is found to be independent of η_s .

Fig 3(a-d) depicts the variation of the critical Rayleigh number based on the mean properties of the porous medium with the interphase heat transfer coefficient *H* for a range of values of the parameters γ, ξ , η_f and η_s . Figure 3(a) shows that $Ra_c\,\gamma/(1+\gamma)$ approaches to a common limit as $H\to\infty$ and this approach to common limit depends strongly on the value of γ . It is also important to note that $Ra_c\,\gamma/(1+\gamma)$, however has no common limit for anisotropic case as $H\to\infty$ (see Figure 3(b–d)) and the effect of ξ , η_f and η_s on the stability of the system is found to be akin to that of the previous case.





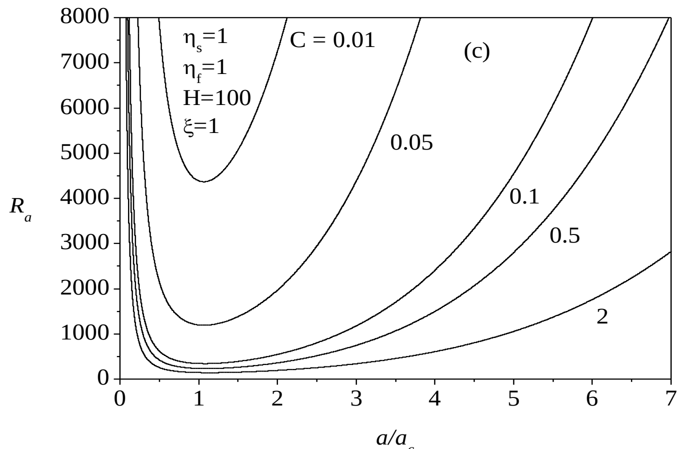
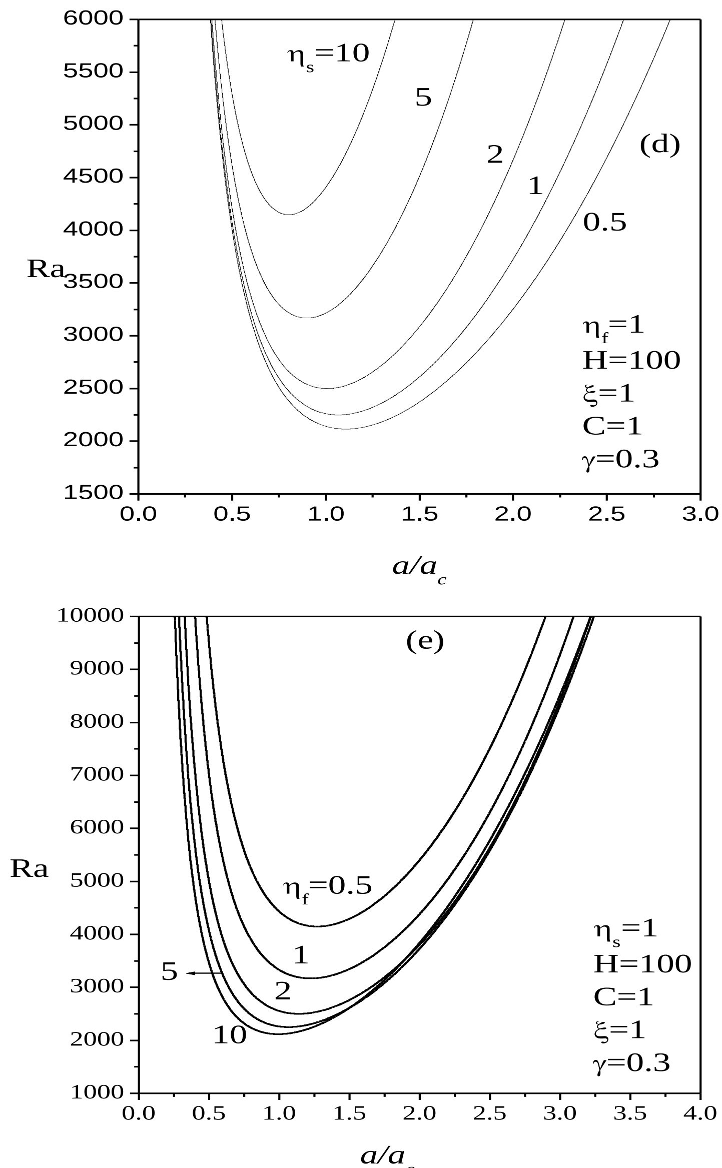


