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Research Article

LINEAR METHOD OF ATTRIBUTE REDUCTION FOR CONCEPT LATTICE

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ABSTRACT
To enrich the study of attribute reduction for concept lattice, by introducing algebraic knowledge into the concept lattice theory, a linear method of attribute reduction is obtained. The formal context corresponding to the concept lattice is regarded as a matrix, and each attribute and its corresponding column are treated as separate vectors, and the elementary transformation of the matrix is performed to obtain the maximal linearly independent group corresponding to the matrix, and then the attribute reduction, core attribute sets, relatively necessary attribute sets, and absolutely unnecessary attribute sets are obtained.

Keywords: formal context; attribute reduction; algebraic theory; maximal linearly independent group

INTRODUCTION

Concept lattice, also known as formal concept analysis, was proposed by Professors Ganter and Wille (1999) of Germany to process information. The attribute reduction of the concept lattice is one of the key contents and has attracted wide attention. By performing attribute reduction on the concept lattice, the same concept and its concept lattice structure can be obtained by using the smallest subset of attributes. Therefore, with the development of big data, the attribute reduction of the concept lattice still has practical research significance.

Zhang *et al*., (2005) studied the decision theorem of uniform set, introduced the discernibility matrix of formal context, and proposed a method of attribute reduction in concept lattice. Mao (2017) investigated all the reducible attributes and concepts in a context with the aid of graph theory. Zhang (2014) studied the problem of attribute reduction in decision formal context from the perspective of tree diagram. Based on the notion of "one-side fuzzy concept", Mao and Miao (2018) proposed a method of attribute reduction combining with the directed graph theory. Two attribute reduction methods based on minimum decision cost are proposed by Bi *et al*., (2016) from the algebraic view and the information theory, respectively. Yao *et al*., (2008) addressed attribute reduction in decision-theoretic rough set models regarding different classification properties, such as: decision-monotocity, confidence, coverage, generality and cost. Ren *et al*., (2016) studied the attribute reduction of three-way concept lattices in order to make the data easily be understood. Konecny (2017) showed how to adapt the clarification and reduction method to the extensions of formal concept analysis considered in the recent papers.

In order to enrich the research of attribute reduction for concept lattice based on previous studies, we propose a linear method for concept lattice attribute reduction.

Preliminaries

In this section, we will give some basic definitions. See document (Ganter and Wille, 1999) and (Pre-algebra Group, 2003) for details.

Definition 2.1 (Ganter and Wille, 1999) A formal context is a triple (U,V,R), which consists of two sets U and V and a relation R between U and V. The elements of U are called the objects and the elements of V are called the attributes of the context. For $x \in U$ and $a \in V$, we write $(x,a) \in R$ as xRa, and say that the object x has the attribute a, or alternatively, the attribute a is possessed by the object x.

Sometimes, note the object x has the attribute a for 1, otherwise for 0.

Definition 2.2 (Ganter and Wille, 1999) Let (U,V,R) be a formal context. $\forall X \subseteq U$ and $A \subseteq V$, pair of operators ${}^{*}: P(U) \to P(V)$ and ${}^{*}: P(V) \to P(U)$ are defined by

$$X^{*} = \{a \in V \mid \forall x \in X (xRa)\},$$
$$A^{*} = \{x \in U \mid \forall a \in V (xRa)\}.$$

Definition 2.3 (Ganter and Wille, 1999) If $X^{*} = A$ and $A^{*} = X$, then a pair (X, A) is called a formal concept. X is called the extension and A is called the intension of the concept (X, A). All concepts in (U,V,R) is denoted by $L(U,V,R)$ and called the concept lattice of (U,V,R).

Definition 2.4 (Zhang WX, *et al*., 2005) Let $L(U,V_1,R_1)$ and $L(U,V_2,R_2)$ be two concept lattices. If for any $(X, A) \in L(U,V_2,R_2)$, there exists $(X^{*}, A^{*}) \in L(U,V_1,R_1)$ such that $X^{*} = X$, then $L(U,V_1,R_1)$ is said to be finer than $L(U,V_2,R_2)$, which is denoted by $L(U,V_1,R_1) \le L(U,$ $V_2,R_2)$. If $L(U,V_1,R_1) \le L(U,V_2,R_2)$ and $L(U,V_2,R_2) \le L(U,V_1,R_1)$, then these two concept lattices are said to be isomorphic to each other, and denoted by $L(U,V_1,R_1) \cong L(U,V_2,R_2)$.

Definition 2.5 (Zhang *et al*., 2005) Let (U,V,R) be a formal context. The set $\{D_i \mid D_i$ is a reduct, $i \in \tau\}$ (τ is an index set) includes all of the reducts in (U,V,R). The attribute set V is divided into 3 parts. $1\rangle$ Absolute necessary attribute (core attribute) $b : b \in \bigcap_{i\in\tau} D_i$. $2\rangle$ Relati- ve necessary attribute $c : c \in \bigcup_{i\in\tau} D_i - \bigcap_{i\in\tau} D_i$. $3\rangle$ Absolute unnecessary attribute $d : d \in V - \bigcup_{i\in\tau} D_i$.

