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Research Article

FUNDAMENTALS OF FLUIDS, FLOW AND THE CONSTITUTIVE EQUATIONS

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ABSTRACT
This paper has discussed about types of fluids like Ideal fluid, Real Fluid, Newtonian and Non-Newtonian fluid their basic property of compressibility. Also, types of Flow and Fundamental Constitutive Equations used in mathematical modelling of flow discussed in brief to provide insight of this field.

Keywords: *Fluid, Newtonian and Non-Newtonian fluid, Flow*

INTRODUCTION
The formation of universe is based on composition of matter, Generally, the matter is classified in three forms viz. solid, liquid, and gas. One further advance stage is plasma. Liquid and gas form of matter combinedly known as fluid. Even in Indian mythology the world is made up of five elements (actually matter). Out of these five constituents, fluids like air and water are in the roots of origin of life on earth. Study of behaviour of fluid, therefore attracted the scientists during the development of science and civilization. The science of fluids is known as fluid dynamics. In the beginning the fluid was considered as Ideal (perfect) fluid. The term 'Ideal fluid' means non viscous (inviscid) fluid. Euler (1755) was first who developed the 'Theory of perfect fluids'. Euler's famous equation of fluid flow for perfect fluid was appeared in the paper "General Principle of the motion of fluids".

The origin of Real fluid of actual fluid theory was an attempt to explain D' Alembert paradox. The concept of Real fluid based on an inherent property of fluid, known as 'Viscosity', this is the resistance in flow, offered by fluid itself. It is also termed as internal friction of the fluid. On introducing viscosity of fluids in perfect fluid, a new field, viscous fluid dynamics came into existence. In this field a law known as Newton's law of viscosity classifies the fluid further in two types, one that follows is termed as Newtonian fluid and the other is non-Newtonian fluid. Navier (1827) and Stokes (1845) have given the equations of motion of viscous Newtonian fluid independently. The equation of motion given by Navier (1827) is applicable only to incompressible flow while Stokes (1845) considered the compressible flow.

The study of the motion of an electrically conducting fluid in the presence of a magnetic field, termed as Magnetofluid dynamics also got an importance in studies. The attracted attention of aerodynamicists, mechanical engineers and applied mathematicians because of the interaction of the two classical disciplines viz., Electromagnetism and fluid dynamics. MHD power generator, MFD flow meters, MFD submarine's etc., are a few applications of MFD. Here when the fluid is incompressible and its other properties such a viscosity, thermal conductivity and electrical conductivity are regarded as constant the word Magnetohydrodynamics is used. Faraday (1832) observed that due to motion of an electrically conducting fluid in a magnetic field, the electric currents are induced in the fluid that produce their own magnetic field, called induced magnetic field, and this modifies the original magnetic field. Besides this the induced currents interact with magnetic field to produce electromagnetic forces perturbing the original motion.

1.2 TYPES OF FLUIDS

All material exhibits deformation under the action of forces. If the deformation in the material increases continually without limit under the action of shearing forces, however small, the material is called "fluid". Fluids available in the nature are broadly classified as follows.

(i) Ideal Fluids

Frictionless and incompressible fluids are known as Ideal fluids, also termed as perfect fluids or inviscid fluids. In the motion of such a perfect fluid, two adjacent layers experience no tangential forces (shearing stresses) because the velocity gradient normal to flow direction is very small, but act on each other with normal stresses (pressures) only. The theory describing the motion of perfect fluid gives satisfactory description of real motion, e.g., the motion of surface waves or the formation of liquid jets in air.

(ii) Real fluids

The fluids that are available in nature are real fluid. They possess all the properties of fluid such as compressibility, viscosity and shearing stress etc. In real fluids tangential and shearing stresses both are present. The existence of inter-molecular attraction causes the fluid to adhere to the contact surface during fluid flow and this gives rise to shearing stresses. The inherent property of fluids viz. the viscosity plays an important role in deciding the behaviour of real fluid flow.

(iii) Newtonian Fluids

Fluids which obey Newton's law of viscosity given by

$$\tau = \mu \frac{\partial u}{\partial y} \qquad ...(1.2.1)$$

where μ is a constant known as viscosity coefficient and is a property of fluid depends to a great extent on its temperature, $\frac{\partial u}{\partial y}$ is the velocity gradient are called Newtonian fluids Equation (1.2.1) explain that for Newtonian fluids the stress components are linear function of the rate of strain components.

