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Research Article

GRAVITATIONAL INERTIA

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ABSTRACT

This paper explains gravitational inertia and the postulate of numerical equality of gravitational mass and inertial mass. It introduces a new perspective on electromagnetic inertia. It infers that time intervals may be finite in gravitational fields.

Keywords: *Inertia, Gravitational Inertia, Electromagnetic Inertia, Gravitational Mass, Inertial Mass, Weak Equivalence Principle, Time.*

INTRODUCTION

The literature is awash with discourses on inertia; however, there is no single universally accepted theory which explains the source of inertia. The well-known idea concerning the physical source of inertia as enshrined in the Mach Principle: The inertia of` a material object – the object's resistance against being accelerated – is not an intrinsic property of matter, but a measure of its interaction with the entities of the universe.

Galileo (*c*. 1610) demonstrated that the speed of a falling body is independent of its weight. Even though confirmed experimentally to a very high degree of accuracy by Eötvös (1890), Roll *et al*. (1964), and several others, there is no single accepted theory which explains the postulate of numerical equality of gravitational mass and inertial mass (that is, the weak equivalence principle).

The literature is not quite clear on electromagnetic inertia.

ASSUMPTIONS

We make three assumptions:

1) An interaction between two objects is communicated by fields along a space-time path.

2) The path of the interaction is continuous.

3) The interaction propagates at a finite speed.

GRAVITATIONAL INERTIA AND ITS SOURCE

Figure 1 shows an object of mass *m* initially at rest at O in a community of objects. The objects in the community are distributed to the "end" points of the X, Y, and Z axes, where each point is assigned an effective mass Ω. (Appendix A shows the details.) Per Assumption (1), gravitational interaction between *m* at O and, for instance, Ω at Y is communicated along the path O-C-Y.

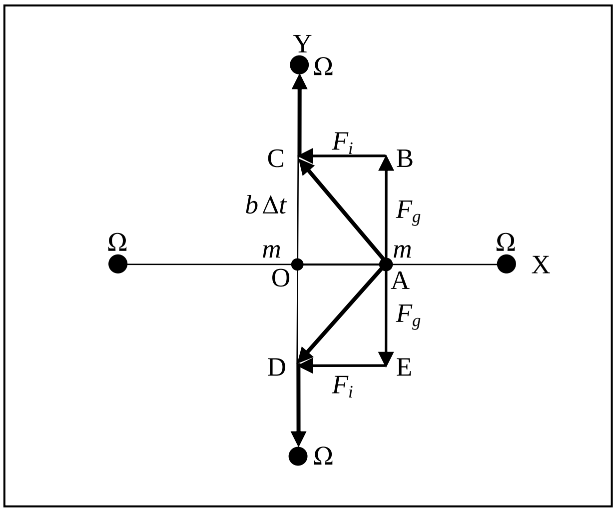

Figure 1: Mass *m* under the gravitational forces of the masses Ω in the community

An impulse is imparted to mass m at O along O-X for duration Δt, which is undetermined at this time. As a result, the mass gains acceleration a at A.

Per Assumption (3), the interaction travels at a finite speed (say, b of gravitational radiation). The effects of the impulse are "felt" only by those masses which are within a sphere of radius $b\Delta t$ by the time the acceleration is completed in Δt; the masses beyond $b\Delta t$ would not even have "known" about the acceleration of m – until after Δt. Therefore, we will consider gravitational forces on m due only to those masses which are within the sphere of radius $b\Delta t$ and use symbol M for those masses.

The impulse displaces segment O-C of the interaction path O-C-Y to A-C. Consequently, according to Assumption (2), path O-C-Y turns into A-C-Y in duration Δt.

Interaction path segment A-C is resolved into A-B, which is the gravitational force F_g, and A-O, which is the force F_i in opposition to the acceleration. (As the force F_i opposes the acceleration a, we name it the force of inertia – in deference to Ernst Mach.) Similarly, segment A-D is resolved into A-E, which is the gravitational force F_g, and A-O, which adds to the force of inertia F_i in opposition to the acceleration. Gravitational forces along AB and AE play no part here in accelerating the mass m.

The impulse affects the two interaction paths in the X-Z plane similarly.

