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Research Article

ARBITRARY AMPLITUDE SOLITONS AND DOUBLE LAYERS OF DUST KINETIC ALFVÉN WAVES IN PLASMA WITH *K*-DISTRIBUTED ELECTRONS

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ABSTRACT
We have studied exact Kinetic Alfvén Solitons and Double Layers in dusty plasma with Kappa distributed electrons. The nonlinear plasma structures are found as super- Alfvénic. For propagation directions nearer to the ambient magnetic field direction the nonthermality due to the non-Maxwellian electrons is found to give no extraordinary influence.

The Mach number of the Double Layers are found as higher in value in comparison to those for the Solitons in case of moderately finite **β** value, and also found to be enhanced by the dust concentration amount in the plasma and the superthermality K of the electrons. K is also seen to be lowered in value for lower value of plasma **β**.

Keywords: *Soliton, Double Layer, Alfvénic Structures, Non-Maxwellian, Dusty Plasma*

INTRODUCTION
Experiments for observations of nonlinear electrostatic pulses as solitary waves at the Earth's Magnetopause lead to detect structures with potentials of ~0.1 to 5 Volts, propagating at speeds comparable to the electron thermal speed (Cattel *et al*, 2002). Similar coherent structures of electrostatic modes of kinetic Alfvén waves (KAWs) are also known to develop in the Earth's auroral region and in near-Earth space regions (Ergun et al, 1998, Bostrom et al.1988). Solitary KAWs (SKAWs) and Double Layers (DLs) are explained in relation to observations by Freja satellites (Dovner *et al*.1994). These SKAWs or solitons and DLs of kinetic Alfvén waves detected by the Freja satellite experiments are identified as density pulses evolved from low-frequency auroral electromagnetic disturbances by Louarn et al (1994). These waves were studied analytically to explain their dynamical behaviour by Hasegawa and Mima (1976) for the first time using the Sagdeev's potential approach. Afterwards many workers have shown different findings within this context (Shukla *et al.*, 1982, Devi *et al.*, 2007, Wu & Chao, 2004, Wu *et al.*, 1995, Roychoudhury and Chatterjee, 1998, Gogoi & Devi, 2008) in both the inertial limitation $(\beta << m_e / m_i)$ and kinetic limitation $m_e / m_i << \beta$ (m_e and m_i are the electron mass and the ion mass, respectively; β is the ratio of the plasma thermal pressure to the magnetic pressure). The results showed that the SKAWs/DLs with the density dip can exist in plasma with $\beta << m_e / m_i$, and the localized waves with the density hump can be excited in plasma with $m_e / m_i << \beta$.

Most astrophysical and space plasmas are observed to have non-Maxwellian high energy tail. Spacecraft measurements of such profiles of electron energy spectra have been successfully modeled with superthermal electrons (Vasyliunas, 1968). Importantly, κ-distribution has been used to analyze in such plasma and interpret spacecraft data on the Earth's magneto-sphere plasma sheet, the solar wind, Jupiter, Saturn and planetary magnetospheres (Hellberg & Mace, 2002). Typically space plasmas are observed to possess a spectral index κ in the range 2-6 (Gogoi and Khan, 2010, Devi and Chaharia, 2017,). The κ-

distribution with κ = 4 yielding good agreement with electron distribution observed in the solar wind (Hellberg *et al.,* 2005).

Moreover it is known that in space plasma the presence of charged dust grains is very common with sizes ranging from micron to submicron (10nm-10μm) having varying masses from 10^{-11} to 10^{-16}g in background plasma of electrons-ions-neutrals etc. Such plasma, known as dusty plasma is a realistic medium in the studies related to interplanetary space, planetary magnetospheres, astronomical phenomenon etc., in addition to the concept of the presence of dust in comets, asteroids, planetary rings, etc. New types of low frequency dust acoustic wave (DAW) in the presence of massive charged dust particles associated with dust number density perturbations with weak coupling between dust particles and plasma particles has been established theoretically (Rao *et al.*, 1990, Shukla and Mamun 2002). Many heuristic works are found to study salient properties of coherent structures occurring in Earth's space region and auororal ionosphere filled up with dusty plasmas (Shukla *et al.*, 1992, Mamun *et al.*, 1996, Pokhotelov *et al.*, 1999, Devi and Gogoi, 2006, Roychoudhury, and Chatterjee 1999 Verheest 2000,Verheest 2000) Side by side the experimental findings (Verheest, 2000) are seen to keep relevance with such wave modes.