(iv) Non-Newtonian fluids

Fluids for which Newton's law of viscosity, defined by equation (1.2.1), do not hold good are called Non-Newtonian fluids. Fluids like Power law fluid, Prandtl fluid, (visco-inelastic fluids) and Oldroyd fluid, Bingham plastic, Rivilin-Ericksen second order fluid, Walters fluid (visco-elastic fluids) are considered as Non-Newtonian fluids and are of importance in industries. The power law

$$\tau = k\left(\frac{du}{dy}\right)^n \qquad ...(1.2.2)$$

is one way to describe the behaviour of Non-Newtonian fluids.

For n <1, fluid is called pseudo-plastic with increasing shear rate, $\frac{du}{dy}$, there is a decrease in effective viscosity. That is an increasing shear rate the fluid is thinning.

For n >1, the fluid is called 'dilatant' here fluid 'thickens' with increasing shear rate.

(v) Compressible fluids

The density of some fluids, particularly gases, changes considerably with the speed of the flow and the temperature of the fluids, such fluids are called as compressible fluids. Liquids generally do not possess this property, therefore these are regarded as incompressible fluid. On the other hand gases are highly compressible. In special case the compressibility of a liquid could be significant for very high pressures. Also, in under water acoustics, the incompressibility of water is important even though the pressure variation and hence compression may be equite small.

(vi) Incompressible fluids

Compressibility as defined above is a measure of the change of volume or density of a fluid under the action of external forces. As the if compressibility of liquid is very small i.e. density is almost constant, the flow of liquids is regarded as incompressible. Also, when the speeds that are not comparable with that of sound, i.e., if Mach number (the ratio of the velocity of flow to the velocity of sound) is small compared with unity, the effect of compressibility on atmospheric air can be neglected and it may be considered as incompressible fluid. In real liquids (water, oil, tec.) density ρ is not exactly constant but considered as almost constant.

1.3 TYPES OF FLOW

Basically there are two types of flow, viz., laminar and turbulent.

(i) Laminar flow

The well-ordered flow in which layers (also known as streamlines) are assumed to slide over one another is termed as 'laminar flow'. In laminar flow, flow is free from macroscopic velocity fluctuations. Mathematically, in laminar flow the curve of shear stress t against the rate of shear S also known as consistency curve for the liquid, is a straight line whose slope is the constant of viscosity μ

i.e. $\frac{\text{Shear Stress } \tau}{\text{Rate of Shear } S} = \mu$ (constant)

(ii) Turbulent flow

When various layers (the stream lines) are interwoven with each other in an irregular manner then the flow is called as turbulent flow. In such flow, at any point of the fluid, random movement give rise to fluctuation in velocity and pressure. This type of flow is not a part of the present work. A non dimensional parameter termed as Reynolds number (Re) is used to denote the imminence of transition from laminar to turbulent flow, i.e., like Re> 2300, in case of pipe flow leads to the transition of laminar to turbulent flow.

(iii) Steady flow

The flow of fluid in which all quantities such as velocity, density, pressure, etc., associated with the flow field remain unchanged with time is said to be steady flow. Consequently, in this type of flow time drops out of the independent variables and the various quantities simply become function of the space co-ordinates only. In the steady flow the path of fluid particles will be along the stream line and streamlines picture does not change with time.

(iv) Unsteady flow

The flow is said to be unsteady when characteristics of fluid, e.g., velocity, density, temperature, etc. changes with time. In unsteady flow particle path, in general, is different from streamline. The simplest unsteady flow, which results due to the impulsive motion of flat plate in its own plane in an infinite mass of fluid which is otherwise at rest and is well known as Stokes flow.

1.4 FUNDAMENTAL EQUATIONS OF THE FLOW OF VISCOUS FLUIDS

(i) Equation of state

Variables that depend only upon the state of a system are called variables of state. The pressure p. the density ρ and the temperature T are variable in fluid flow which depends only upon the state of the system and so known as variables of state. A relationship in these defined by

$$F\,(p, \rho, T) = 0 \qquad ...(1.4.1)$$

is known as equation of state.