The geometrical ratio of F_i to F_g is:

$$\frac{F_i}{F_g} = \frac{\text{AO}}{\text{AB}} = \frac{(1/2)\,a\,(\Delta t)^2}{b\,\Delta t} = \frac{a\,\Delta t}{2b}, \tag{1}$$

where the gravitational force F_g is given by:

$$F_g = \frac{G\,M\,m}{(\text{AB})^2} = \frac{G\,M\,m_g}{b^2\,(\Delta t)^2}, \tag{2}$$

where G is the classical gravitational constant, and M and m ($\equiv m_g$) are *gravitational* masses to match gravitational force F_g. Gravitational mass participates in gravitational interactions and, so, is an intrinsic property of matter.

Substituting (2) into (1) and adding force A-D and the forces in the X-Z plane, we have:

$$F_i = \left(\frac{2\,G\,M\,m_g}{b^3\,\Delta t}\right) a\ , \tag{3}$$

where the dimension of the term in the bracket is of mass, which is designated *inertial* mass m_i to match inertial force F_i :

$$m_i = \frac{2\,G\,M\,m_g}{b^3\,\Delta t} \tag{4}$$

Inertial mass does not participate in gravitational interactions. Rearranging (4), we get:

$$\frac{m_i}{m_g} = \frac{2G}{b^3}\,\frac{M}{\Delta t}, \tag{5}$$

that is, the ratio m_i/m_g varies with $M/\Delta t$ but is independent of acceleration a.

Equations (3) and (4) represent gravitational inertia and inertial mass respectively, and we make the following inferences:

1) The impulse and the acceleration caused by it are resisted by the force of inertia.

2) An object has inertia because: it is participating in a community of mutually interacting objects; interactions produce accelerations; accelerations adjust lines of interactions; and interactions and associated adjustments propagate at a finite speed.

3) The inertial mass of an object is proportional directly to its gravitational mass and the community's mass (within a radius of $b\Delta t$) and inversely to the duration of acceleration and cube of the propagation speed of gravitational interaction.

4) As $M \to 0$, $F_i \to 0$ and $m_i \to 0$. That is, an object in isolation would have no inertia. That is, the objects (M), which are within a sphere of radius of $b\Delta t$, endow the object (m) with inertia for Δt.

5) As $b \to \infty$, $F_i \to 0$ and $m_i \to 0$. That is, if interactions were to propagate at infinite speed, there would be *no* inertia. (In a chamber containing gas particles, the time it takes for the information to travel the mean free path is negligible. That is, a particle has virtually no inertia by other gas particles in the chamber.)

We introduced the concepts of inertia and inertial mass without using the Newton's laws of motion.

GRAVITATIONAL MASS AND INERTIAL MASS

We address the numerical equality of gravitational mass and inertial mass.

An object of gravitational mass m_g and inertial mass m_i is at a distance r from the center of the earth (mass M). The object is falling toward the earth with acceleration a due to gravitational force F_g exerted by the earth. From the Newtonian mechanics, we have:

$$\frac{G\,M\,m_g}{r^2} = F_g = m_i\,a \tag{6}$$

Rearranging (6), we get:

$$a = \left(\frac{G\,M}{r^2}\right)\frac{m_g}{m_i}, \tag{7}$$

that is, at any point $r = r$ during the fall, the acceleration a of an object is proportional to its m_g/m_i. Previously, from (5), we inferred that m_i/m_g is independent of acceleration a. These two seemingly contradictory statements may be reconciled if m_i/m_g is a *numeral*.

The classical treatment of electromagnetic radiation pressure shows that an electromagnetic wave has linear momentum $p = E/c$ in the direction of propagation (E denotes energy and c speed). That is, the inertial mass of the electromagnetic wave would be E/c^2. As the electromagnetic wavelength ranges from about 10^{-18} m through 10^5 m, the inertial mass (m_i) of electromagnetic waves ranges from about 10^{-47} kg through 10^{-24} kg. Their gravitational mass (m_g) is zero and inertial mass (m_i) infinitesimal. That is, for *zero-mass objects*:

$$m_i \approx \varepsilon, \tag{8}$$

where ε is an infinitesimal number.