Solitary Kinetic Alfvén Waves (SKAW's) propagating in an oblique direction to ambient magnetic field are of much importance in the interpretation of low-frequency electromagnetic fluctuations observed in auroral zone of the Earth by the satellite Freja (Dovner and Holmgreen, 1994; Lourn *et al.*, 1994). Recently, some authors have studied the SKAW's in low-β dusty plasmas (Gogoi *et al.*, 2011, Jatenco-Pereira *et al*., 2014).

BASIC EQUATION

Here we assume plasma consisting of superthermal electrons, ions and negatively charged massive dust grains, embedded in an ambient magnetic field $B_0\hat{z}$ where $\hat{z}$ is the unit vector of z-direction. The mathematical representation of the present study requires the expression of the physical situation created by means of the non-Maxwellian electron distribution considered in our model. The universal Newton's equation for $j = i, e$ respectively for the ions and the electrons goes as

$$m_j n_j[\partial_t \vec{u}_j + \vec{u}_j . \nabla\vec{u}_j + \nabla\vec{p}] = q_j n_j[\vec{E} + \vec{u}_j \times \vec{B}].$$

This equation is further simplified for our plasma model consisting of non-Maxwellian electrons. The relativistic effect for the electron motion through the momentum term has been neglected due to the observational fact of existence of low energy profile at intermediate speed (Vasyliunas, 1968). For low-β $(0 << \beta << 1)$ plasma, the compressive component of the magnetic field perturbation can be ignored. The normalized equations governing the dynamics of the kinetic Alfvén waves in the presence of dust species are as follows.

The dust grains keep static due to their heavy mass and carry negative charges. In the case of our consideration, the quasi-neutrality condition reads

$$\delta_e n_e + Z\delta_d - n_i = 0$$

Where n_e and n_i are the electron and ion densities, normalized by n_{e0} and n_{i0}, respectively; n_{e0} and n_{i0} are the electron and ion densities in equilibrium, respectively. And $\delta_e = n_{e0}/n_{i0}$, $\delta_d = n_d/n_{i0}$, n_d is the dust grain density. Z is the number of charge on a dust grain. Obviously, in equilibrium, $\delta_e = 1 - Z\delta_d$.

For the electrons,

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z} n_e v_{ez} = 0 \qquad (1)$$

For the ions,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial z} n_i v_{ix} = 0 \qquad (2)$$

$$v_{ix} = \alpha Q \frac{\partial E_x}{\partial t} \qquad (3)$$

From Maxwell's equations,

$$\frac{\partial B_y}{\partial t} = -\alpha Q\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) \tag{4}$$

$$\frac{\partial B_y}{\partial x} = -\delta_e n_e v_{ez} \tag{5}$$

Charge neutrality is expressed as $\delta_e n_e + z\delta_d - n_i = 0$ (6)

For the κ-distributed electrons, $n_e = \left(1 - \frac{\psi}{(\kappa - (3/2))}\right)^{-(\kappa - (1/2))}$ (7)

Charge neutrality at equilibrium gives, $\delta_d = 1 - \delta_e$ (8)

Here Q = m_e/m_i (electron to ion mass ratio), (two potentials ϕ, ψ are included to justify a low -β plasma model) and $\alpha = \beta/2Q$.

DERIVATION OF THE SAGDEEV POTENTIAL EQUATION

For plasma in a uniform ambient magnetic field B_0 along the z-direction, the stationary independent variable η is considered as $\eta = xk_x + zk_z - Mt$ with $k_x^2 + k_z^2 = 1$ (9)

Where, M is Mach number of the wave in the unit of the Alfvén velocity V_A. Using this set up for stationary frame, we have the transformation as

$$k_x\frac{\partial}{\partial \eta} = \frac{\partial}{\partial x},\ k_z\frac{\partial}{\partial \eta} = \frac{\partial}{\partial z},\ -M\frac{\partial}{\partial \eta} = \frac{\partial}{\partial t} \tag{10}$$

Now, from equations (1) - (8) using these transformation relation and under the boundary conditions $n_e \to 1,\ v_{ez} \to 0$, the following is obtained.

