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Research Article

STABILITY OF EIFFEL TOWER IS ON THE BASIS OF STRUCTURAL DESIGN OF HUMAN FEMUR AND ITS MATHEMATICAL ANALYSIS

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ABSTRACT

Eiffel Tower is an epitome of *light-weight beauty* and *gigantic structure* whereas human femur is a *light-weight living structure* with static as well as dynamic efficiency of body-weight transmission. Here, analogy between Eiffel's design of tower and Nature's design of human femur has been discussed. Economic use of materials with their roots in rigorous engineering principles in comparison with human femur has been discussed and explained on the basis of structural engineering and anatomical view point with the use of mathematical tool.

Keywords: *Eiffel Tower, Human Femur, Mathematical Analogy*

INTRODUCTION

Figure 1: First Design of Eiffel Tower by Maurice Koechlin

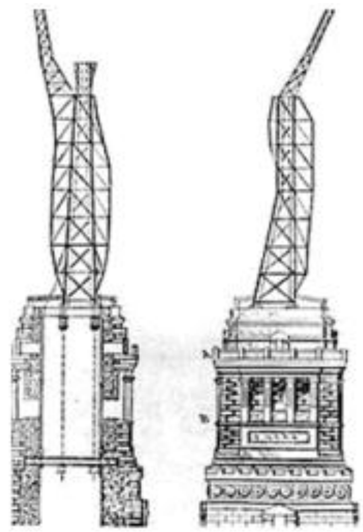

Figure 2: Interior Structural Design of the Statue of Liberty Designed by Gustave Eiffel

Initially, it was decided to install the *Statue of Liberty* at the place. So, they contacted with Auguste Bartholdi (2 August 1834 – 4 October 1904; he was a French sculptor who is best known for designing *Liberty Enlightening the World*, commonly known as the Statue of Liberty). On that basis Bartholdi employed Eugène Viollet-Le-Duc (27 January 1814 – 17 September 1879; who was a French architect and theorist, famous for his interpretive "restorations" of medieval buildings. Born in Paris, he was a major Gothic Revival architect. He was the architect hired to design the internal structure of the Statue of Liberty, but died before the project was completed) for the construction of statue. Some work had already been carried out by Eugène but he had died in 1879. Then Bartholdi felt need of an engineer to complete the left work or new construction of *Statue of Liberty* and contacted with Eiffel in 1881 as because of his experience with wind stresses. Eiffel devised a structure consisting of a four-legged pylon to support the copper sheeting which made up the body of the statue [Figure 2].
In 1884 French republic had announced a contest for a spectacular centerpiece for the Paris World-Fair in 1889 and Gustave Eiffel's (15 December, 1832 – 27 December, 1923; French civil engineer and architect,

best known for world famous Eiffel Tower. He made research on meteorology and aerodynamics) firm took the opportunity and was selected to erect the tower prior to world fair. Two young engineers Émile Nouguier (17 February, 1840 CE – 23 November, 1897; French Civil Engineer and Architect) assisted chief engineer to design Eiffel Tower and Maurice Koechlin (8 March, 1856 – 14 January, 1946; Swiss structural engineer) first designed the Eiffel Tower as chief engineer and an architect Stephen Sauvestre (26 December, 1847 – 18 June, 1919; a French Architect) helped in designing Eiffel Tower and were included in this project under Eiffel's Engineering Firm *(Compagnie des Establissements Eiffel)*. They created an initial design of a one-thousand feet high iron tower. In May 1884 Koechlin, working at his home, made an outline drawing of their scheme, described by him as *"a great pylon, consisting of four lattice girders standing apart at the base and coming together at the top, joined together by metal trusses at regular intervals"* [Figure-1] which pleased Eiffel where he made further refinements and improvements to promote World-Fair's monument as Eiffel Tower (Eiffel Tower's structure first appeared for public on 22 October, 1884 in the back page of *Le Figaro*. The newspaper noted *"one of the most extraordinary project is certainly a 300-metres iron tower that M. Eiffel proposes to build"*.

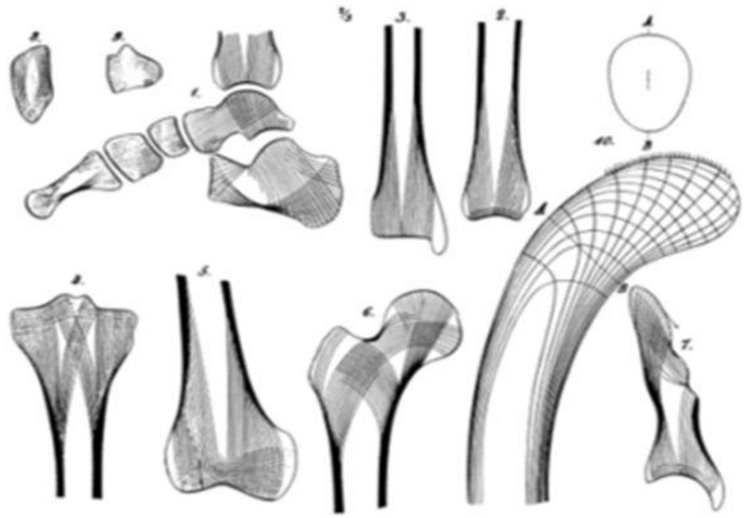

Figure 3A: G. H. von Meyer Showed Composite Illustrations on Sections of Various Human Bones with Stylized Arching Trabecular Pattern (Numbers 1-7) and Culmann-Crane Structure of Human Femur AB (Meyer, 1867)

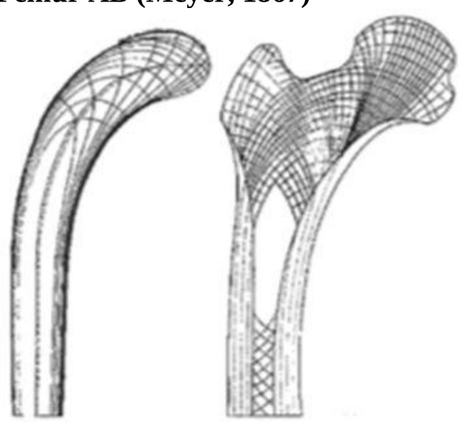

Figure 3B: Culmann's Crane in Comparison with Human Femur; Comparing its Trabecular Structure and Lines of Forces within the Crane

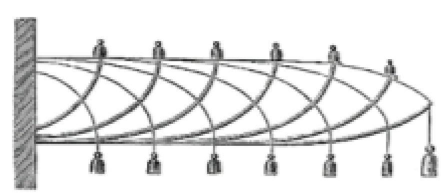

Figure 3C: Culmann's Cantilever Using Trabecular Arch within Proximal Human Femur as Per von Meyer (Culmann, 1866); Here all the Trabecula are Weight Bearing