We re-express (8) for a *nonzero-mass object* of infinitesimal gravitational mass m_g as follows:

$$m_i - m_g \approx \varepsilon - m_g \approx \varepsilon' \tag{9}$$

Rearranging (9), we get:

$$\frac{m_i}{m_g} = 1 + \frac{\varepsilon'}{m_g} \tag{10}$$

The ratio m_i/m_g is a numeral; so, as m_g increases, m_i increases. As m_g increases, m_i/m_g gets closer to 1.

So far, we have found that: ratio m_i/m_g is a *numeral*; difference $(m_i - m_g)$ is positive and infinitesimal; and as m_g increases, m_i/m_g gets closer to 1. Based on these findings, we may make the *numeral* approximately, but not exactly, equal to one:

$$\frac{m_i}{m_g} \approx 1, \tag{11}$$

this is the postulate of numerical equality of gravitational mass and inertial mass.

The inertial mass and gravitational mass of an object are not exactly equal; the inertial mass is greater than gravitational mass; and the numerical difference between them varies from about 10^{-24} kg for zero-mass objects to about zero for massive objects. The weak equivalence principle holds but only approximately.

We infer from (7) and (11): All objects fall at the same rate – as observed by Galileo.

We addressed the (so-called) postulate of numerical equality of gravitational mass and inertial mass (or the weak equivalence principle) without invoking the observations of Galileo and of Eötvös.

TIME IN GRAVITATIONAL FIELD

We have from (5) and (11):

$$\Delta t = \frac{2G}{b^3} M \tag{12}$$

As long as M is fixed, Δt is fixed. That is, Δt is the *finite* interval of time in which impulses may be imparted and accelerations produced in the gravitational field of a mass.

The speed of gravitational radiation has not been *measured* yet. The literature considers that the speed of gravitational radiation (b) is numerically equal to the speed of electromagnetic radiation (c). A recent theory (Singh, 2017) inferred that the speed of gravitational radiation (b) is less than the speed of electromagnetic radiation (c); that is, $b = 0.2222\, c$.

We now use (12) to calculate the magnitude of Δt for the aforesaid speeds of gravitational radiation:

1) $b \equiv c$:

Substituting in (12) the magnitudes of G and c from Appendix B, we have:

$$\Delta t \approx (4.952 \text{ x } 10^{-36})\, M \tag{13}$$

Near a mass of 1 kg, $\Delta t \sim 10^{-36}$ sec; near Earth, $\Delta t \sim 10^{-12}$ sec; near Sun, $\Delta t \sim 10^{-3}$ sec; and near the black hole at the Milky Way's center, $\Delta t \sim 40$ secs.

2) $b = 0.2222\, c$:

Substituting in (12) the magnitudes of G and b from Appendix B, we have:

$$\Delta t \approx (4.512 \text{ x } 10^{-34})\, M \tag{14}$$

Near a mass of 1 kg, $\Delta t \sim 10^{-34}$ sec; near Earth, $\Delta t \sim 10^{-9}$ sec; near Sun, $\Delta t \sim 10^{-6}$ sec; and near the black hole at the center of Milky Way, $\Delta t \sim 10^{3}$ secs (16.7 minutes).

ELECTROMAGNETIC INERTIA

We take a lattice formed with ionized atoms, which are under mutual electrostatic interactions. An electromagnetic radiation (or a photon) accelerates an ionized atom in the lattice.

We will re-interpret Figure 1 and Equations (1) through (5) for the lattice, which is now a community of ionized atoms. We will use the following symbols: e (charge), F_e (electrostatic force), c (the speed of light), Q (Coulomb's constant), and E (total charge within a sphere of radius $c\Delta t$).

Equation (1) becomes:

$$\frac{F_i}{F_e} = \frac{\text{AO}}{\text{AB}} = \frac{(1/2)\, a\, (\Delta t)^2}{c\, \Delta t} = \frac{a\, \Delta t}{2c}\ , \tag{15}$$

where the electrostatic force F_e is given by:

$$F_e = \frac{Q\, E\, e}{(\text{AB})^2} = \frac{Q\, E\, e}{c^2\, (\Delta t)^2} \tag{16}$$

Substituting (16) into (15) and adding the force AD and the forces in the X-Z plane, we have:

$$F_i = \left(\frac{2Q\, E\, e}{c^3\, \Delta t} \right) a\ \ , \tag{17}$$

where the dimension of the term in the bracket is of mass, which is designated inertial mass m_i to match inertial force F_i:

$$m_i = \frac{2Q\, E\, e}{c^3\, \Delta t} \tag{18}$$

Rearranging (18), we get:

$$\frac{m_i}{e} = \frac{2Q}{c^3} \frac{E}{\Delta t}\ , \tag{19}$$

that is, e/m_i of an object is independent of acceleration a.