Definition 2.6 (Zhang WX, *et al*., 2005) Let (U,V,R) be a formal context. If there exists an attribute set $D \subseteq V$ such that $L(U,D,R_D) \cong L(U,V,R)$, then D is called a consistent set of (U,V,R). And further, if $\forall d \in D$, $L(U, D-\{d\}, R_{D-\{d\}}) \ncong L(U,V,R)$, then D is called a reduct of (U,V,R). The intersection of all the reducts is called the core of (U,V,R).

Definition 2.7 (Pre-algebra Group, 2003) If a vector in an vector group can be linearly represented by the remaining vectors, then this vector group is called linear correlation.

Definition 2.8 (Pre-algebra Group, 2003) A vector group is called linear independence, if $k_1, k_2, \cdots, k_s$ that are not all zeros do not exist and satisfy $k_1\vec{\alpha}_1 + k_2\vec{\alpha}_2 + \cdots + k_s\vec{\alpha}_s = \vec{0}$.

Definition 2.9 (Pre-algebra Group, 2003) A partial group of a vector group is called a maximally linearly independent group. If the partial group itself is linearly independent, and any vector (if any) is added from the vector group, the resulting partial vector group is linear correlation.

Attribute reduction of formal context

This section will solve the attribute reduction problem of the formal context by solving the maximally linearly independent set.

Definition 3.1 Let (U,V,R) be a formal context, $U=\{u_1,u_2,...,u_n\}(n\in Z^+)$, $V=\{v_1,v_2,...,v_k\}(k \in Z^+)$. For $\forall v_i \in V(i=1,2,\cdots,k)$, we note the value of the object $u_j \in U(j=1,2,\cdots,n)$ based on the attribute v_i for v_{iu_j} in the formal context (U,V,R). For example, if the object u_j has the attribute v_i, the $v_{iu_j}=1$, otherwise $v_{iu_j}=0$.

Definition 3.2 Let (U,V,R) be a formal context, For $\forall v_i \in V$, $\vec{v}_i=(v_{iu_1},v_{iu_2},\cdots,v_{iu_k})$. The vector group corresponding to the formal context consists of all $\vec{v}_i$.

Example 1

Table 1 is the formal context (U,V,R) with $U=\{1,2,3,4\}$ and $V=\{a,b,c,d,e\}$.

Table 1: Formal context (U,V,R)

	a	*b*	*c*	*d*	*e*
1	1	1	0	1	1
2	1	1	1	0	0
3	0	0	0	1	0
4	1	1	1	0	0

According to the above formal context (U,V,R), it can be known that $\vec{a}=(1,1,0,1),\vec{b}=(1,1,0,1),\vec{c}=(0,1,0,1),\vec{d}=(1,0,1,0),\vec{e}=(1,0,0,0)$. $\vec{a},\vec{b},\vec{c},\vec{d},\vec{e}$ is the vector group corresponding to the formal context (U,V,R).

Theorem 1: Let (U,V,R) be a formal context, $V=\{v_1,v_2,...,v_k\}(k\in Z^+)$, $\vec{v}_1,\vec{v}_2,\cdots,\vec{v}_s(s\le k)(II)$ is the extremely irrelevant group of the vector group $\vec{v}_1,\vec{v}_2,\cdots,\vec{v}_k(I)$. Therefore, the attribute set $D=\{v_1,v_2,\cdots,v_s\}$ is the attribute reduction set of formal context (U,V,R).

Proof: Because $\vec{v}_1,\vec{v}_2,\cdots,\vec{v}_s(s\le k)$ is the extremely irrelevant group of the vector group $\vec{v}_1,\vec{v}_2,\cdots,\vec{v}_k$. Therefore, it satisfies the following properties: ①$\vec{v}_1,\vec{v}_2,\cdots,\vec{v}_s$ are linearly independent. ②Add any vector $\vec{v}_t(v_t\in V,t\le k)$ in $(I)-(II)$ to the vector group (I), so $\vec{v}_1,\vec{v}_2,\cdots,\vec{v}_s,\vec{v}_t$ are linearly related. According to the defination of linear correlation, it can be known that $\vec{v}_t$ can be linearly represented by $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_s$. This shows that attribute v_t can be obtained by attribute set $D = \{v_1, v_2, \cdots, v_s\}$. According to the property of the extremely irrelevant group, the extremely irrelevant group is the minimal set that can able to linearly represent any vector in a vector group. Therefore, the attribute set $D = \{v_1, v_2, \cdots, v_s\}$ is the attribute reduction.

