International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2020 Vol. 10, pp. 50-58/Mathur and Parihar
Research Article

STEADY LAMINAR FREE CONVECTION FLOW OF AN ELECTRICALLY CONDUCTING FLUID ALONG A MOVING POROUS HOT VERTICAL NON-CONDUCTING PLATE IN THE PRESENCE OF CONSTANT HEAT FLUX

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ABSTRACT
In this paper steady laminar free convection flow of an electrically conducting fluid along a moving porous vertical non-conducting hot plate in the presence of sink and constant heat flux is investigated. The governing equations of motion are solved by using a regular perturbation technique. Velocity distribution, Temperature distribution flow past a heated plate and cooled plate is presented through figures and values of skin friction coefficient with respect to non-dimensional parameter is presented in table

Keywords: *Free convection flow, Electrically conducting fluid, Heat flux*

INTRODUCTION

The problem of free convection flow of an electrically conducting fluid past a vertical plate under the influence of a magnetic field attracted many scientists, in view of its applications in Astrophysics, Geophysics, Engineering and Aerodynamics etc.

Laminar natural convection flow and heat transfer in fluid flows with and without heat source in channels with constant wall temperature was discussed by Ostrach (1952). An analysis of laminar free convection flow and heat transfer on a flat plate parallel to the direction of generating body force was studied by Ostrach (1953). Combined natural and forced convection laminar flow and heat transfer in fluids, with and without heat source through channels, with linearly varying wall temperature was investigated by Ostrach (1954). Free convection effects on the Stokes problem for infinite vertical plate was investigated by Soundalgekar (1977). Raptis *et al.*, (1981) discussed the flow of Walter's fluid past an infinite plate with suction, constant heat flux between fluid and plate, taking into account the influence of visco-elastic fluid on the energy equation.

Sharma (1991) investigated free convection effect on the flow past an infinite vertical, porous plate with constant suction and constant heat flux. The aim of the present study to investigate MHD free convective flow long an infinite vertical porous plate in the presence of sink and constant heat flux.

1.2 FORMUATION OF THE PROBLEM

The x*-axis is taken along a moving porous plate in the upwards direction, y*-axis is normal to it and a transverse constant magnetic field is applied i.e. in the direction of y*-axis. Since the motion is two-dimensional and the length of the plate is large, therefore, all the physical variables are independent of x*. The governing equations of continuity, motion and energy for a steady laminar free convection flow of an electrically conducting fluid Bansal (1994), along a hot, non-conducting porous moving vertical plate in the presence of heat sink and heat flux are given by

Equation of continuity

$$\frac{\partial v^*}{\partial y^*}=0 \qquad \Rightarrow \qquad \text{v* is independent of y*}$$

i.e. $v^*=-v_0$ (constant), $v_0>0$ …..(1.2.1)

Equations of motion

$$\rho v^*\frac{\partial u^*}{\partial y^*}=\mu\frac{\partial^2 u^*}{\partial y^{*2}}+\rho g\beta\left(T^*-T_\infty\right)-\sigma B_0^2 u^* \qquad \text{…..(1.2.2)}$$

$$\frac{\partial p^*}{\partial y^*}=0 \qquad \Rightarrow \qquad \text{p* is independent of y*} \qquad \text{…..(1.2.3)}$$

i.e. p* is constant

Equation of energy

$$\rho C_p v^*\frac{\partial T^*}{\partial y^*}=k\frac{\partial^2 T^*}{\partial y^{*2}}+\mu\left(\frac{\partial u^*}{\partial y^*}\right)^2+S^*\left(T^*-T_\infty\right) \qquad \text{…..(1.2.4)}$$

Where u* and v* are the velocity components of fluid along x*- and y*-axes respectively, ρ the density of the fluid, v_0 the cross-flow velocity, μ the coefficient of viscosity, g the acceleration due to gravity, β the coefficient of volumetric expansion σ the magnetic permeability, B_0 the magnetic field coefficient, p* the pressure, T* the temperature, T_∞ free stream temperature and S* sink parameter.

The corresponding boundary conditions are

$$y^*=0;\ u^*=U,\ \frac{\partial T^*}{\partial y^*}=-\frac{q}{k},$$

$$y^*\to\infty;\ u^*\to 0,\ T^*\to T_\infty, \qquad \text{…..(1.2.5)}$$

Where, U is the plate velocity and q is the heat flux.