$$k_z^2 k_x^2 \frac{\partial}{\partial \eta}\left(\frac{\left(\kappa - \frac{3}{2}\right)}{\left(\kappa - \frac{1}{2}\right)} n_e^{\frac{\kappa + \frac{1}{2}}{-\kappa + \frac{1}{2}}} \frac{\partial n_e}{\partial \eta}\right) = k_x k_z \frac{1}{\alpha Q}\left(1 - \frac{1}{1 + \delta_e(n_e - 1)}\right) - \frac{\delta_e M^2}{\alpha Q}(n_e - 1) \tag{11}$$

Using mathematical technique for integration and simplifications, we have

$$\Rightarrow \frac{1}{2}\left(\frac{dn_e}{d\eta}\right)^2 = \left(\frac{\left(\kappa - \frac{3}{2}\right)}{\left(\kappa - \frac{1}{2}\right)} n_e^{\frac{\kappa + \frac{1}{2}}{-\kappa + \frac{1}{2}}}\right)^{-2} \left(\begin{array}{l} -\frac{1}{k_x k_z}\frac{1}{\alpha Q}\left(\kappa - \frac{3}{2}\right)\left(n_e^{\frac{1}{-\kappa + \frac{1}{2}}} - 1\right) n_e^{\frac{1}{-\kappa + \frac{1}{2}}} - \frac{1}{k_x k_z}\frac{1}{\alpha Q}\frac{\left(\kappa - \frac{3}{2}\right)}{\left(\kappa - \frac{1}{2}\right)}\int \frac{n_e^{\frac{\kappa + \frac{1}{2}}{-\kappa + \frac{1}{2}}}}{1 + \delta_e(n_e - 1)} dn_e \\ -\frac{\delta_e M^2}{\alpha Q k_z^2 k_x^2}\left(n_e^{\frac{-\kappa + \frac{3}{2}}{-\kappa + \frac{1}{2}}} - 1\right) - \frac{\delta_e M^2}{\alpha Q k_z^2 k_x^2}\left(\kappa - \frac{3}{2}\right)\left(n_e^{\frac{1}{-\kappa + \frac{1}{2}}} - 1\right) \end{array}\right)$$

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Where, $\int \frac{n_e^{\frac{\kappa+\frac{1}{2}}{-\kappa+\frac{1}{2}}}}{1+\delta_e(n_e-1)}dn_e = \frac{1}{2(-1+\delta_e)}\left((-1+2k)n^{1-\frac{1}{2k-1}-\frac{2k}{2k-1}} \; Hypergeometric2F1\left[-\frac{2}{2k-1},1,1-\frac{2}{2k-1},-\frac{n\delta_e}{1-\delta_e}\right]\right)$

The function Hypergeometric 2F1 can be obtained as a solution of linear ordinary differential equation of the second order, like $\frac{d^2w}{dz^2}+p\frac{dw}{dz}+qw=0$

In our case, we have $2F1\left(a,b;c;\frac{n_e\delta_e}{-1+\delta_e}\right)=\sum_{k=0}^{\infty}\frac{a_k b_k}{c_k}\frac{z^k}{\angle k}$

Here, $a=-\frac{2}{2k-1}$, $b=1, c=1-\frac{2}{2k-1}$.

Thus the required S. P. Equation is obtained as

$$\Rightarrow \frac{1}{2}\left(\frac{dn_e}{d\eta}\right)^2+K(n_e,\alpha,Q,\delta_e,k_x,k_z,\kappa,M)=0 \tag{12}$$

Where, $K(n_e,\alpha,Q,\delta_e,k_x,k_z,\kappa,M)=$

$$-\left(\frac{\left(\kappa-\frac{3}{2}\right)}{\left(\kappa-\frac{1}{2}\right)}n_e^{\frac{\kappa+\frac{1}{2}}{-\kappa+\frac{1}{2}}}\right)^{-2}\left(\begin{array}{l} -\frac{1}{k_xk_z}\frac{1}{\alpha Q}\left(\kappa-\frac{3}{2}\right)\left(n_e^{\frac{1}{-\kappa+\frac{1}{2}}}-1\right)-\frac{1}{k_xk_z}\frac{1}{\alpha Q}\frac{\left(\kappa-\frac{3}{2}\right)}{\left(\kappa-\frac{1}{2}\right)}\frac{(-1+2k)}{2(-1+\delta_e)} \\ \left(\begin{array}{l} n_e^{\frac{2}{1-2k}}Hypergeometric2F1\left[1,\frac{2}{1-2\kappa},1+\frac{2}{1-2\kappa},\frac{\delta_e n_e}{-1+\delta_e}\right] \\ -Hypergeometric2F1\left[1,\frac{2}{1-2\kappa},1+\frac{2}{1-2\kappa},\frac{\delta_e}{-1+\delta_e}\right]\end{array}\right) \\ -\frac{\delta_e M^2}{\alpha Q k_z^2 k_x^2}\left(n_e^{\frac{-\kappa+\frac{3}{2}}{-\kappa+\frac{1}{2}}}-1\right)-\frac{\delta_e M^2}{\alpha Q k_z^2 k_x^2}\left(\kappa-\frac{3}{2}\right)\left(n_e^{\frac{1}{-\kappa+\frac{1}{2}}}-1\right)\end{array}\right)$$