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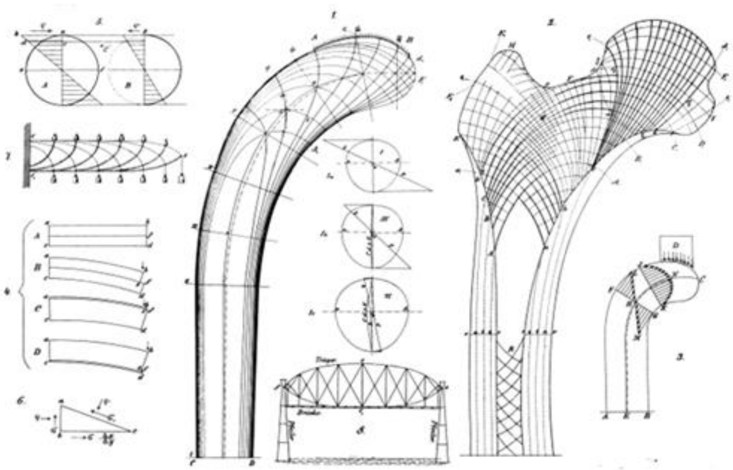

Figure 3D: Julius Wolff's Composite Diagram: Reproduction of Culmann's Cantilever Beam and Crane (no.7 & 1); Wolff got the Drawing of Crane and most other Structures from Culmann (Wolf, 1892; 1873); No.1 of the Diagram Illustrate Forces and Trajectories Acting within Bone may be Used in Crane Structure where no.1a - 1c Depict the Force Layout for Selected Cross Section Examples I, II and VI in Solid Interior (Culmann's Technique); No. 2 of Diagram Represents the Graphical Static Lines of Forces within the Human Femur; No.8 of the Diagram Gave Schematic Illustration of Bridge

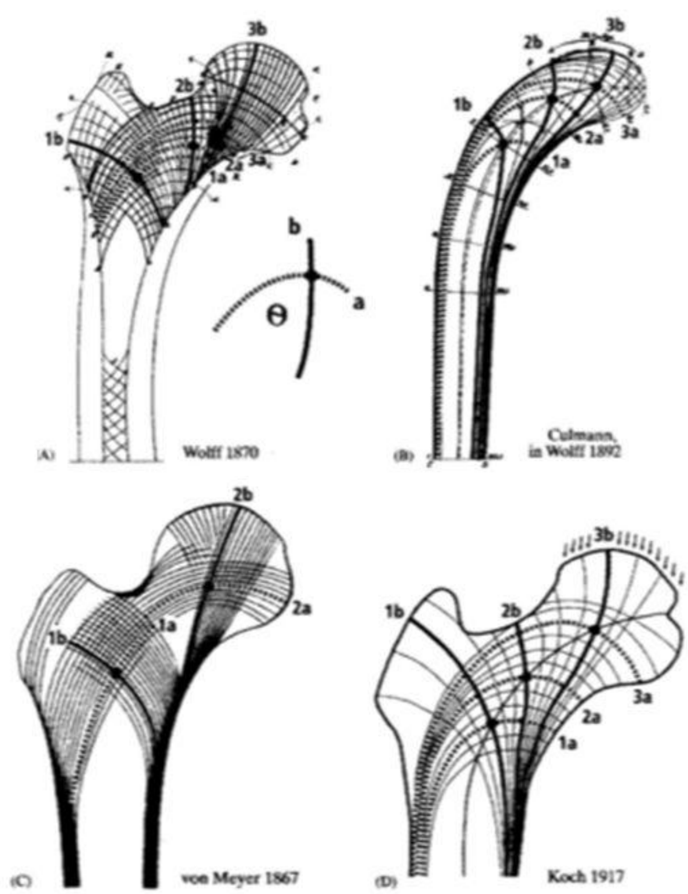

Figure 3E: Some Trajectorial Structures of Femur; These Figures Shows Stress Transfer; θ indicates the Angle between Lines of Forces (Skedros and Baucom, 2007)

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Koechlin being a student of Karl Culmann (A famous German Structural Engineer and Mathematician; 10 July, 1821 – 9 December, 1881) interacted with him about the structure, discussed with Meyer about possible mechanical relevance of bone. In his text book *"Graphical Statik" he described 'how transmission of stresses in structures could be determined with use of graphical analysis.*) and intended to use Culmann's illustrated text on solid structures showing calculation of stress trajectories [Figures - 3B and 3C] by the influence of von Meyer's (16 August, 1815 – 21 July, 1892; a German Anatomist, in 1860, first detected *Architecture of the trabecular bone and spongy substance in it.* In his books *"Statics and Mechanics of the Human Skeleton"* he discussed the matter) ideas about the mechanical relevance of trabecular architecture within the human femur. Koechlin used Culmann's approach in designing and engineering structure of Eiffel Tower.
According to Richard A. Brand clear statement of Wolff's Law: *"The law of bone remodelling is that mathematical law according to which observed alterations in the internal architecture and external form of bone occur as a consequence of the change in shape and / or stressing of bone"* (Skedros and Baucom, 2007).
Wolff made observations, measurements on two-dimensional, cantilever beams and curved beams having intersecting compression / tension stress trajectories. Results showed in calcanei:
a) the same non-linear equation best described the dorsal (compression) and plantar (tension) trabecular tracts;
b) these tracts could be exactly superimposed on the corresponding compression / tension stress trajectories of the cantilevered beams;
c) trabecular tracts typically formed orthogonal intersections.
Wolff also proved that *the human proximal femur is a trajectorial structure.*
So, *Trabecular bone maintains strength and architectural (connectivity) properties.*
Mathematical description of natural phenomena never reflect causation, but rather merely describe and often accurately predict. Confusing explanatory descriptions of cause and effect often lead to misunderstanding about mathematical laws governing the natural phenomena (Levangie and Norkin, 2001).

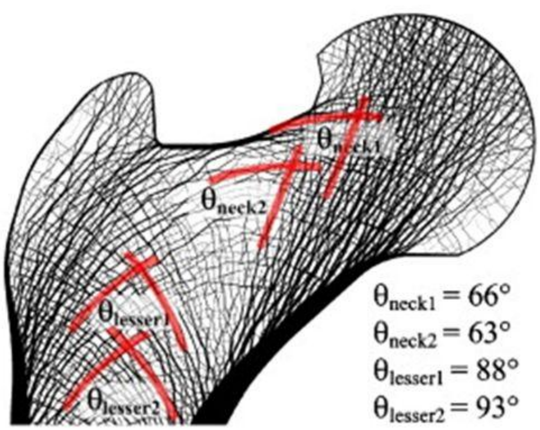

Figure 3F: Angle of Intersection of Trabeculae within Proximal Femur

Opposing calcified trabecular tracts typically form in orthogonal and quasi-orthogonal intersection intersections.
Trabecular tracts in the human femoral neck are quasi-orthogonal. Their shapes often differ from the trabecular tracts and stress trajectories of simply loaded beams [Figure 3F].
Bone is hard and compact (Compact bone is known as *cortical bone*) outside whereas spongy (Spongy bone is known as *trabecular bone*) inside. Compact part acts as pillar and spongy part plays an important role in carrying the pushing & pulling forces that our bone constantly endure.