1.3 METHOD OF SOLUTION

Introducing the following dimensionless quantities

$$y=\frac{y^* v_0}{\upsilon},\ u=\frac{u^*}{U},\ V=\frac{v^*}{v_0}\ \ \theta=\frac{(T^*-T_\infty)kv_0}{q\upsilon},\ M=\left(\frac{\sigma B_0^2\upsilon^2}{\mu v_0^2}\right)^{1/2},$$

$$Gr=\frac{q^2\beta\upsilon^2}{\mu v_0^3 U},\ \ Ec=\frac{kv_0 U^2}{\upsilon C_p q},\ \ \Pr=\frac{\mu C_p}{k},\ \ S=\frac{S^*\upsilon^2}{kv_0^2} \qquad \text{…..(1.3.1)}$$

Into the equations (1.2.2) and (1.2.4) we get

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}-M^2u+Gr\theta=0 \qquad \text{…..(1.3.2)}$$

$$\frac{\partial^2\theta}{\partial y^2}+\Pr\frac{\partial\theta}{\partial y}+S\Pr\theta+\Pr Ec\left(\frac{\partial u}{\partial y}\right)^2=0 \qquad \text{…..(1.3.3)}$$

The corresponding boundary conditions are

$$y=0;\ \ u=0,\ \frac{\partial\theta}{\partial y}=-1,$$

$$y \to \infty; \quad u \to 0, \ \theta \to 0 \qquad .....(1.3.4)$$

where M is the Hartmann number, Pr is the Prandtl number, Ec is the suction Eckert number and S is the sink parameter. Since the suction Eckert number-Ec is very small for incompressible flow, therefore following Soundalgekar (1977) the physical variables u and θ can be expended in the powers of Ec, as given by

$$u(y) = u_0(y) + Ecu_1(y) + O(Ec^2) \text{ , and} \qquad .....(1.3.5)$$

$$\theta(y) = \theta_0(y) + Ec\theta_1(y) + O(Ec^2) \text{ ,} \qquad .....(1.3.6)$$

Using (1.3.5) and (1.3.6) into the equations (1.3.2) and (1.3.3) and equating the coefficient of the like powers of Ec, we have

Zeroth order

$$u''_0 + u'_0 - M^2 u_0 = -Gr\theta_0 \text{,} \qquad .....(1.3.7)$$

$$\theta''_0 + \Pr\theta'_0 + S\Pr\theta_0 = 0 \text{,} \qquad .....(1.3.8)$$

First order

$$u''_1 + u'_1 - M^2 u_1 = -Gr\theta_1 \text{,} \qquad .....(1.3.9)$$

$$\theta''_1 + \Pr\theta'_1 + S\Pr\theta_1 + \Pr u'^2_0 = 0 \text{,} \qquad .....(1.3.10)$$

Where, prime denotes the derivative with respect to *y*.
Now, the corresponding boundary conditions are

$$y = 0: \quad u_0 = 1, \ u_1 = 0, \ \frac{\partial\theta_0}{\partial y} = -1, \ \frac{\partial\theta_1}{\partial y} = 0,$$

$$y \to \infty: \quad u_0 \to 0, \ u_1 \to 0, \ \theta_0 \to 0, \ \theta_1 \to 0 \qquad .....(1.3.11)$$

The equation (1.3.7) to (1.3.10) are ordinary coupled differential equations in u_0, θ_0, u_1 and θ_1 with the boundary conditions (1.3.11). Through straight forward algebra, the solutions of u_0, u_1, θ_0 and θ_1 are known and given by

$$u_0(y) = A_6 e^{-A4y} - A_5 e^{-A_1 y} \qquad .....(1.3.12)$$

$$u_1(y) = A_{22} e^{-A4y} - A_{18} e^{-A1y} + A_{19} e^{-A8y} + A_{20} e^{-A10y} - A_{21} e^{-A12y} \qquad .....(1.3.13)$$

