Research Article

ANISOTROPIC BIANCHI TYPE-III STRING COSMOLOGICAL MODEL WITH VARYINGGRAVITATIONAL CONSTANT AND COSMOLOGICAL CONSTANT IN GENERAL RELATIVITY

***Harpreet Kaur**
Sant Baba Bhag Singh Institute of Engineering & Technology, Department of Applied Sciences, Khiala, Padhiana, Jalandhar- 144030, Punjab, India
**Author for Correspondence*

ABSTRACT
In this paper we have investigated anisotropic Bianchi type-III string cosmological models in presence of perfect fluid with varying gravitational constant G and cosmological constant Λ(t) in general relativity. We assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma$, the condition leads to $B = C^n$. Thus, the physical and geometrical aspects of the model are also discussed.

***Keywords:** Anisotropic Bianchi Type-III, Gravitational Constant, Shear Scalar, Hubble Parameter, Decelerating, Cosmological Constant*

INTRODUCTION

The Einstein's field equations have two parameters, the gravitational constant G which plays important role of coupling constant between matter and geometry and cosmological constant Λ. Both constants are taken as a function of time (Singh, 2008; Pradhan, 2004, 2007, 2009; Pradhan and Pandey, 2003, 2006). Many researchers observed that expansion of universe is under late time acceleration (Perlmutter *et al.*, 1997-1999; Riess *et al.*, 1998, 2000; Garnavich *et al.*, 1998; Schmidt *et al.*, 1998; Efstathiou *et al.*, 2002; Spergel, 2003; Allen *et al.*, 2004; Sahni and Starobinsky, 2000; Peebles and Ratra, 2003; Padmanabhan, 2003; Lima and Maia, 1994). The cosmological constant cannot explain huge difference between observation and the vaccum energy density of quantum field theories. To solve this problem cosmological constant was introduced such that cosmological was large in earlier universe and later on decayed with evolution. During last two decades many researchers studied different models observing different decay laws (Lima and Maia, 1994; Lima and Trodden, 1996; Chen and Wu 1990; Pavon, 1991; Carvalho *et al.*, 1992; Arbab and Abdel-Rahaman 1994; Vishwakarma, 2001; Cunha and Santos, 2004; Carneiro and Lima, 2005). Dirac (1937, 1938, 1975), Hoyle and Narlikar (1964), Canuto *et al.,* (1977a, 1977b), Dicke (1961) and Dersarkissian (1985) suggested a possibility of time variation in Gravitational constants. Researchers found link between variation of Gravitational constant and Cosmological constant (Beesham 1986; Berman, 1991a; Abdel-Rahman, 1990; Kallingas *et al.*, 1992).
The Cosmological models Bianchi type-I and Bianchi type-III with varying Gravitational constant and Cosmological constants have been recently studied by various researchers (Arbab, 2003; Sistero, 1991; Sattar and Vishwakarma, 1997; Pradhan and Chakrabarty, 2001; Pradhan and Yadav, 2002; Yadav *et al.*, 2012; Singh *et al.*,2007; Singh *et al.*, 2008, Singh and Kale 2009; Bali and Tinker, 2008; Bali *et al.*, 2010).
In this paper, I consider space-time of the Bianchi Type III model in a general form with variable G and Λ. To obtain an explicit solution, I assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma$, the condition leads to $B = C^n$.

Metric and Field Equations

We consider the space time of Bianchi Type III metric as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2, \qquad (1)$$

where A, B and C are the function of time t, and α is also constant.

Einstein's field equations with variable gravitational constant G(t) and variable cosmological term Λ(t) in suitable units are

R_{ij} - ½ Rg_{ij} = -8 π GT_{ij} + Λ(t) g_{ij} (2)

The energy momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} \qquad (3)$$

Where p is isotropic pressure and ρ is energy density.