Figure 1: Neutral curves for different values of γ, ζ, C, η_s and η_f


(d)
η_s=10
5
2
1
0.5
η_f=1
H=100
ξ=1
C=1
γ=0.3
Ra
a/a_c
(e)
η_f=0.5
1
2
5
10
η_s=1
H=100
C=1
ξ=1
γ=0.3
Ra
a/a_c

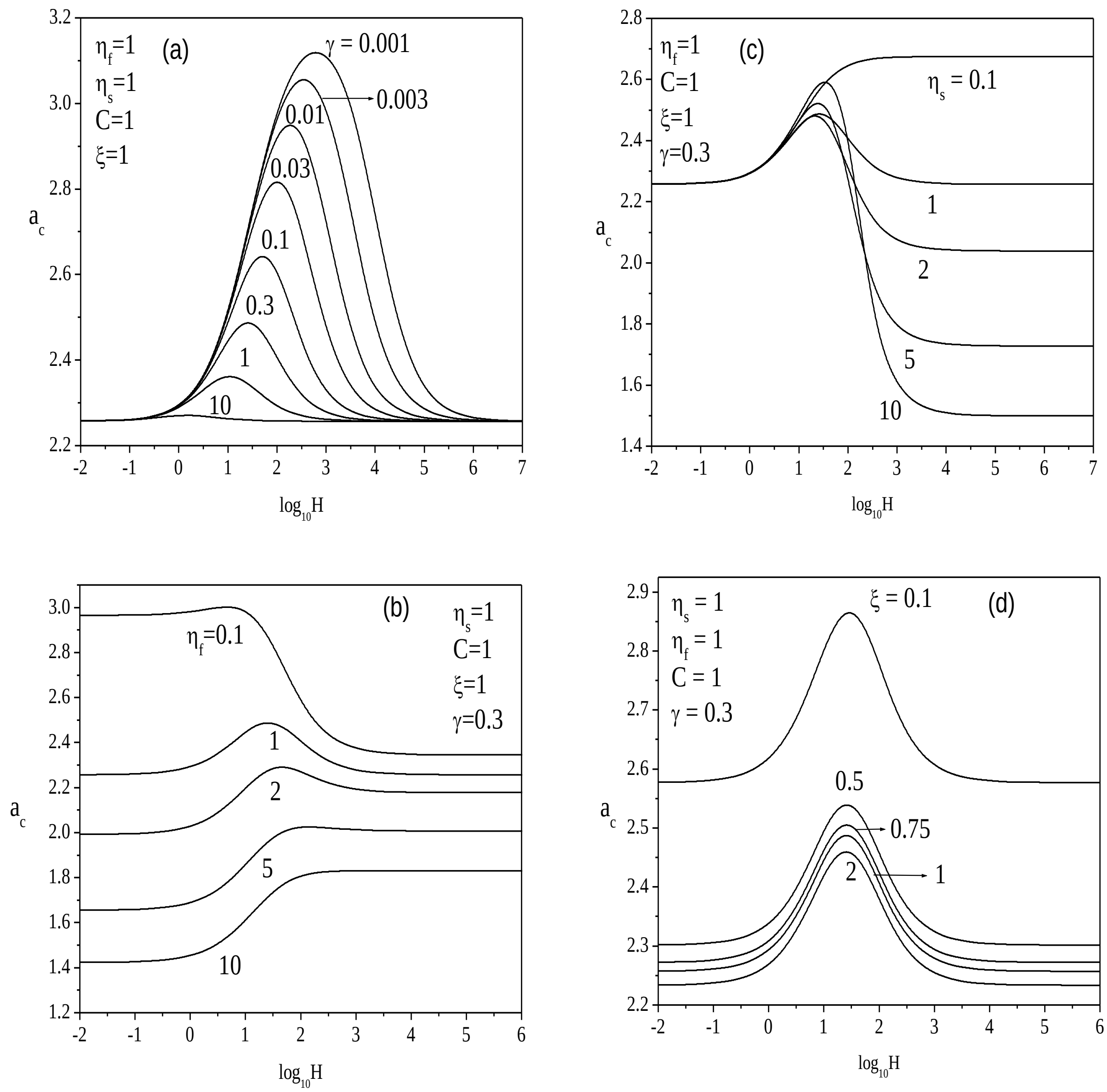


Figure 2. Variation of critical wave number with interphase heat transfer coefficient H for different values of γ, ζ, C, η_s and η_f