Figure 3A: Start of Erection of Metal-Work on 28 January, 1887

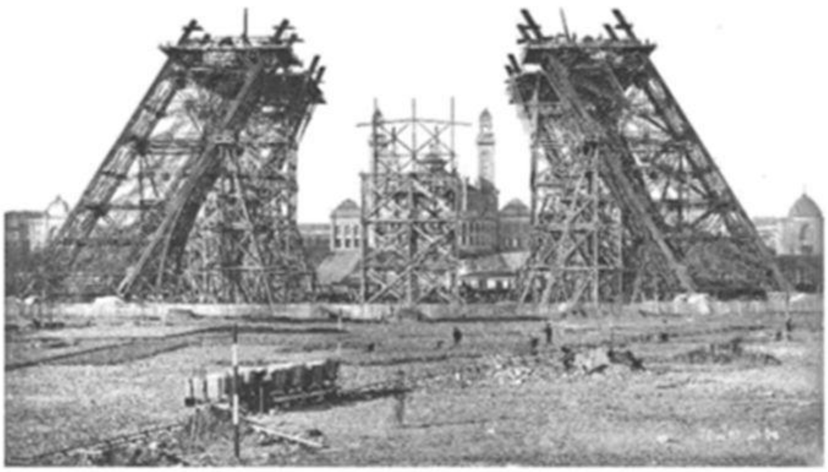

Figure 3B: Construction of Legs with Scaffolding, 7 December, 1887

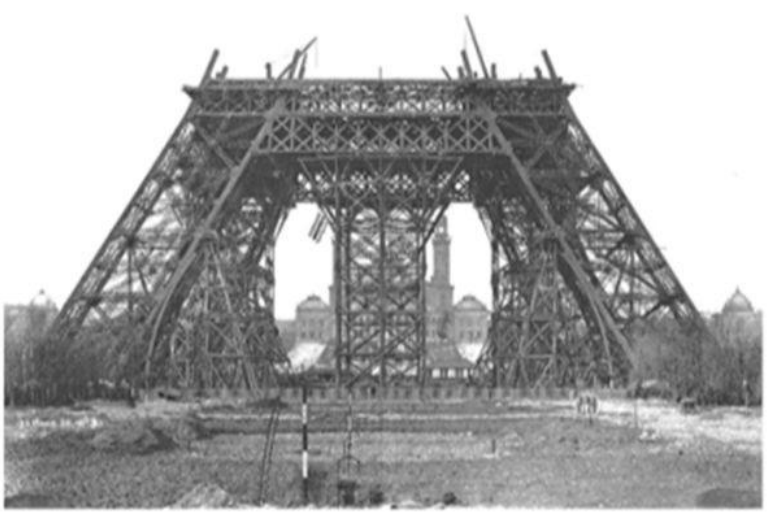

Figure 3C: Completion of First Level, 20 March, 1888

Figure 3D: Start of Construction of Second Level, 15 May, 1888

Figure 3E: Completion of Second Level, 21 August, 1888

Figure 3F: Construction of Upper Level, 26 December, 1888

Figure 3G: Construction of Cupola, 15 March, 1889

Figure 3H: Top of Eiffel Tower

RESULTS AND DISCUSSION

Discussion

Figure 4: Eiffel Tower

Figure 5: Human Femur

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EIFFEL Tower is an epitome of light-weight beauty and gigantic structure where as Human Femur is also light-weight static as well as dynamic efficiency of body weight.

Let us start from the base construction [Figure – 6A]. At the erection base structure Eiffel had constructed it at an angle 30^0 with vertical line i.e. at an angle of 60^0 with horizontal whereas the base is along inclined straight line. He took the opportunity of the consideration of his structural engineer Maurice Koechlin's idea on the basis human femoral condylar angle with the vertical [Figure – 6B].

Figure 6A: Erection of Base

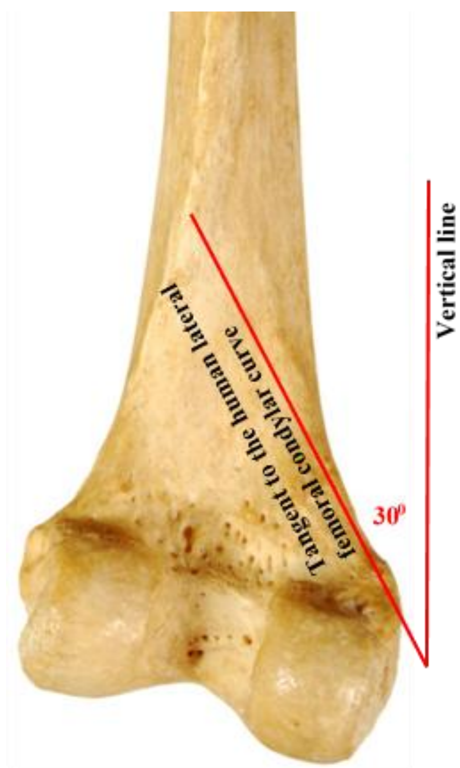

Figure 6B: Angle Made by the Tangent to the Side Curve of Condyle with Vertical

Figure 7A: Arch between the Inclined Pillars

Figure 7B: Arch between the Femoral Condyles

The arches within two inclined pillars, at the base of Eiffel Tower [Figure – 7A], was parabolic structure and arch between condyles of the femur is also parabolic [Figure – 7B]. These geometrical figures tally with the Figures – 7C & 7D respectively.
In both the cases parabolic arches assumes the mathematical formula $x^2 = -y$ as because both parabolas were considered and drawn by multiplying the graph-unit i.e. one is five-times the other (One arbitrary unit is 0.1 and other is 0.5). Opening of a parabola could be manipulated. Here, vertex (0, 0), axis of the parabola is negative direction of *y*-axis, tangent to the vertex is *x*-axis, focus $\left(\frac{1}{4}, 0\right)$ and equation of Directrix: $y - \frac{1}{4} = 0$.