The u^i is the cloud four velocity vectors and x^i represents a direction of anisotropy, i.e. the direction of string. They satisfy the relations (Wang, 2005)

$$u^i\, u_i = -1 \qquad (4)$$

The expressions for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H, \qquad (5)$$

The expressions for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H, \qquad (6)$$

The non-vanishing component of shear tensor σ_{ij} defined by $\sigma_{ij} = u_{i;j} + u_{j;i} - \frac{2}{3} g_{ij} u;^k_k$ are obtained as

$$\sigma_{11} = \frac{A^2}{3}\left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) \qquad (7)$$

$$\sigma_{22} = \frac{B^2 e^{-2\alpha x}}{3}\left(\frac{2\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right) \qquad (8)$$

$$\sigma_{33} = \frac{C^2}{3}\left(\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) \qquad (9)$$

Thus the shear scalar σ is obtained as

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right), \qquad (10)$$

where H is the Hubble parameter.

The average anisotropy parameter A_m as

$$A_m = \frac{1}{3}.\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, where \Delta H_i = H_i - H \qquad (11)$$

Where i=1--3

For the metric (1) Einstein's field equation can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -8\pi G p + \Lambda \qquad (12)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda \qquad (13)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G + \Lambda \qquad (14)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{A^2}=8\pi G\rho+\Lambda \quad (15)$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \quad (16)$$

Where, dot denotes the ordinary differentiation with respect to t.
From equation (16) we have

$$A = KB \quad (17)$$

Where, K is constant of integration.
In view of vanishing of the divergence of Einstein tensor, we get

$$8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]+8\pi\rho\dot{G}+\dot{\Lambda}=0 \quad (18)$$

The usual energy conservation equation $T^{j}_{i;j}=0$ gives

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \quad (19)$$

From equations (18) and (19), we get

$$8\pi\rho\dot{G}+\dot{\Lambda}=0 \quad (20)$$

Implying that Λ is a constant whenever G is constant and increase and decreases depending on one another.
We defines spatial volume of metric (1) as

$$V^3 = ABC\, e^{-ax} \quad (21)$$

The Hubble parameter in anisotropic models may be defined as

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \quad (22)$$

Thus,

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right) \quad (23)$$

Where $H_1=\frac{\dot{A}}{A},\ H_2=\frac{\dot{B}}{B}\ and\ H_3=\frac{\dot{C}}{C}$ are directional Hubble factors in direction of x, y and z respectively.
The deceleration parameter is given as

$$q=-\frac{V\ddot{V}}{\dot{V}^2} \quad (24)$$

We assume that the matter content obeys an equation of state

$$p=\omega\rho,\ 0\le\omega\le 1 \quad (25)$$

Solution of the Field Equations

There are five independent equations (12) - (16) in the seven unknowns A, B, C, ρ,p , G and Λ. Thus, two more relations needed to solve the system completely. We assume that scalar of expansion is proportional to the shear scalar $\theta\propto\sigma$, the condition leads to

$$B=C^n \quad (26)$$

On solving equation (13) and (14), we get

$$\dot{B}C = K_1 \tag{27}$$

Using equation (26) and (23), we get

$$C = \left[\frac{n+1}{n}\right]^{1/n+1} \left[K_1 t + nK_2\right]^{1/n+1} \tag{28}$$

On substituting equation (28) in (26), we get

$$B = \left[\frac{n+1}{n}\right]^{n/n+1} \left[K_1 t + nK_2\right]^{n/n+1} \tag{29}$$

$$A = K\left[\frac{n+1}{n}\right]^{n/n+1} \left[K_1 t + nK_2\right]^{n/n+1} \tag{30}$$

By using equation (28), (29) and (30) in the metric (1) we get

$$ds^2 = -dt^2 + \left[\left[\frac{n+1}{n}\right]^{n/n+1} \left[K_1 t + nK_2\right]^{n/n+1}\right]^2 \left[K^2 dx^2 + e^{-2\alpha x} dy^2\right] + \left[\left[\frac{n+1}{n}\right]^{1/n+1} \left[K_1 t + nK_2\right]^{1/n+1}\right]^2 dz^2, \tag{31}$$