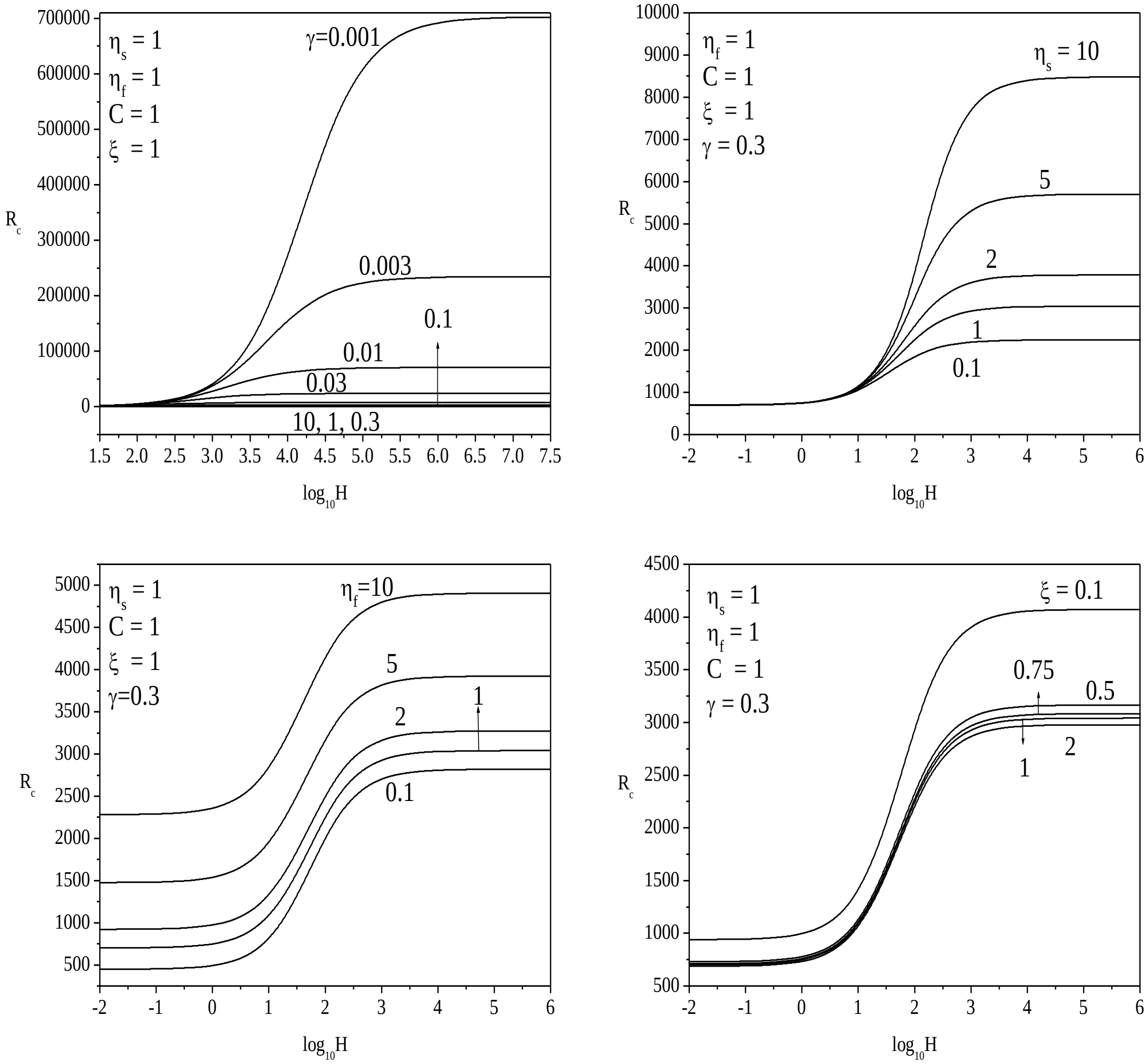


Figure 3. Variation of critical Rayleigh number with interphase heat transfer coefficient H for different values of γ, ζ, C, η_s and η_f