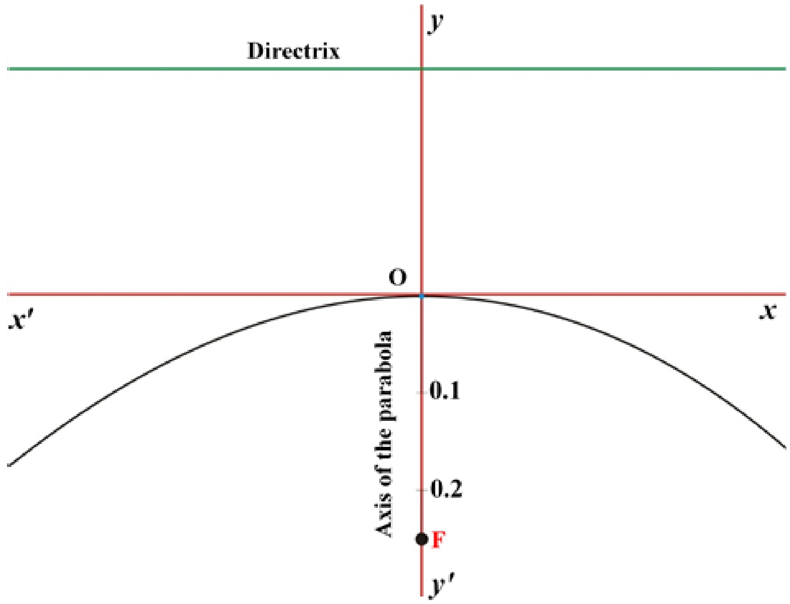

Figure 7C: Parabolic Arch between the Inclined Pillars

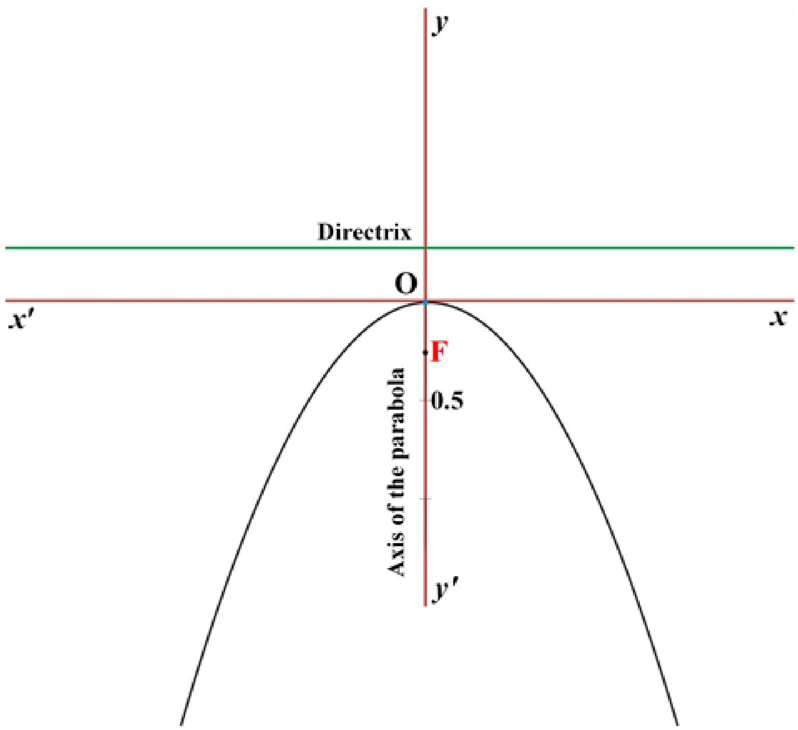

Figure – 7D: Parabolic arch between the femoral condyles

Opening of the parabolas are different according to units of the axes. Unit of axes considered in parabola under Figure – 7C is 5-times that in [Figure – 7D]. This multiplication make the parabolic arch so flat-type [Figure – 7A] than parabolic arch within the femur [Figure – 7C]. Engineers considered arches between the inclined pillars of the tower to bind them and to distribute weight of the tower among the four legs. They took the idea of bearing the weight of upper part of the body by condyles including an arch within it. On the basis of weight bearing device within human body at femoral condyles the base of Eiffel tower was erected.

Arches in the Eiffel Tower can be looked upon as parabolic (Parabola: It is a locus of a point equidistance from a single point, called the focus of the parabola, and a line, called the Directrix of the parabola) in nature. (A parabola may be transformed into a flat-parabola from a narrow-parabola and vice-versa by multiplying its graph units uniformly or reverse keeping the same status of the parabola i.e. focus, vertex to be same only Directrix will be nearer or further i.e. equation of the parabola remains same. Since Directrix of the parabola is an infinite line then moving the focus of the parabola has no effect on the parabola. It only zooming in and out).
General mathematical conception on above demonstration used in Eiffel Tower may be expressed by following figure:

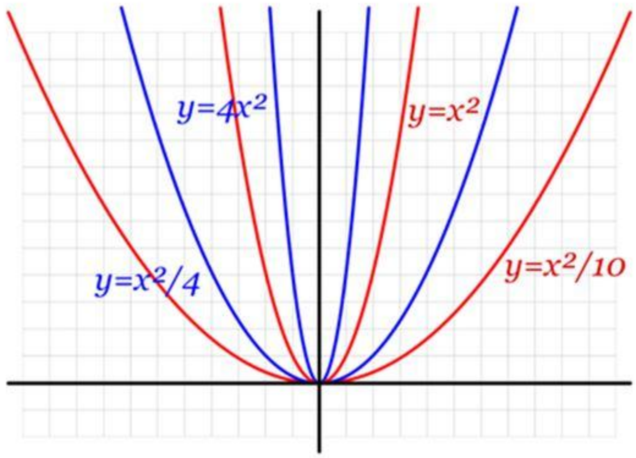

Figure 7E: Parabolas with Different Spans

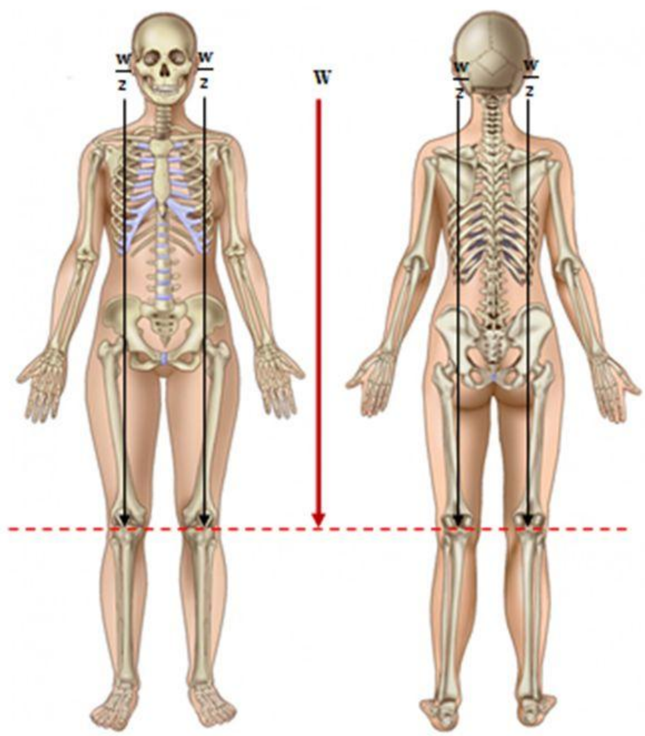

Figure 8: Transmission Body-Weight of Upper-Body; Left One is Anterior View and Right One is Posterior View

Here, we have considered the equation of the parabola as: $y = ax^2$ where *'a'* is the parameter and *'a'* has assumed values $4, 1, \frac{1}{4}, \frac{1}{10}$ which show that reducing the values of the parameter indicating increasing span.
Therefore, we see parabola may be transformed into flat or narrow either by multiplying its graph units or by multiplying the parameter *'a'*.
This idea has been introduced in the Eiffel Tower in choosing base arch for weight bearing as well as weight distributing. Parabolic arch between the condyles share different weights in different activities of human body. The giant structure of Eiffel Tower is not experiencing different activities and stands still. So, one type of arch was used.
Let us consider the body-weight above the femoral condyles = W then this 'W' will be transmitted through condyles in both legs equally as $\frac{W}{2}$ [Figure-8].