On using (28), (29), (30) and (25) in (19), we get

$$\rho = \frac{K_3}{\left[K_1 t + nK_2\right]^{(1+w)(2n+1)/n+1}} \tag{32}$$

On using (28), (29), (30) and (15) in (20), we get

$$\Lambda = \left[\frac{K_1^2 n(2+n)\left(w(2n+1)-1\right)}{(1+w)(2n+1)(n+1)^2}\right] \cdot \left[\frac{1}{(K_1 t + nK_2)^2}\right] - \left[\frac{\alpha^2\left(w(2n+1)+1\right)}{(1+w)(2n+1)K^2\left(\frac{n+1}{n}\right)^{2n/n+1}}\right] \cdot \left[\frac{1}{(K_1 t + nK_2)^{2n/n+1}}\right]$$

(33)

$$8\pi G = \left[\frac{2K_1^2 n(2+n)}{K_3(1+w)(2n+1)(n+1)}\right] \cdot \left[\frac{1}{(K_1 t + nK_2)^{w(2n+1)-1/n+1}}\right] - \left[\frac{2n\alpha^2}{K_3(1+w)(2n+1)K^2\left(\frac{n+1}{n}\right)^{2n/n+1}}\right] \cdot \left[\frac{1}{(K_1 t + nK_2)^{w(2n+1)+1/n+1}}\right] \tag{34}$$

Substituting equation (28) --(30) in equation (5), we have

$$\theta = \left[\frac{K_1(2n+1)}{n+1}\right] \left[\frac{1}{K_1 t + nK_2}\right] \tag{35}$$

Using equation (28) --(30) in equation (10), we get

$$\sigma = \frac{1}{3^{1/2}}\left[\frac{K_1}{n+1}\right] \left[\frac{1}{K_1 t + nK_2}\right] \tag{36}$$

The volume V of the model is given by

$$V = K\left[\frac{(n+1)}{n}\right]^{2n+1/n+1} \cdot \left[\frac{1}{K_1 t + nK_2}\right]^{2n+1/n+1} \quad (37)$$

$$\frac{\sigma}{\theta} = \frac{1}{(2n+1)(3)^{1/2}} \quad (38)$$

The deceleration parameter is given as

$$q = \frac{n+2}{2n+1} \quad (39)$$

The model has singularity at

$$t = -\frac{nK_2}{K_1} = t_0(say) \quad (40)$$

Conclusion

We have studied anisotropic Bianchi type-III string cosmological models in presences of perfect fluid with varying gravitational constant G and cosmological constant Λ(t) in general relativity. We assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma$, the condition leads to $B = C^n$. Thus, the physical and geometrical aspects of the model are also discussed. When $t = t_0 \to 0$, the spatial volume, G and scale factor A, B, C is zero and the expansion scalar, ρ, Λ, σ and Hubble parameter is infinite which implies that the big-bang starts evolving. The density, the coefficient of shear scalar and the cosmological term diverges at the initial singularity. Hence, the model has a "point type singularity" at the initial epoch. Thus, rate of expansion slows down with increase in time. When $t = t_0 \to \infty$, the spatial volume, G and scale factor A, B, C is infinity and the expansion scalar, ρ, Λ, σ and Hubble parameter is zero thus tend to isotropic. When $t = t_0$ increases then spatial volume also increases. The gravitational constant i.e. G>0 then $t = t_0 > 0$. We observe that cosmological constant decrease as time increase and vice versa, thus tends to small positive value at present epoch. As we know $\lim_{t\to\infty} \frac{\sigma}{\theta}$ = constant, the model does not approach isotropy for large value of t. The model describes a shearing non-rotating continuously expanding universe with a big-bang start. The model is decelerating i.e. q>0 for n<-1/2 and accelerating when q<0 for n>-1/2. The model is accelerating and decelerating due to combined effect of gravitational constant and cosmological constant. Many observers found type Ia supernovae Perlmutter *et al.*, (1997,1998,1999), Riess *et al.*, (1998, 2000) is in a accelerating and decelerating phase in $-1 < q \le 0$ .Thus, our model is in consistent with above observations made by researchers.

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