MATHEMATICAL CONDITIONS FOR THE EXISTENCE OF SOLITON SOLUTION

The conditions for solitary waves are

(i) $K(n_e)<0$ between $n_e=1$ and $n_e=N$ so that $dn_e/d\eta$ is real, here N gives the amplitude of the solitary wave. N can be both greater than 1 and less than 1. In the former case we have compressive solitary waves and in the latter case rarefactive solitary waves.

(ii) K (n_e) must be a maximum at $n_e=1$ which means $\left.\frac{\partial K(n_e)}{\partial \eta}\right|_{n_e=1}=0$ and $\left.\frac{\partial^2 K(n_e)}{\partial \eta^2}\right|_{n_e=1}<0$.

(iii) $K(n_e)$ should cross the 'n_e' axis from below near $n_e=N$; and $K(n_e)>0$ for $n_e>1$.

And the conditions for the double layer solutions appears through the occurrence of double roots so that K (n_e) is maximum for $n_e=N\ (\neq 1)$ leading to K (n_e)=0 at $n_e=N$.

Also for double layer solution it is required that $\frac{\partial K(n_e)}{\partial \eta}$ vanishes at $n_e = 1$.

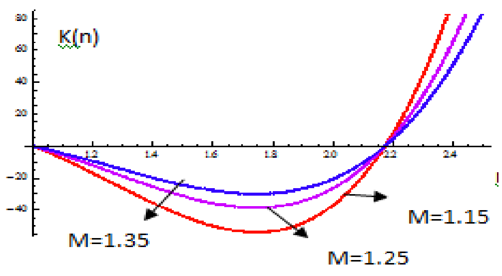

Fig. 1. Shows Sagdeev potential *K(n)* verses *n* with β=.09, δ_e=.099, k_x=.07, κ=4 for different values of *M*=1.15(R),1.25(P),1.35(B).

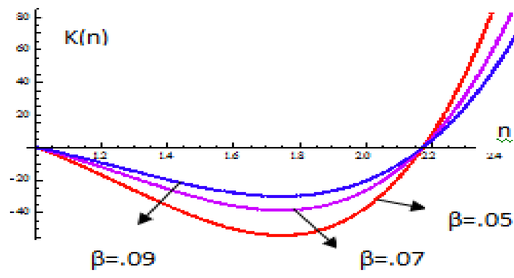

Fig. 2. Shows Sagdeev potential *K(n)* verses *n* with δ_e=.099, *M*=1.15, k_x=.07, κ=4 for different values of β=.05(R), .07(P), .09(B).

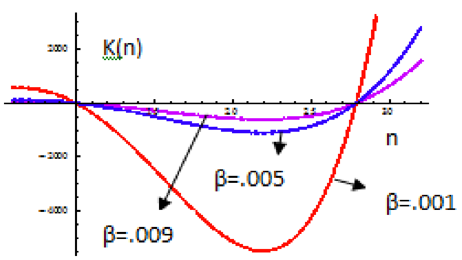

Fig. 3. Shows Sagdeev potential *K(n)* verses *n* with δ_e=.07, *M*=1.15, k_x=.07, κ=3 for different values of β=.001(R), .005(B), .009(P)

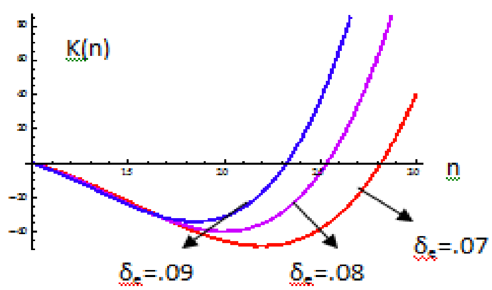

Fig. 4. Shows Sagdeev potential *K(n)* verses *n* with β=.09, *M*=1.15, k_x=.07, κ=4 for different values of δ_e=.07(R), .08(P), .09(B).