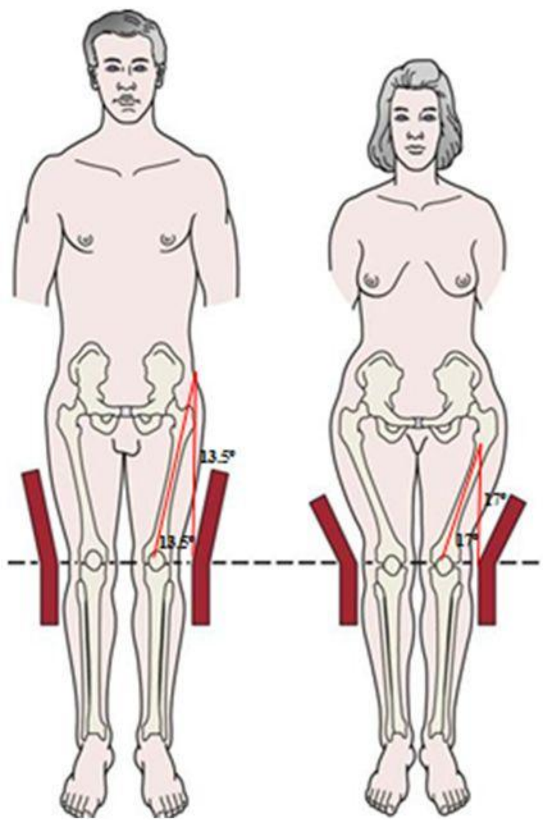

Figure 9: Quadricep Angle in Male and Female

In standing position (erect position) human femur inclined at some angle with the vertical line i.e. with the line of weight bearing.
1. This angle is known as *Quadricep Angle* or *Q-Angle* (The Quadricep-angle or Q-angle of the knee is a measurement of the angle between the quadriceps muscles and the patella tendon). This provides useful information about the alignment of the knee-joint.
a) A line representing the resultant line of force of the quadriceps, made by connecting a point near the ASIS (anterior-superior iliac spine) to the mid-point of the Patella (Patella is not shown but space for patella has been shown in the picture).
The Q-angle can be measured in laying or standing. Standing usually more suitable, due to the normal weight-bearing forces being applied to the knee-joint as occurs during daily activity.
In male it is 13.5^0 and in female it is 17^0 in general. But in some cases it may vary [Figure 9].

Here, average Quadricep Angles among male and female = $(13.5^0 + 17^0) \div 2 = 15.25^0$.

Figure 10A: Quadricep Angle

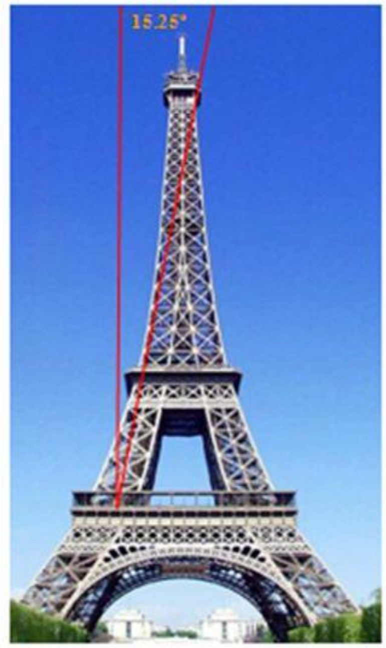

Figure 10B: Quadricep Angle Used in Eiffel Tower

In Eiffel Tower, tower above the base was build keeping curve alongside and this curvature was considered in such a way that tangent to the curve keeps 15^0 [Figure – 10 B] with the vertical and this angle is an average of quadricep angles in male and female human beings [Figure – 10A].
Curvature of the tower maintained above the base structure in parabolic curvature turning at angle of 15^0 [Figure – 10B].

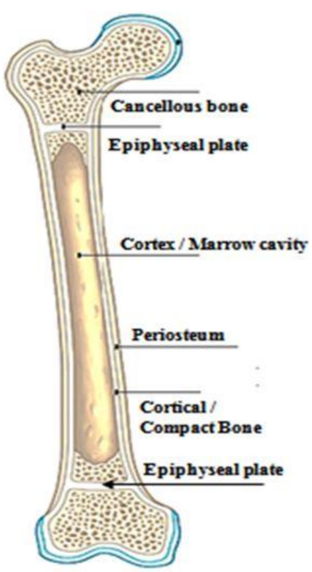

Figure 11A: Internal Structure of Human Femur

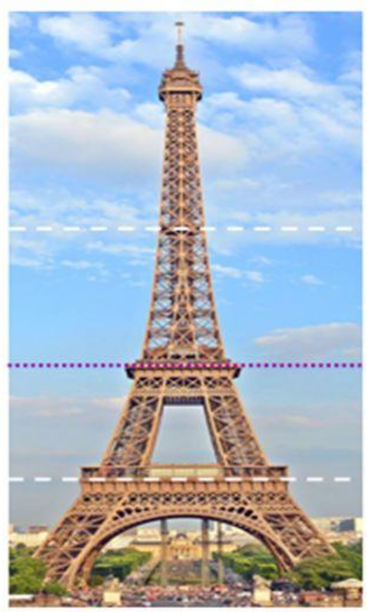

Figure 11B: Binding in the Structure

We see there are bindings in the femur by *Epiphyseal Plates* [Figure – 11A]. These bindings had been employed in the tower mentioned by white dotted lines [Figure – 11B].

Cortex / Marrow cavity [Figure – 11A] was also maintained within the tower structure as pyramidal curved cavity between upper and lower bindings [Figure – 11B] mentioned between two white dotted lines.

For security and stability an additional binding had been introduced, mentioned in violet coloured dotted line [Figure – 11B].