Equations (17) and (18) represent electromagnetic inertia and inertial mass respectively, and we make the following inferences:

1) The impulse and the acceleration caused by it are resisted by the force of inertia.

2) A charge has inertia because: it is participating in a community of mutually interacting charges; interactions produce accelerations; accelerations adjust lines of interactions; and interactions and associated adjustments propagate at a finite speed (c).

3) The inertial mass of a charge is proportional directly to its charge and the community's charge (within a radius of $c\Delta t$) and inversely to the duration of acceleration and cube of the propagation speed of electromagnetic interaction.

4) As $E \to 0$, $F_i \to 0$ and $m_i \to 0$. That is, a charge in isolation would have no inertia. That is, the charges (E), which are within a sphere of radius of $c\Delta t$, endow the charge (e) with inertia for Δt.

5) As $c \to \infty$, $F_i \to 0$ and $m_i \to 0$. That is, if interactions were to propagate at infinite speed, there would be *no* inertia.

CONCLUSION

The inertia of an object is its resistance against being accelerated. An object has *inertia* because: it is participating in a community of mutually interacting objects; interactions produce accelerations; accelerations adjust lines of interactions; and interactions and associated adjustments propagate at a finite speed. The inertia of an object is *not* its inherent (self) property. Inertia is an endowed property; an object is endowed with inertia as it interacts with all other objects (in its sphere of influence). Based on the underlying interaction, there are two types of *inertia*: gravitational and electromagnetic.

There are two aspects of *mass*: gravitational and inertial. Gravitational mass participates in gravitational interactions; inertial mass does not. Gravitational mass is greater than or equal to zero; inertial mass is greater than zero. The difference between inertial mass and gravitational mass is numerically infinitesimal; the ratio of inertial mass to gravitational mass is virtually equal to one. Not all objects have gravitational mass, but all objects have inertial mass.

It takes longer to accelerate a mass in a stronger gravitational field.

Time intervals in which impulses are imparted and accelerations produced are finite.

REMARKS

We addressed gravitational and electromagnetic inertia, because those are the only two forces known at present at the macroscopic levels. Inertia due to the strong and the weak interactions – at the microscopic levels – are neither entertained nor known.

Inertial mass is greater than zero, but we could not explain *why* that is so. One plausible explanation would be: Each and every object in the universe is in motion and, so, has momentum relative to, besides other objects, the space-time "point" where the universe originated. In the universe, each and every object, whether of zero-mass or nonzero-mass, possesses momentum and, so, has inertial mass.

Equation (12) may be used to estimate b, provided Δt can be measured. Such an experiment should reveal whether and why the speed of gravitational radiation is equal or unequal to the speed of electromagnetic radiation. At this time, we are unable to suggest an experiment to measure Δt.

Equation (12) may as well be used to estimate Δt, provided b can be measured. Singh (2017) suggested an experiment to measure b.

Gravity affects the passage and the interval of *time*. In an earlier paper (Singh, 2017), we found that the gravitational dilation of time periods of atomic clocks relative to infinity is of the order of 10^{-9} sec at the earth and 10^{-6} sec at the sun. In this paper, we inferred that in the time interval of the order of 10^{-9} sec at the earth and 10^{-6} sec at the sun, impulses may be imparted and accelerations produced. The time interval of the order of 10^{-9} sec at the earth may be of significance.

Some inferences could not be drawn satisfactorily, even though this paper hints at them. We advance them here as *hypotheses*:

1) The inertia of an object in the interior of the universe is not the same as that near its "edge."
2) Electromagnetic induction is a sign of electromagnetic inertia.
3) Time intervals are finite (quantum) in gravitational fields.
4) Time intervals are finite (quantum) in electromagnetic fields.

Inertia and the weak equivalence principle will be addressed again in a separate paper but within the framework of the new gravitational model (Singh 2017).

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APPENDIX A: A COMMUNITY OF MASSES

Figure 1 shows a quadrant of a sphere containing objects. The radius of the sphere is *R* and of the quadrant *r* (≤ *R*). The objects in the sphere are of total mass μ and are homogenously distributed.