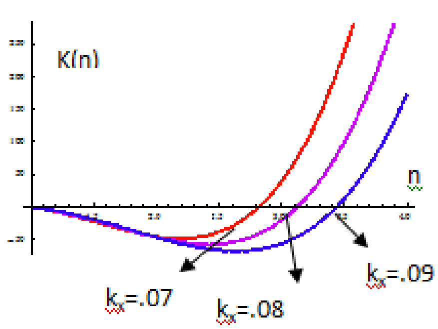

Fig. 5. Shows Sagdeev potential *K(n)* verses *n* with β=.09, δ_e=.07, *M*=1.15, κ=4 for different values of k_x=.07(R), .08(B), .09(P).

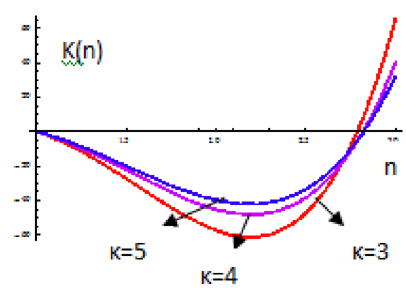

Fig. 6. Shows Sagdeev potential *K(n)* verses *n* with β=.09, δ_e=.07, *M*=1.15, k_x=.07 for different values of κ=3(R), 4(P), 5(B).

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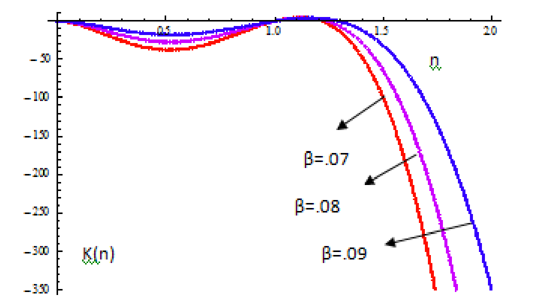

Fig. 7. Sagdeev potential *K(n)* profile plotted against density *n* for three different values of β = .07 (red line), .08(pink line), .09 (blue line) and k_x = .045, *M*=2.15, δ_e =.09, κ=4.

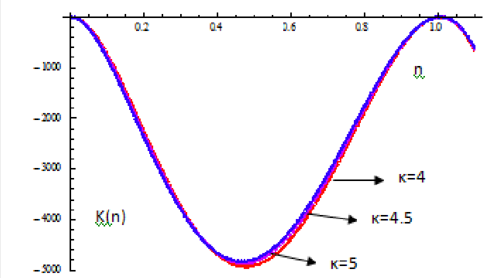

Fig. 8. Sagdeev potential *K(n)* profile plotted against density *n* for three different values of κ = 4(red line), 4.5(pink line), 5(blue line) and k_x = .003, *M*=2.15, δ_e =.09, β=.09.

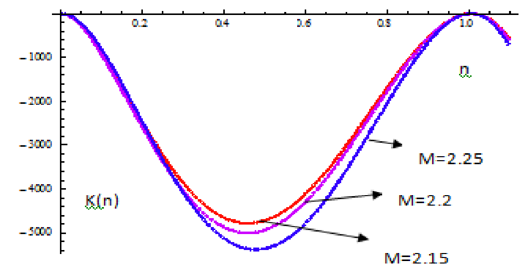

Fig. 9. Sagdeev potential *K(n)* profile plotted against density *n* for three different values of *M* = 2.15 (red line), 2.2(pink line), 2.25(blue line) and k_x = .003,, δ_e =.09, κ=4, β=.09.

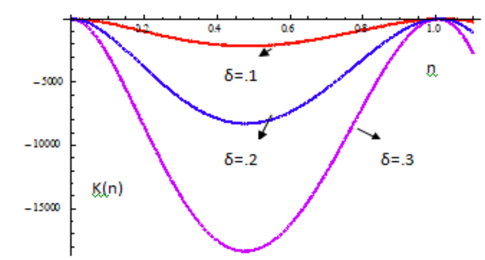

Fig. 10. Sagdeev potential *K(n)* profile plotted against density *n* for three different values of δ_e = .1 (red line), .3 (pink line), .2 (blue line) and k_x = .003, κ=4, *M*=1.25, β=.09.