$$\theta_0(y) = A_2 e^{-A1y} \qquad .....(1.3.14)$$

$$\theta_1(y) = A_{17} e^{-A1y} - A_{14} e^{-A8y} - A_{15} e^{-A10y} + A_{16} e^{-A12y} \qquad .....(1.3.15)$$

where A_1 to A_{22} are constants, and their expressions are as follows

$$A_1 = \frac{\Pr + \sqrt{\Pr^2 - 4S\Pr}}{2}, \qquad A_2 = \frac{1}{A_1},$$

$$A_3 = \frac{1}{2}\left(1 - \sqrt{1 + 4M^2}\right), \qquad A_4 = \frac{1}{2}\left(1 + \sqrt{1 + 4M^2}\right),$$

$$A_5 = \frac{GrA_2}{(A_3 - A_1)(A_4 - A_1)}, \qquad A_6 = 1 + A_5,$$

$$A_7 = A_6^2 A_4^2, \qquad A_8 = 2A_4,$$

$$A_9 = A_5^2 A_1^2, \qquad A_{10} = 2A_1,$$

$$A_{11} = 2A_6 A_4 A_5 A_1, \qquad A_{12} = A_4 + A_1,$$

$$A_{13} = \frac{\Pr - \sqrt{\Pr^2 - 4S\Pr}}{2}, \qquad A_{14} = \frac{\Pr A_7}{(A_{13} - A_8)(A_1 - A_8)},$$

$$A_{15} = \frac{\Pr A_9}{(A_{13} - A_{10})(A_1 - A_{10})}, \qquad A_{16} = \frac{\Pr A_{11}}{(A_{13} - A_{12})(A_1 - A_{12})},$$

$$A_{17} = \frac{1}{A_1}(A_{14}A_8 + A_{15}A_{10} - A_{16}A_{12}), \qquad A_{18} = \frac{GrA_{17}}{(A_3 - A_1)(A_4 - A_1)},$$

$$A_{19} = \frac{GrA_{14}}{(A_3 - A_8)(A_4 - A_8)}, \qquad A_{20} = \frac{GrA_{15}}{(A_3 - A_{10})(A_4 - A_{10})},$$

$$A_{21} = \frac{GrA_{16}}{(A_3 - A_{12})(A_4 - A_{12})}, \quad \text{and} \quad A_{22} = A_{18} - A_{19} - A_{20} + A_{21}.$$

1.4 Skin-Friction

The skin-friction coefficient at the plate is given by

$$C_r = \left(\frac{\tau_w}{\rho U_0 V_0}\right)_{y*=0} = \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$= -A_6A_4 + A_5A_1 + Ec(-A_{22}A_0 + A_{18}A_1 - A_{19}A_8 - A_{20}A_{10} + A_{21}A_{12}) \ldots\ldots(1.4.1)$$

1.5 NUSELT NUMBER

The rate of heat transfer in terms of Nusselt number at the plate is given by

$$Nu = -\left(\frac{V_0/\upsilon}{T_w^* - T_\infty}\right)\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = 1.0 \qquad \ldots..(1.5.1)$$

RESULTS AND DISCUSSION

When Gr > 0 i.e. flow past a heated plate

It is observed from the Fig-1 that the fluid velocity increases due to increase in Grashoff number, while it decreases with the increase of Hartmann number, suction Eckert number or Prandtl number keeping other parameters fixed which is qualitatively in agreement with the physical requirements of the phenomenon, It is noted from the figure Fig-2 that the fluid temperature increases with the increase of Hartmann number, while it decreases with the increase in suction Eckert number, Prandtl number of Grashoff number keeping other parameters fixed. It is seen from the Table-1 that the skin-friction coefficient increases with the increase of Grashoff number, while it decreases wue to increase of Hartmann number, suction Eckert number or Prandtl number keeping other parameters fixed.

When Gr < 0 i.e. flow past a cooled plate

Fig-3 depicts that the fluid velocity increases with the increase of suction Eckert number or Prandtl number, while it decreases with the increase of Hartmann number or Grashoff number. The fluid temperature decreases with the increase of Grashoff number or Hartmann number, suction Eckert number or Prandtl number keeping other parameters fixed as observed from the Fig-4. It is noticed from the Table-1 that the skin-friction coefficient increases with the increase of suction Eckert number or Prandtl number, while it decreases with the increase of Hartmann number or Grashoff number keeping other parameters fixed.