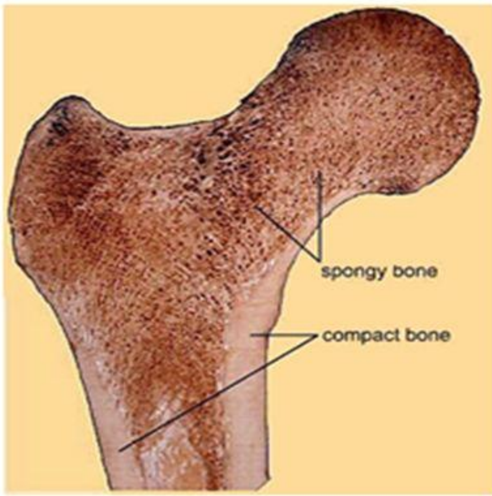

Figure 12A: Internal Structure of Upper Part of Femur Showing (a) Trabecular / Spongy Bone, (b) Cortical / Compact Bone

Figure 12B: 50 Times Demonstration of Trabecular / Spongy Bone within the Upper Part of Femur

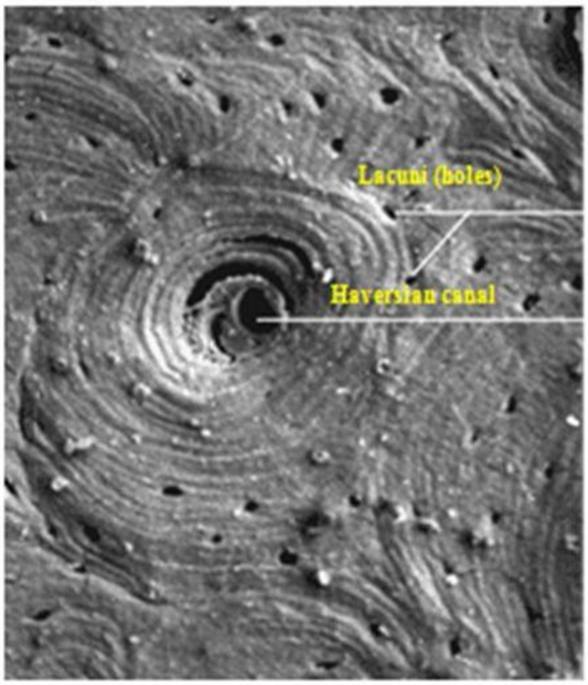

Figure 12C: 800 Times Scanning Electron Micrograph (SEM) of Haversian Canal in Compact Bone

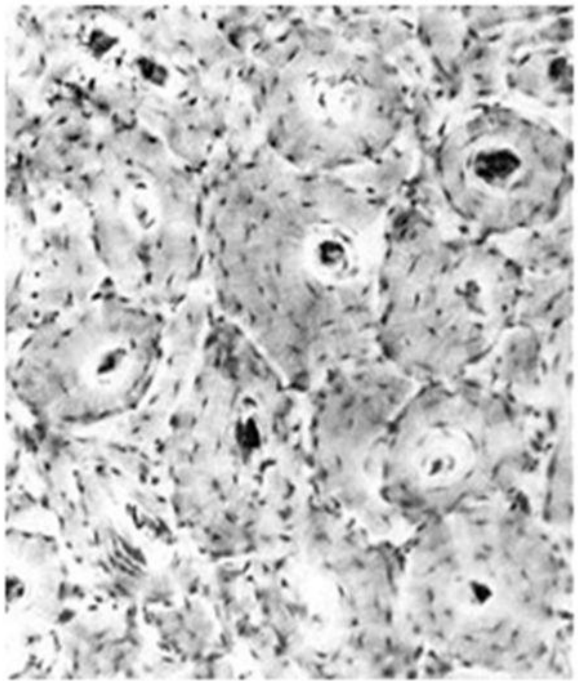

Figure 12D: 350 Times Scanning Electron Micrograph (SEM) of Haversian Canal in Compact Bone

Compact bone or cortical bone is made up many rod-like units called osteons or Haversian systems which run longitudinally within the bone [Figure – 12C]. Haversian systems have a central Haversian canal which carries blood and lymphatic vessels and nerve branches [Figure – 12D]. The central dark circle is the Haversian canal, the canal is surrounded by rings a concentric lamellae of calcified bone matrix. The lamellae contain dark spots, these are the lacuni (holes) where osteocytes (a type of bone cell) occur [Figures – 12C & 12D]. Compact bone forms the outer shell of all bone and also the shafts in long bones [Figure – 12A].
It is dense bone with solid bony matrix filled with organic ground substance and inorganic salts, leaving only tiny spaces (lacunae) that contain the osteocytes or bone cells. The function of compact bone is structural support, both for overall body structure and the protection of cancellous bone, which contains marrow. Mature compact bone is layered and very dense, with the mineral calcium phosphate embedded in collagen proteins, with tiny spaces for the living, bone-producing cells.

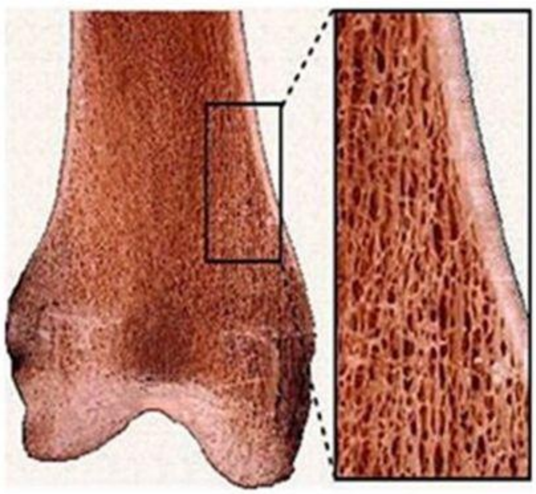

Figure 12E: Bone Structure at the Distal End of Femur; Right: 40 Times Scanning Electron Micrograph (SEM) Showing Trabecular (Spongy) and Cortical (Compact) Bone

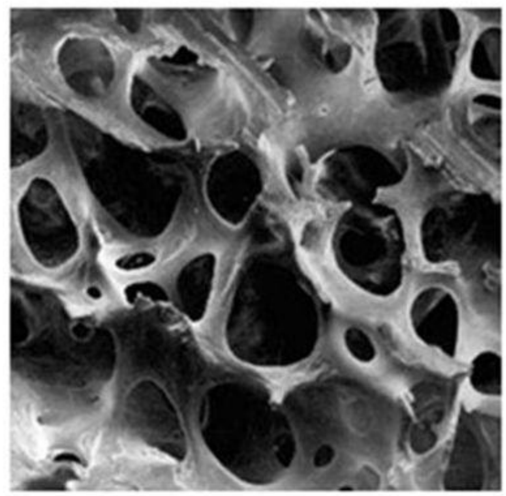

Figure 12F: 800 Times Scanning Electron Micrograph (SEM) Showing Spongy Bone Structure