Property 1: Let (U, V, R) be a formal context, $V = \{v_1, v_2, ..., v_k\}(k \in Z^+)$, $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_s (s \le k)(II)$ is the extremely irrelevant group of the vector group $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_k (I)$. Let $D = \{v_1, v_2, \cdots, v_s\}$. Therefore, $1\rangle$ Absolute necessary attribute (core attribute) $b : b \in \bigcap_{i \in \tau} D_i$. $2\rangle$ Relative necessary attribute $c : c \in \bigcup_{i \in \tau} D_i - \bigcap_{i \in \tau} D_i$. $3\rangle$ Absolute unnecessary attribute $d : d \in V - \bigcup_{i \in \tau} D_i$.

This paper is an exposition of the theory of attribute reduction method for formal context. For the method of maximally irrelevant group, see document (Pre-algebra Group, 2003), which is not described here. Hereinafter, the method will be described by way of the following example.

Example 2

In this example, the attribute reduction of the formal context in example 1 will be solved using the method herein.

According to the formal context in example 1, $\vec{a} = (1,1,0,1), \vec{b} = (1,1,0,1), \vec{c} = (0,1,0,1), \vec{d} = (1,0,1,0), \vec{e} = (1,0,0,0)$ are available. So you can get:

$$\begin{array}{c}\begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \\ \begin{matrix} \vec{a} & \vec{b} & \vec{c} & \vec{d} & \vec{e} \end{matrix}\end{array}$$

The matrix is subjected to elementary transformation and converted into a stepped matrix. The detailed process is as follows:

$$\begin{array}{c}\begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \\ \begin{matrix} \vec{a} & \vec{b} & \vec{c} & \vec{d} & \vec{e} \end{matrix}\end{array} \to \begin{array}{c}\begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix} \\ \begin{matrix} \vec{a} & \vec{b} & \vec{c} & \vec{d} & \vec{e} \end{matrix}\end{array} \to \begin{array}{c}\begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{matrix} \vec{a} & \vec{b} & \vec{c} & \vec{d} & \vec{e} \end{matrix}\end{array}$$

From the above results, we know: on the one hand, $\vec{a}, \vec{c}, \vec{d}$ are linearly independent，adding $\vec{b}$ or $\vec{e}$ will be linearly related. Therefore, the extremely irrelevant group is $\vec{a}, \vec{c}, \vec{d}$. On the other hand, $\vec{b}, \vec{c}, \vec{d}$ are linearly independent，adding $\vec{a}$ or $\vec{e}$ will be linearly related. Therefore, the extremely irrelevant group is $\vec{b}, \vec{c}, \vec{d}$. So, there are two attribute reductions in the formal context in example 1, which are

$D_1=\{a,c,d\}, D_2=\{b,c,d\}$. Absolute necessary attributes are c and d. Relative necessary attributes are a and b. Absolute unnecessary attribute is e.

CONCLUSION

In this paper, the algebraic idea is used to process the formal context. According to the formal context, the matrix corresponding to the vector group consisting of attributes is obtained. Through the matrix processing, the maximal irrelevant group of the vector group is obtained, and the attribute reduction of the formal context is obtained. ①This paper enriches the relevant theory of attribute reduction solution for formal context. ②The use of this method for attribute reduction does not require that the formal context should be purified first. ③The same solution can also get the object reduction result. The application of the above theory in the multi-valued formal context remains to be explored later.

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REFERENCES

Bi Z, Xu F, Lei J, et al (2016). Attribute reduction in decision-theoretic rough set model based on minimum decision cost. *Concurrency & Computation Practice & Experience* 28 (15): 4125-4143.

Ganter B and Wille R (1999). Formal Concept Analysis: *Mathematical Foundations*, (Springer-Verlag Berlin, Heidelberg, Germany).

Konecny J (2017). On attribute reduction in concept lattices: Methods based on discernibility matrix are outperformed by basic clarification and reduction. *Information Sciences* 415–416: 199-212.

Mao H (2017). Representing attribute reduction and concepts in concept lattice using graphs. *Soft Computing* 21(24): 7293-7311.

Mao H, Miao HR (2018). Attribute reduction based on directed graph in formal fuzzy contexts. Journal of Intelligent & Fuzzy Systems 34(6): 4139-4148.

Pre-algebra Group, Department of Geometry and Algebra, Department of Mathematics, Peking University (2003). Advanced Algebra. 3rd Edition. *Higher Education Press*.

Ren RS, Wei L (2016). The attribute reductions of three-way concept lattices. *Knowledge- Based Systems* 99: 92-102.

Yao Y, Zhao Y (2008). Attribute reduction in decision-theoretic rough set models. *Information Sciences* 178(17): 3356-3373.

Zhang WX, Wei L, Qi J (2005). Attribute reduction theory and approach to concept lattice. *Science China Information Sciences* 48(6): 713-726.

Zhang T, Lu J, Ren HL (2014). An attribute reduction algorithm based on tree graph. *Minicomputer System* 35 (1): 177-180.