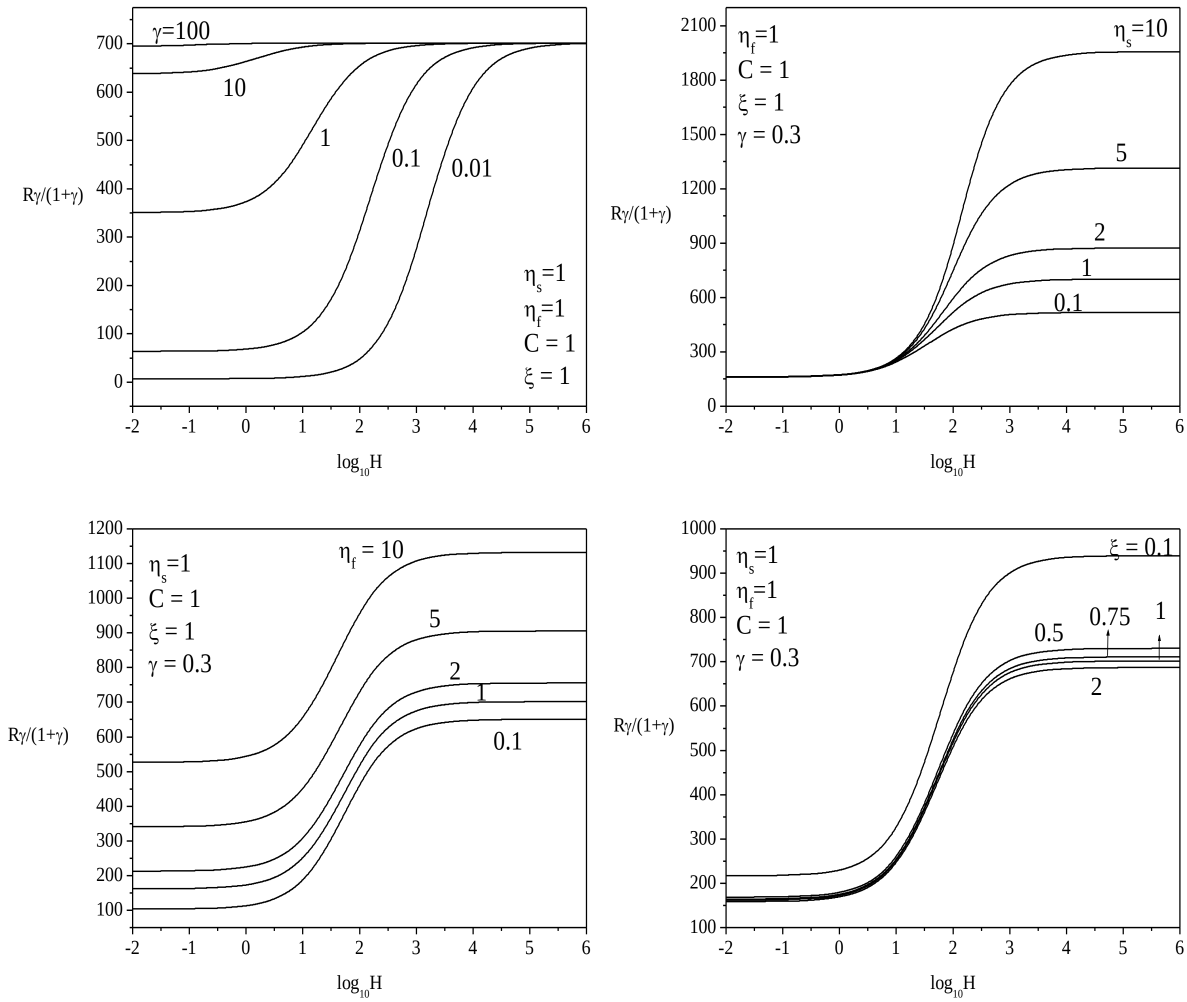


Figure4. Variation of critical Rayleigh number based on mean properties of the porous medium with interphase heat transfer coefficient H for different values of γ, ζ, C, η_s and η_f