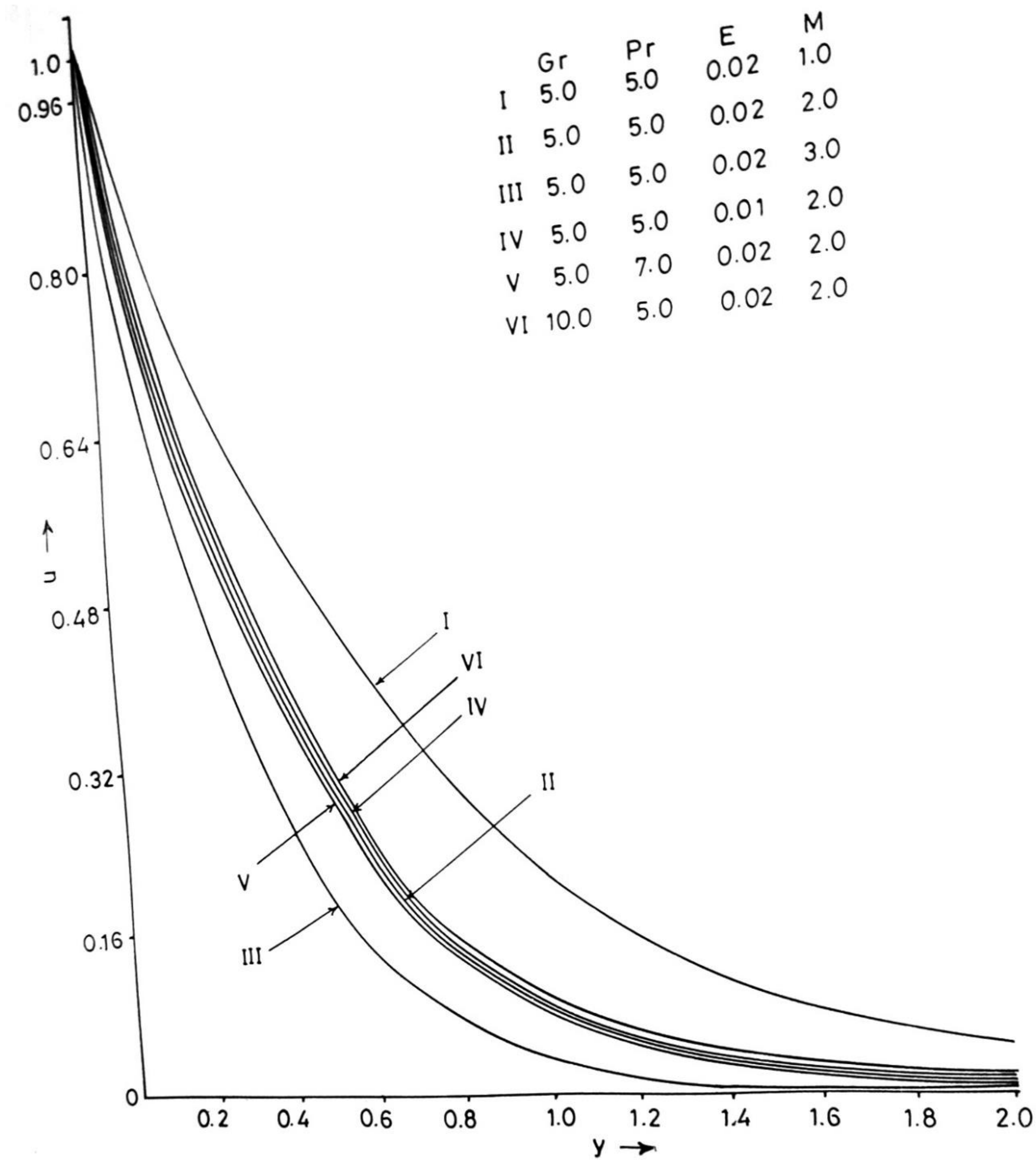

Fig-1: Velocity distribution versus y when S= – 0.5

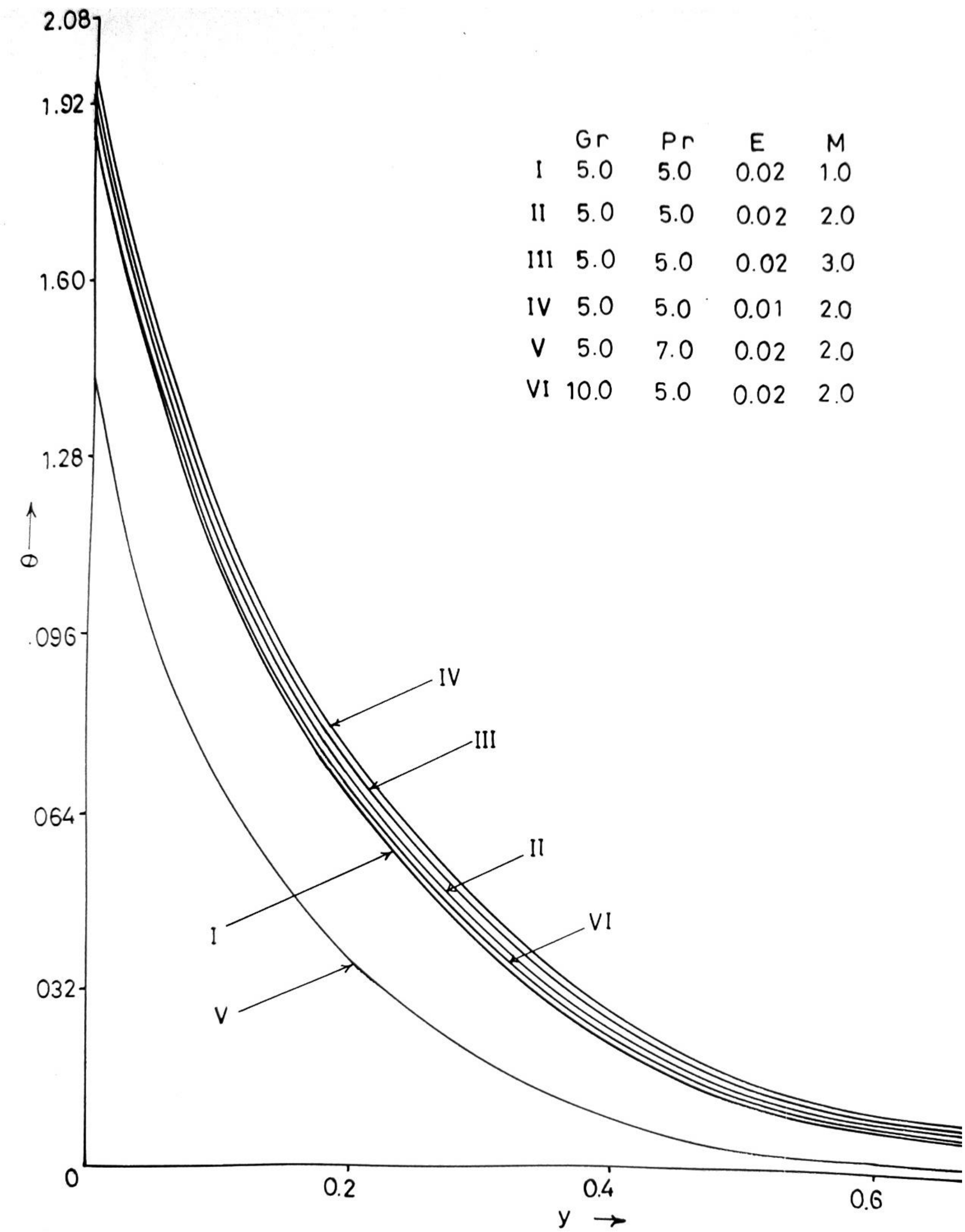

Fig-2: Temperature distribution versus y when S= – 0.5

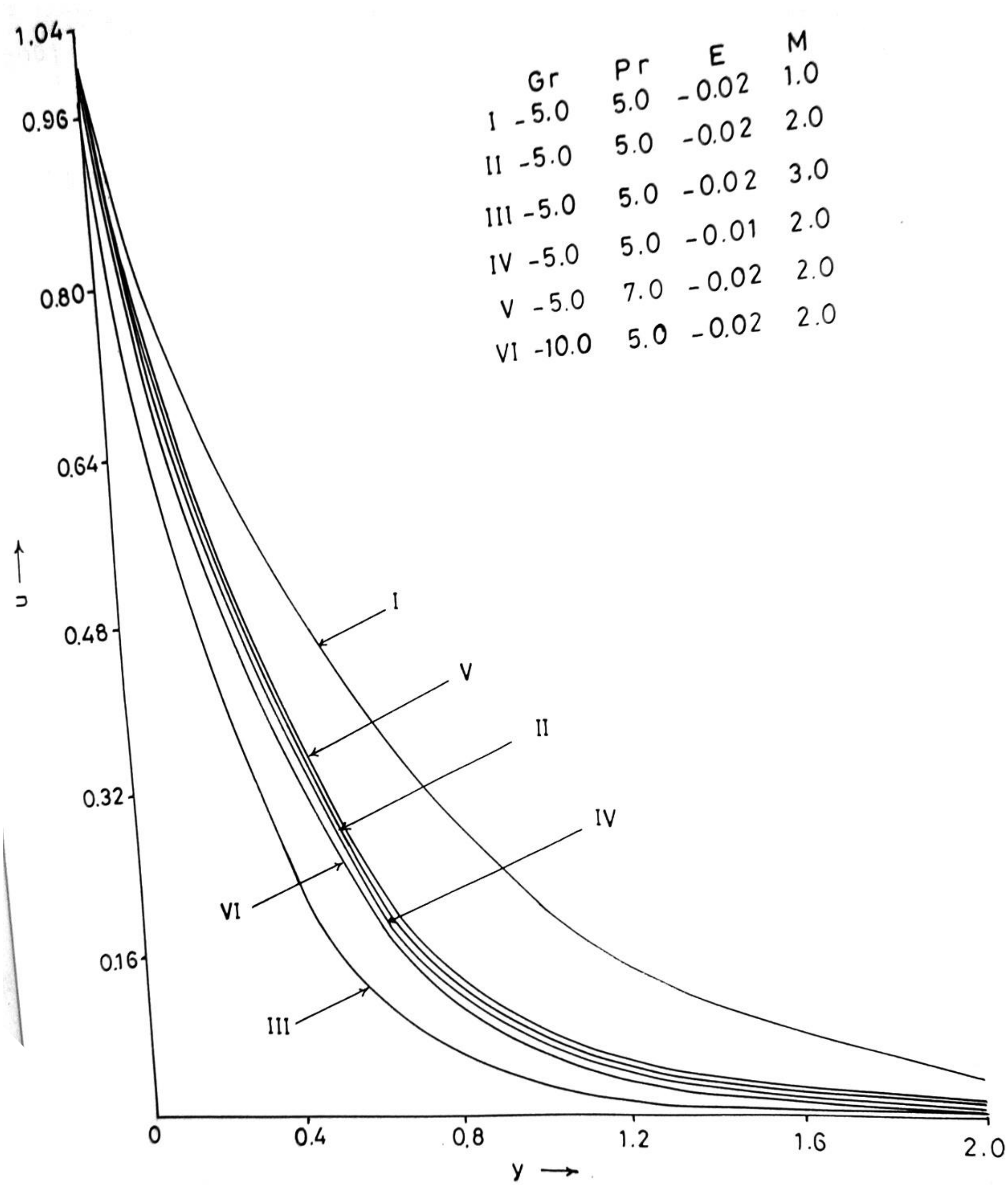