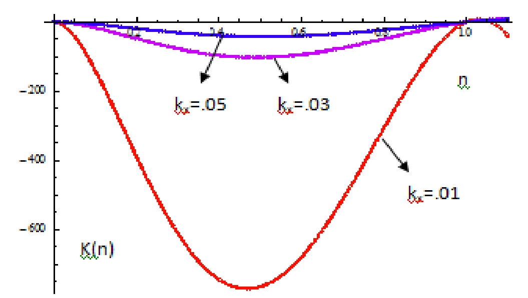

Fig. 11. Sagdeev potential *K(n)* profile plotted against density *n* for three different values of k_x = .01 (red line), .03 (pink line), .05 (blue line) and β=.09 , δ_e = .2, κ=6, *M*=1.25.

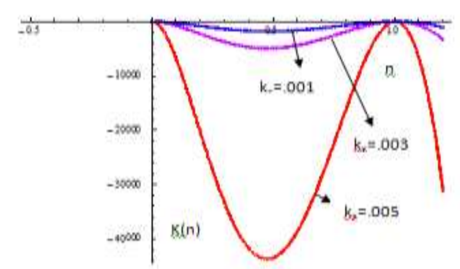

Fig. 12. Sagdeev potential *K(n)* profile plotted against density *n* for three different values of k_x = .005 (red line), .003 (pink line), .001 (blue line) and β=.09 , δ_e = .2, κ=6, *M*=1.25.

RESULTS AND DISCUSSION

Our results are dominated by the conditions $0<\beta<1$ and κ=3-4 for spectral index of the energetic electrons. For $0<\beta<<1$, widths of the solitary waves become smaller. Significant effect of coupling between dust and plasma cyclotron frequencies can be seen through the computation of the Sagdeev Potential $K(n_e)$. Critical value of κ appears to be around 4 for propagation of solitons near the magnetic field. .

For the findings to be useful we try to analyze the variation of the width (i.e., the wave amplitude divided by the corresponding potential depth) in relation to the above mentioned parameters namely M, δ_d, k_z, β, κ we have in plasmas. Through the figures (1) to (6) we depict the following:

The width of solitary waves increases with increase in the value of Mach number M =1.15 (red line), 1.25 (purple line), 1.35 (blue line) in figure (1), the value of β =.05 (red line), .07 (purple line), .09 (blue line) and β =.001(red line), .005 (purple line), .009 (blue line) in figure (2) and (3) and the value of δ_e=.07(red line), .08 (purple line), .09 (blue line) in figure (4).

The amplitude of solitary waves increases with increase in the value of k_x=.07 (red line), .08 (purple line), .09 (blue line) in figure (5) and decreases with increase in the value of κ =3 (red line), 4 (purple line), 5 (blue line) in figure (6).

We have derived the Sagdeev potential equation (12) for the pseudo-potential $K(n)$, where the plasma potential n is designated by n_e. The potential function in (12) is where we have replaced n by n_e. Use of the charge neutrality condition (6) is being made during our mathematical calculation to arrive at this potential in terms of Z=1 for the presence of dust particles in the plasma. The parameters for the study of double layers are the Mach number M, dust density δ_d, direction of propagation k_z, β, the spectral index κ for superthermal electrons.

Rarefactive double layers are found to form which are governed by the Sagdeev potential equation (12), since we have obtained the values of the amplitudes as smaller than unity (i.e., n<1). Without the effect of dust concentration, some findings show the formation of compressive double layers with similar ranges of kappa values between 4 and 6. For example in Gogoi and Khan (2010), the adiabatic character of the ions plays a different role. The widths of the double layers become bigger for higher concentration of dust density δ_d in the plasma. The propagation of the electrostatic double layers is found to be directed significantly close to the direction of the ambient magnetic field, i.e., for bigger k_z get their widths significantly great. The kappa index of the superthermality of the non-Maxwellian electrons is in good agreement with the double layer formations in space and astrophysical plasmas. Because in our computations for double layers kappa values are seen ranging between 4 and 6.

It is found that the values of the plasma beta parameter are obtained as finite to have good signatures of the propagating structures. The width measures are in terms of kilometric orders. This fact is very well established in cases of different space plasma probes. Thus our findings show exciting plasma enhancement in such regions.

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