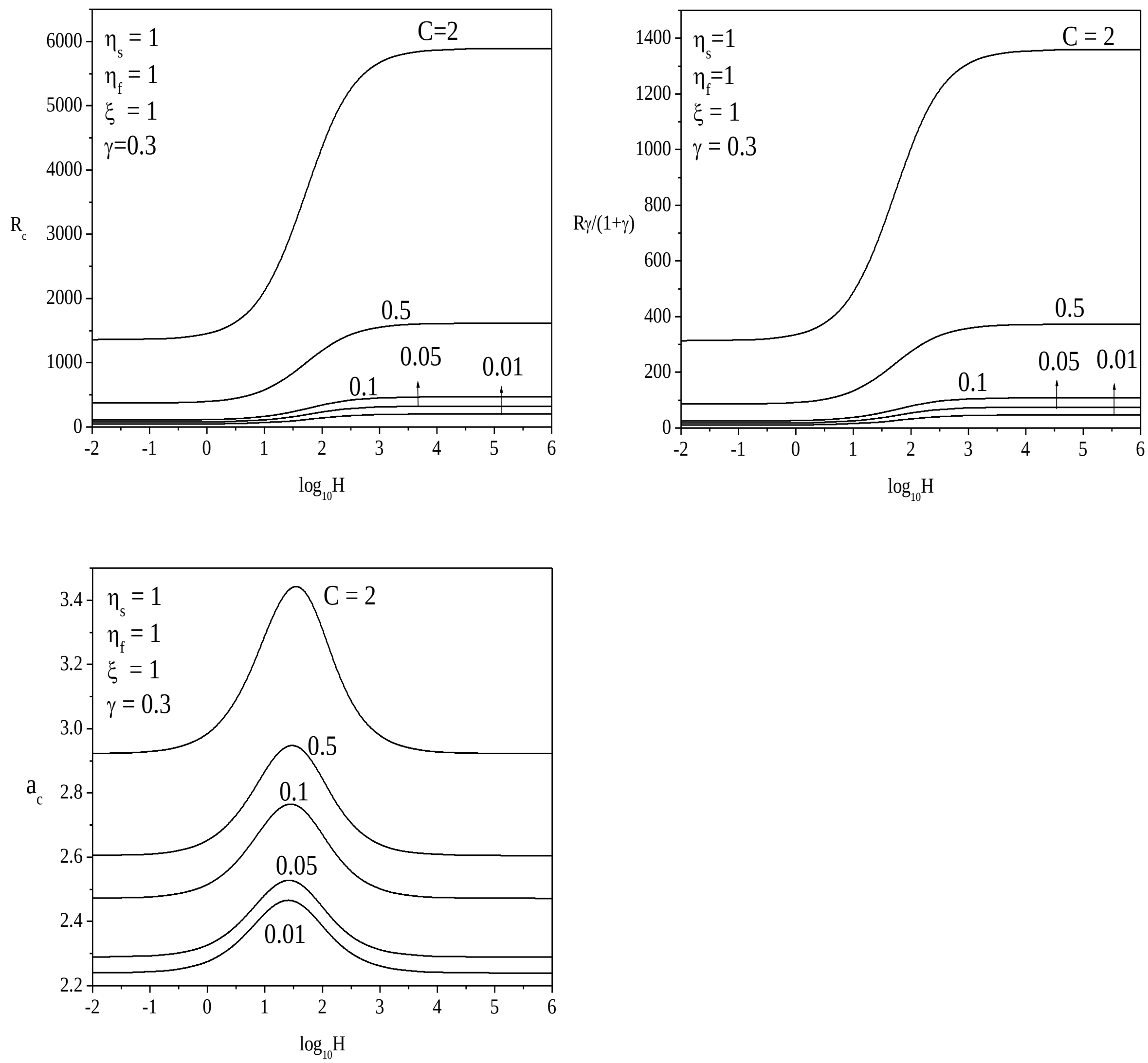


Figure5. Variation of critical Rayleigh number , critical Rayleigh number based on mean properties of the porous medium and critical wave number with interphase heat transfer coefficient H for different C

In Figures 4 (a–d) we display the critical wave number for a range of values of the parameters γ, ξ, η_f , η_s and C. We see that the critical wave number remains constant for very small and very large values of the interphase heat transfer coefficient *H*. The reason is that the solid phase ceases to affect the thermal field of the fluid when $H \to 0$ and on the other hand, the solid and fluid phases will have identical temperatures when $H \to \infty$. For the intermediate values of *H*, the critical wave number attains the maximum value for each values of the parameters γ, ξ, η_f and η_s . We also observe that in the absence of anisotropic effects

the critical wave number approaches a common limit $H \to 0$ and as $H \to \infty$ (see Figure 4(a)), while in the presence of anisotropic effects the critical wave number does not approach to a common limit.
The effect of the couple stress parameter on critical Rayleigh number, Rayleigh number based on mean properties of the porous medium and critical wave number is displayed in Fig 5(a)-(b). It is shown that the effect of increasing couple stress parameter is to increase the critical Rayleigh number indicating that the couple stress parameter has a stabilizing effect. Further, we observe that the effect of couple stress parameter is significant for moderate and large values of H and less significant for small values of H. The effect of the couple stress parameter on the critical wave number is shown in Figure 5(c). We find from this figure that critical wave number decreases with increase in the value of couple stress parameter.