We select one object of mass *m* and place it at the center of the sphere. We will show that the masses in the sphere may be distributed to the six "end" points of the X, Y, and Z axes so that net gravitational forces of the masses on *m* is mathematically equivalent to the gravitational forces of the distributed masses on *m*.

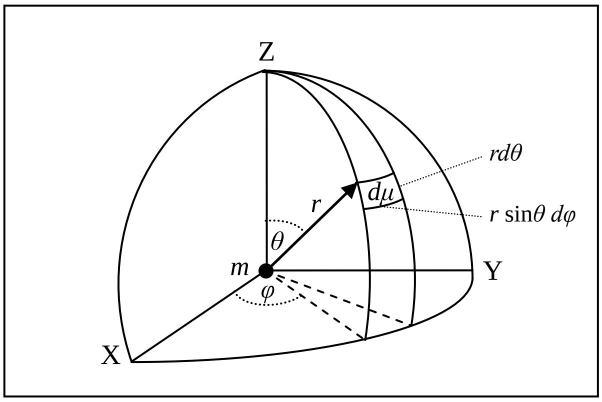

Figure 2: A quadrant (radius *r*) of the sphere containing objects

We start with an infinitesimal mass $d\mu$ at coordinates (r, θ, φ) in a small volume $d\upsilon$ bounded by line elements dr (not shown), $rd\theta$, and $r sin\theta d\varphi$. Volume $d\upsilon$ is given by:

$$d\upsilon = r^2 \sin\theta d\theta d\varphi dr \tag{A1}$$

Mass density ρ of the sphere is given by:

$$\rho = 3\mu / 4\pi R^3 \tag{A2}$$

From (A1) and (A2), $d\mu$ is given by:

$$d\mu = \rho r^2 \sin\theta d\theta d\varphi dr \tag{A3}$$

Gravitational force $d\mathbf{F}$ on m due to $d\mu$ is given by:

$$d\mathbf{F} = \frac{Gm\,d\mu}{r^3}\,\mathbf{r} \tag{A4}$$

From (A4) and Figure 2, the *x*, *y*, *z* components of $d\mathbf{F}$ are given by:

$$dF_x = \frac{Gm\,d\mu}{r^2}\sin\theta\cos\varphi;\quad dF_y = \frac{Gm\,d\mu}{r^2}\sin\theta\sin\varphi;\quad dF_z = \frac{Gm\,d\mu}{r^2}\cos\theta \tag{A5}$$

The *x*, *y*, *z* components of the gravitational force on *m* by all the masses in the sphere is given by substituting (A3) in (A5) and then integrating them as follows:

$$F_x = G\rho m\int_0^R \int_{-\pi/2}^{\pi/2} \int_0^{\pi} \sin^2\theta d\theta \cos\varphi d\varphi dr = \frac{3Gm\mu}{4R^2} \tag{A6}$$

$$F_y = G\rho m\int_0^R \int_0^{\pi} \int_0^{\pi} \sin^2\theta d\theta \sin\varphi d\varphi dr = \frac{3Gm\mu}{4R^2} \tag{A7}$$

$$F_z = G\rho m\int_0^R \int_0^{2\pi} \int_0^{\pi/2} \sin\theta\cos\theta d\theta d\varphi dr = \frac{3Gm\mu}{4R^2} \tag{A8}$$

The magnitudes of the forces along -x, -y, and -z are the same as above.

From (A6), (A7), and (A8), we infer that a mass of $3\mu/4 \equiv \Omega$ may be assigned at each of the six "end" points of the X, Y, and Z axes at a distance of *R* from *m* in order to get the equivalent effects.

We note that the magnitude of neither Ω nor *R* would be needed in calculations in this section.

APPENDIX B: PHYSICAL DATA

Speed of gravitational radiation (b): 6.661×10^7 m/s (Singh, 2017)
Speed of light (c): 2.998×10^8 m/s;
Newton's constant (G): 6.672×10^{-11} nt-m^2/kg^2;
Sun's mass: 1.989×10^{30} kg;
Earth's mass: 5.976×10^{24} kg;
Milky Way's black hole's mass: 3.2 million suns. (Singh, 2017)