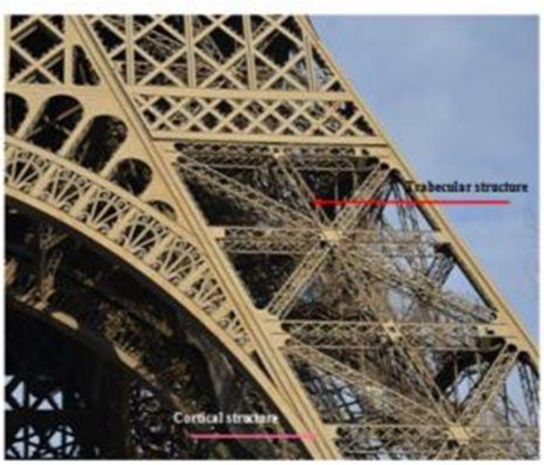

Figure 13A: Structure of Eiffel Tower

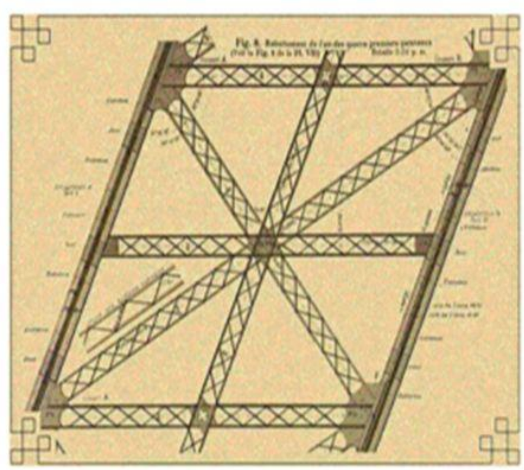

Figure 13B: Construction of Trabecular Structure along with Cortical Structure on both Sides as Femur

Spongy bone is also known as cancellous bone or trabecular bone. It provides structural support and facilitates movement of the joints and limbs to absorb mechanical stress. Spongy bone is light and porous Spongy bone is found at the inner part of long bones and fills most irregular bones. Spongy bones essentially act as shock absorbers; the human body endures high volumes of impacts each day through movements such as walking, skipping, running and jumping. Each stride and jump sends tiny shock waves through the skeletal system, and spongy bones help to absorb those impacts, which in turn prevents bones from breaking and prevents damage to their delicate frames.
Structure of spongy bone had been incorporated in the Eiffel Tower.
In Figures 13A & 13B we see the use of concept of structure within human femur bone used in Eiffel tower.

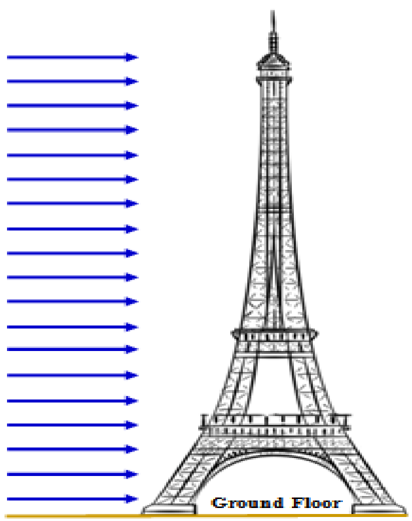

Figure 14A: Air Flowing from Side from Top to Bottom

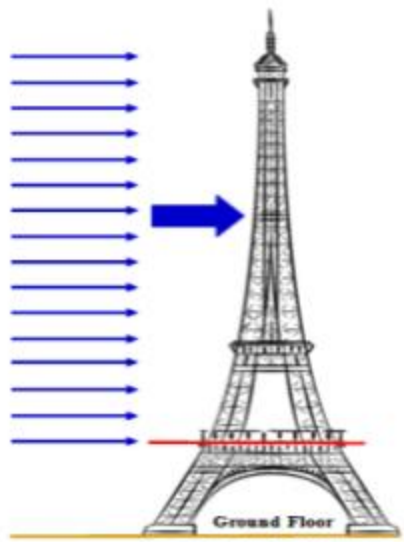

Figure 14B: Effective Air Flowing from Side from Top to Bottom

After construction of the tower Mr. Gustave Eiffel took the problem of wind flow and found O. K.
In modern time we can think it following way:
N: Axial force along the base of the tower [Figure – 15]; V: Vertical component of N; H: Horizontal component of N.
Here, $\frac{V}{N} = \cos\alpha$ and $\frac{H}{V} = \tan\alpha$

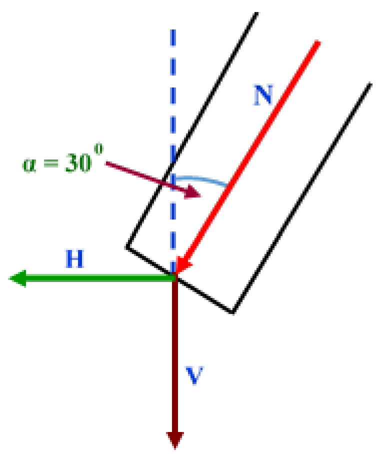

Figure 15: Axial Force and its Components at Base

This horizontal for 'H' tends to push the base of the tower away but it will be resisted by fixing at the ground. As H<V stability is possible.
Now let us consider the effect of wind flow, causing wind pressure, with uniform density, indicated by blue arrows, from the bottom to the top [Figure – 14A]. But base is so solid wind cannot effect the lower part. So, effective wind force will be started above lowest part i.e. from red-line and upwards [Figure – 14B]. Because horizontal wind force is able to tilt a structure creating bending moment thereof.