CONCLUSION

Analytical analysis is done to determine the stability of an anisotropic, fluid-saturated, horizontal coupling stress porous layer that is heated from below and chilled from above when the solid and fluid phases are not in local thermal equilibrium. The momentum equation is represented by the Darcy model, while the energy equation is represented by the two-field model, which each independently represents the solid and fluid phases. Analytical methods are used to determine the prerequisite for convection to begin. For a variety of values of $\gamma, \eta_f, \eta_s, \xi, H \, and \, C$, we show the outcomes in Figures 1-4. It has been discovered that raising the conductivity ratio has the impact of lowering both the critical Rayleigh number and the critical wave number. Destabilizing the system is the result of raising. The critical Rayleigh number is independent of H for γ =10. The critical Rayleigh number is independent of γ for very small H while for large H, it decreases with increasing γ . The critical Rayleigh number decreases and the critical wave number rises as the mechanical anisotropy parameter is increased. The consequence of increasing the thermal anisotropy parameters is to raise the critical Rayleigh number for a certain value of H. The critical Rayleigh number is unaffected by the thermal anisotropy of the solid phase for very low values of H. For both small and large values of H, the critical wave number is unchanged, and for intermediate values, it reaches its maximum. The system becomes more stable as a result of the effect of increasing the couple stress parameter, which delays the commencement of convection.

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