Fig-3: Velocity distribution versus y when S= – 0.5

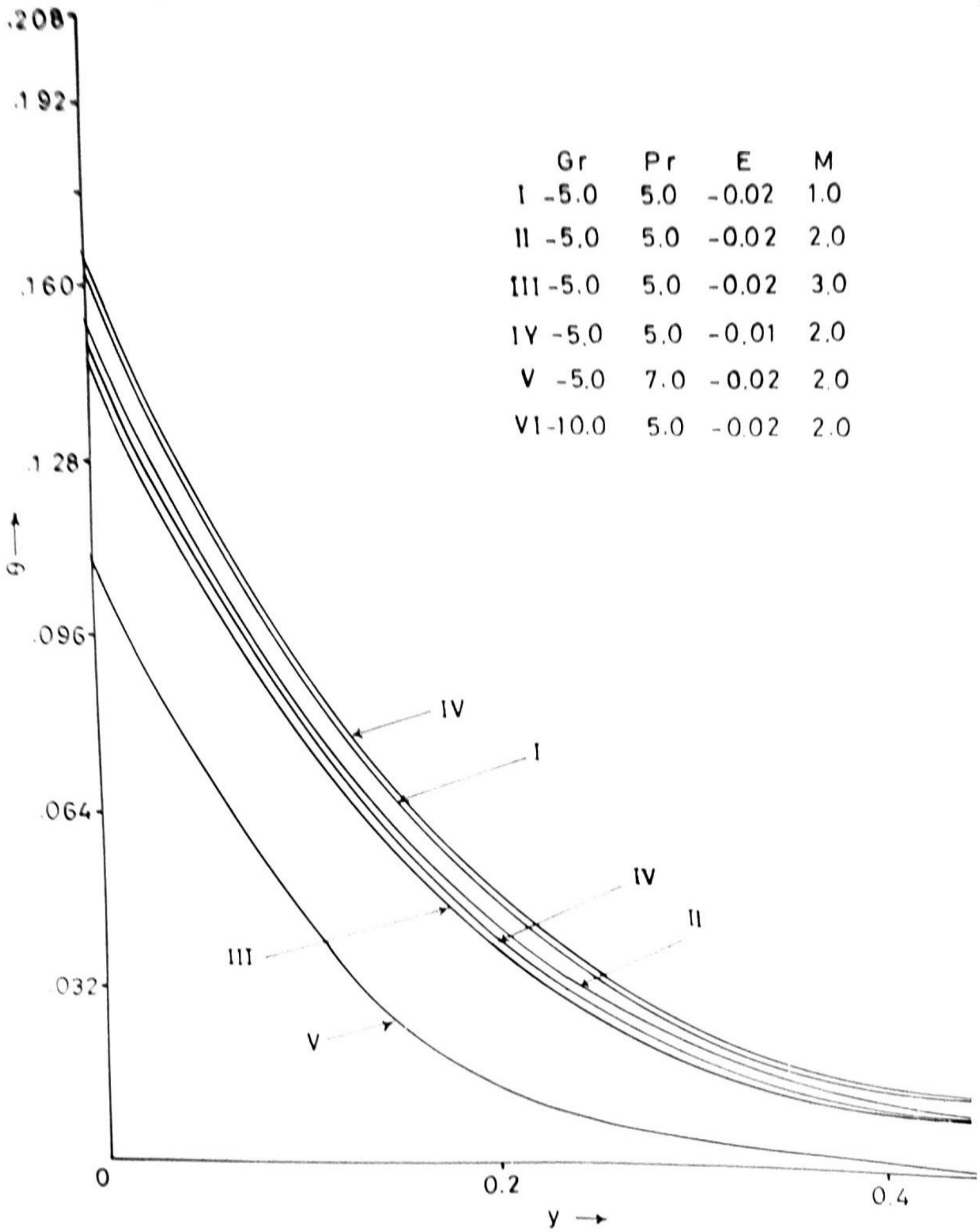

Fig-4: Temperature distribution versus y when S= – 0.5

Table 1: Values of Skin Friction Coefficient C_f at the plate when S = – 0.5

S.No.	Gr	Pr	Ec	M	Cr
1	5.0	5.0	0.02	1.0	-1.454512
2	5.0	5.0	0.02	2.0	-2.417371
3	5.0	5.0	0.02	3.0	-3.412210
4	5.0	5.0	0.01	2.0	-2.403693
5	5.0	7.0	0.02	2.0	-2.474601
6	10.0	5.0	0.02	2.0	-2.273665
7	-5.0	5.0	-0.02	1.0	-1.742604
8	-5.0	5.0	-0.02	2.0	-2.663635
9	-5.0	5.0	-0.02	3.0	-3.625489
10	-5.0	5.0	-0.01	2.0	-2.677845
11	-5.0	7.0	-0.02	2.0	-2.609738
12	-10.0	5.0	-0.02	2.0	-2.764524

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