If molecules of a fluid are presumed to have a mutual effect stems from perfectly elastic collisions, then the kinetic theory of gases indicates that for such a fluid, called perfect gas, there exists a simple formulation for equations of state given by

$$p = \rho R\, T \qquad ...(1.4.2)$$

Where R is gas constant, depends only on the molecular weight of the fluid. When the fluids are considered as incompressible the equation of state is reduced to

$$\rho = \text{constant}. \qquad ...(1.4.3)$$

(ii) Equation of continuity (conservation of mass)

The mathematical expression of the law of conservation of mass is known as the equation of continuity. When the region of fluid contains neither source nor sinks (i.e., region of creation or annihilation of the fluid) then the amount of fluid within the region is conserved in accordance with the principle of conservation of matter. By the principle of continuity, we have

Rate of mass accumulation = {Rate of mass flowing in region} − (Rate of mass flowing out of region}.

Mathematically it is given as

$$\frac{\partial \rho}{\partial t} = -[\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)] \quad ...(1.4.4)$$

Where v_x, v_y, v_z are velocity components of the fluid in X, Y and $Z-$ direction respectively, ρ is density and t is the time. In vector notation

$$\frac{\partial \rho}{a t} + div\ (\rho\ \vec{v}) = 0 \quad ...(1.4.5)$$

In case of steady compressible flow, the equation of continuity reduces to

$$div(\rho\ \vec{v}) = 0 \quad ...(1.4.6)$$

If the fluid is incompressible then the equation of continuity reduces to

$$div(\vec{v}) = 0 \quad ...(1.4.7)$$

(iii) Equations of motion (Navier - Stokes' equations)

The equations of motion are derived from Newton's second law of motion (Conservation of momentous) which states that -

Total force = Rate of change of linear momentum

If S is closed surface enclosing a volume V in the region occupied by the moving fluid, then

Total force = {Body forces acting on the enclosed volume V} + {Surface forces acting on the controlled surface S}

$$= \int_v \rho f_i\, dv + \int_S P_i\, dS \quad ...(1.4.8)$$

where ρ is density, f_i the body forces per unit mass and P_i, the forces on the boundary per unit area.

Rate of change of linear momentum

= {Rate of increase of momentum in the enclosed volume V}

+ {Rate of out flow of momentum through controlled surface S}

$$= \frac{\partial}{\partial t}\int_v \rho v_i\, dv + \int_S v_i(\rho v_j n_j) dS \quad ...(1.4.9)$$

(1.4.8) and (1.4.9) implies that

$$= \frac{\partial}{\partial t}\int_v \rho v_i\, dv + \int_S v_i(\rho v_j n_j) dS = \int_v \rho f_i\, dv + \int_S P_i\, dS \quad ...(1.4.10)$$

Where P_i is given by

$$P_i = \sigma_{ij} n_j \quad ...(1.4.11)$$

and

$$\sigma_{ij} = -p\ \delta_{ij} + \tau_{ij} \quad ...(1.4.12)$$

for isotropic Newtonian fluid these expressions are given by the constitutive equation

$$\tau_{ij} = 2\mu e_{ij} - \frac{2}{3}\mu e_{kk}\delta ij \quad ...(1.4.13)$$

where

$$e_{ij} = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_j}) \quad ...(1.4.14)$$

substituting these equations, using Gauss theorem and noting that V is an arbitrary chosen volume, we get the equations of motion as

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}(\rho v_i v_j) = \rho f_i - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \quad ...(1.4.15)$$

using equation of continuity (1.4.4) the equation (1.4.15) finally reduces to

$$\rho\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right)=\rho f_i-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_j} \qquad ...(1.4.16)$$

which is valid for any continuous fluid medium.
For isotropic Newtonian fluid (1.4.14) becomes

$$\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}=\rho f_i-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}[\mu(\frac{\partial v_i}{\partial x_j}-\frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k}) \qquad ...(1.4.17)$$

These are known as Navier - Stokes equations of motion for a viscous compressible fluid and are three in number.

(iv) Equation of Energy (conservation of Energy)

The law of conservation of energy for fluid flow states that = the difference in the rate of supply of energy to a controlled surface S enclosing a volume V in the region occupied by a moving fluid and the rate at which the energy goes out through S must be equal to the net rate of increase of energy in the enclosed volume V.
Mathematically it can be expressed as

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial x_j}\left(v_i\sigma_{ij}\right)-\frac{\partial}{\partial x_j}\left(E_t\rho v_j\right)-\frac{\partial}{\partial x_j}q_j-\frac{\partial}{\partial t}(E_t+\rho)=0 \qquad ...(1.4.18)$$

where E_t = total energy of the system per unit mass
= Kinetic energy + Potential energy + Internal Energy
$=\frac{1}{2}v_iv_i+K+I$

and
$q_j=-k\frac{\partial T}{\partial x_j}$ is called heat flux vector.

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