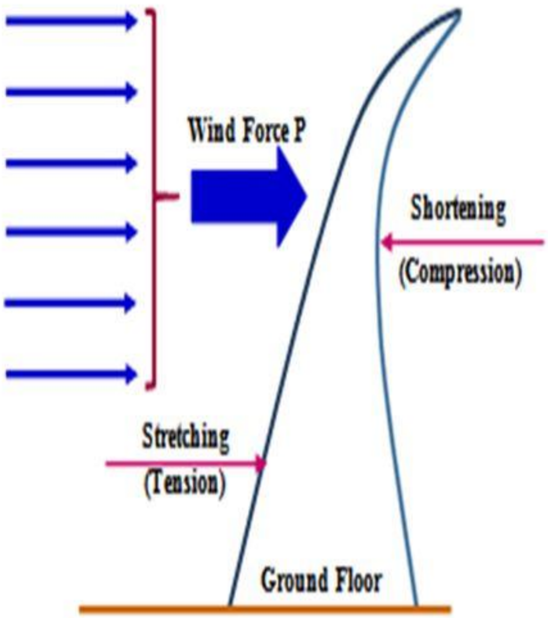

Figure 16A: Bending of Tower due to Wind-Force

Figure 16B: Artificial Image of Bending of Tower due to Wind-Force

Even if, we consider due to wind-force (P) the tower bends [Figures – 16A & 16B] then we can say:
1) The horizontal wind-force (P) will produce Stretching / Tension (T) along one side and in corresponding opposite side will experience shortening / Compression (C).
2) Then the bending moments will be the resulting tension and compression in the columns.
But on the bending moment Gustave Eiffel said: *At any height on the tower, the moment of the weight of the higher part of the tower, up to the top, is equal to the moment of the strongest wind on this same part.*
Horizontal wind force will be effective to create *bending moment* on the tower above the second platform of the Eiffel Tower.
In [Figure – 17] let us take wind force per unit length is to be *'p'* then wind force effective above height *'x'* from the first-floor to produce bending moment will have magnitude = *p(l – x)* unit. This force will act at a point with height $\frac{(l-x)}{2}$.
Therefore, bending moment = $M = p(l-x) \times \frac{(l-x)}{2} = \frac{p(l-x)^2}{2}$
Where, we have taken non-effective height from first floor to be *x* as it vary from second floor above.
Considering $M = \frac{p(l-x)^2}{2}$ in variable form we get $y = \frac{p}{2}(l-x)^2$ i.e. $(l-x)^2 = \frac{2}{p}y$ which represent a parabola with vertex at (0, *l*), axis parallel to *y*-axis and opening towards *x*-axis having focus $\Big(0,\ l+$ $12p$ length of the latus-rectum = $12p$ [Figure – 18] where equation of the Directrix is $y+l+12p=0$.

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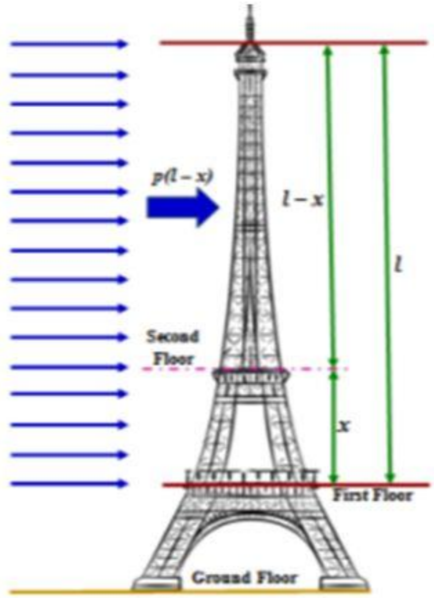

Figure 17: Effective Bending Moment of the Tower due to Wind-Force

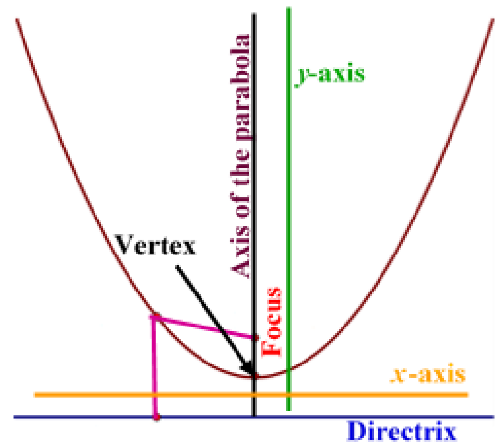

Figure 18: Parabola

Angle of inclination over first floor decreases to 10^0. Let $\beta = 15^0$ [Figure – 10B, 19]. Here H′ < H, N′< N and V′ <N as N′, H′ and V′ are above first floor where as N, H and V are of total tower.
Now, axial force N decreases with height of the tower as the vertical load angle of inclination decreases ($\cos\alpha < \cos\beta$ for $\alpha > \beta$ from equation with [Figure – 15]).
Therefore, due bending moment at the upper part of the tower may bend it and that bend will be in the form of a parabola [Figure – 16B].
The compression forces will be identical in all four columns each of them are taking one-fourth of the vertical load $\left(\frac{V}{4}\right)$. Eiffel had considered that cross-sectional area at higher point to decrease for smaller compression force would create a small compression stress to make the tower stable.
The bending moment created by wind-load, turn it to parabolic bending (Horizontal wind flow ultimately took the path of a parabola under gravity following the path of projectile) at any point can be found so easily if the entire structure is treated as single unit. As the horizontal force, creating the bending moment, is the wind simplified as a point load acting at the mid-point of the area under consideration [Figure 14B].

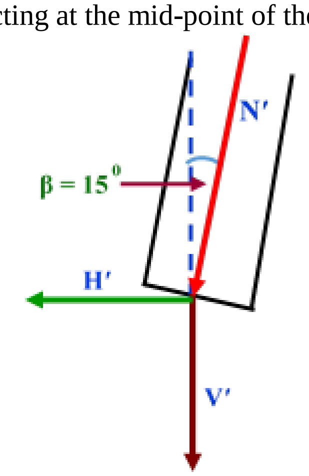

Figure 19: Axial Force and its Components (Upper Part)

We see horizontal wind-force creates bending moment in parabolic path. So, to keep tower stand erect essentiality of insertion of parabolic arches (Wind creating additional force above second floor of the tower initiates additional force at legs upward ejecting force and that follow the path of a projectile i.e. parabola under gravitational effect as the legs are angular not vertical. Curve inserted within the legs are parabolic, then this force will be transmitted and will be absorbed by the legs. So, possibility of uprooting becomes null and void) were needed at the base structure of the Eiffel tower for binding the base pillars as well as to resist against bending moment. This idea was inculcated from the parabolic arch between the condyles of the human femur.

Figure 20: Eiffel Tower Covered by a Hollow Cylinder

Figure 21: Air Enclosing Eiffel Tower is Heavier than Weight of Eiffel Tower

Dimension of Eiffel Tower: Height – 324 meters and Width – 125 meters. If any melt the iron structure into plate of dimension: *125 meters × 125 meters × 0.06 meters* with Volume ≈ 930 m^3 (Equivalent to an iron ball of diameter 12 meters) weighing 7300 metric tons or 7.3 million kilograms.
Volume of smallest cylinder that wraps the Eiffel Tower being of diameter 176 meters = $\pi \times 88^2 \times 324$ $meter^3$ ≈ 8 million cubic meters. This much air would have mass nearly 10 million kilograms.
The air in a cylinder that around the EIFFEL TOWER would weigh more than the EIFFEL TOWER itself.

Conclusion

Above discussion shows that Eiffel Tower structure has one-to-one correspondence with human femur. There is only difference between the two that human femur (Length of human femur is 17 inches to 18 inches for a normal man of height 6 feet and diameter of the shaft is nearly 1-inch weighing a) for male average 290 grams; b) for female 260 grams. Femur holds the capacity to carry 30-times of its weight) experiences forces from both dynamic and static efficiencies whereas Eiffel Tower experiences force to keep it static. To establish the word *light-weight-beauty* i.e. the idea of using minimum crisscross steel structure to erect such giant tower, Eiffel could prove the point successfully.

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