Research Article

ANISOTROPIC LRS BIANCHI TYPE-I COSMOLOGICAL MODEL IN F(R,T) THEORY OF GRAVITY WITH RADIATING FLUID

*Santhi Kumar. R and Lakshmi. K

Aditya Institute of Technology and Management, K.Kotturu, Tekkali Srikakulam District, Andhra Pradesh, India-532203
*Author for Correspondence

ABSTRACT
Field equations in a modified theory of gravitation proposed by Harko *et al.,* (2011) are obtained with the aid of a spatially homogeneous and anisotropic LRS Bianchi type-I metric. Cosmological models corresponding to false vacuum, radiating fluid and dust distributions are obtained. Some physical and kinematical properties of each of the models are also studied.

Keywords: Modified Gravity, Radiating Fluid

INTRODUCTION
In recent years there has been an increasing interest in modified theories of gravity in view of the direct evidence of late time acceleration of the universe and the existence of the dark matter and dark energy (Reiss *et al.,* 1980; Permutter *et al.,* 1999; Bennet *et al.,* 2003). In particular, f(R) theory of gravity formulated by Nojiri and Odinstov (2003a) and f(R,T) theory of gravity proposed by Harko *et al.,* (2011) are attracting more and more attention. It has been suggested that cosmic acceleration can be achieved by replacing Einstein-Hilbert action of general relativity with a general function f(R) (R being the Ricci Scalar curvature). Carroll *et al.,* (2004), Nojiri and Odinstov (2003b, 2004, 2007) and Chiba *et al.,* (2007) are some of the authors who have investigated several aspects of f(R) gravity. Very recently, Adhav(2012), Reddy *et al.,* (2012a, 2012b, 2013), Satyannarayana *et al.,* (2016) have investigated Bianchi type-I and III and Kaluza-Klein perfect fluid cosmological model in f(R,T) theory of gravity. Also we have investigated LRS Bianchi type-I universe, by using a special law of variation for Hubble's parameter which represents cosmological model with a constant deceleration parameter in f(R,T) gravity.

Motivated by the above investigations we study spatially homogeneous LRS Bianchi type-I anisotropic cosmological models corresponding to physically important matter distribution namely false vacuum, radiating fluid and dust distributions in f(R,T) gravity. These models are very important in the discussion of large scale structure and to realize real picture of the universe in its early stages. They are also necessary to study the evolution of the universe.

This paper is organized as follows, in the section 2, the field equations of f(R,T) gravity are derived with the help of LRS Bianchi type - I metric in the presence of perfect fluid distribution. Section 3, is devoted to the solution of the field equations. Cosmological models corresponding to radiating fluid is studied. The last section contains some conclusions.

Metric and Field Equations
We consider a homogeneous LRS Bianchi type–I space–time given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2) \tag{1}$$

Where, A and B are functions of cosmic time t.
The field equations of f(R,T) gravity are

$$f(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)\,T_{ij} - f_T(R,T)\,\theta_{ij} \tag{2}$$

Here, $\theta_{ij} = -2\,T_{ij} + g_{ij}\,L_m - 2g^{lk}\dfrac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}$ $\qquad\qquad$ (3)

Here, $f_R = \dfrac{\delta f(R,T)}{\delta R}$, $\qquad f_T = \dfrac{\delta f(R,T)}{\delta T}$ $\Box = \nabla^i\nabla_i$, ∇_i is the co-variant derivative and T_{ij} is the standard matter energy-momentum tensor derived from the Lagrangian L_m. It may be noted that when f(R,T) ≡f(R) the equation (2) yield the field equations of f(R) gravity.

Research Article

The problem of the perfect fluids described by an energy density ρ, pressure p and four velocity u^i is complicated since there is no unique definition of the matter Lagrangian. However, here, we assume that the stress energy tensor of the matter is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{4}$$

and the matter Lagrangian can be taken as $L_m = -p$ and we have

$$u^i \nabla_j u_i = 0, \quad u^i u_i = 1 \tag{5}$$

Then, with the use of Equations (5) we obtain for the variation of stress-energy of perfect fluid the expression

$$\theta_{ij} = -2T_{ij} - p g_{ij} \tag{6}$$

Generally, the field equations also depend through the tensor θ_{ij}, on the physicsl nature of the matter field. Hence, in the case of f(R,T) gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of f(R,T). Assuming

$$f(R,T) = R + 2f(T) \tag{7}$$

As a first choice where f(T) is an arbitrary function of the trace of stress-energy tensor of matter, we get the gravitational field equations of f(R,T) gravity from Equation (2) as

$$R_{ij} - \frac{1}{2}R g_{ij} = 8\pi T_{ij} - 2f'(T) T_{ij} - 2f'(T)\theta_{ij} + f(T) g_{ij} \tag{8}$$

Where, the prime denotes differentiation with respect to the argument.
If the matter source is a perfect fluid,

$$\theta_{ij} = -2T_{ij} - p g_{ij}$$

Then, the field equations become

$$R_{ij} - \frac{1}{2}g_{ij} R = 8\pi T_{ij} + 2f'(T) T_{ij} + [2pf'(T) + f(T)] g_{ij} \tag{9}$$

Using co-moving coordinates and equations (4)-(6) f(R, T) gravity field equations (8) with the particular choice of the function (Harko *et al.,* 2011).

$f(T) = \lambda T$, λ is constant
For the metric (1) take the form

$$2\left(\frac{\ddot{B}}{B}\right) + \left(\frac{\dot{B}}{B}\right)^2 = (8\pi + 3\lambda)p - \lambda\rho \tag{10}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = (8\pi + 3\lambda)p - \lambda\rho \tag{11}$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = -(8\pi + 3\lambda)\rho + \lambda p \tag{12}$$

Where, an over head dot denotes differentiation with respect to cosmic time t.
The spatial volume for the metric (1) is given by

$$V = AB^2 \tag{13}$$

We define $a = (AB^2)^{\frac{1}{3}}$ as the average scale factor of the space-time (1) so that the Hubble's parameter is

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{14}$$

We define the generalized Hubble's parameter H as

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right) \tag{15}$$

Where $H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{B}, H_z = H_x$ are the directional Hubble's parameters in the direction of x,y,z respectively.
The scalar expansion θ, shear scalar σ^2 and the average anisotropy parameter A_α are defined by

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{16}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{17}$$

$$A_\alpha = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{18}$$

Where $\Delta H_i = H_i - H$ (i = 1,2,3)

Research Article

Solutions and the Models

When, $\rho = 3p$ (Radiating Model)

The field equations (10)-(12) are a system of three independent equations in four unknowns A,B, p and ρ. Hence, to obtain determinate solutions, we consider the following physically important cases and discuss the corresponding cosmological models.

In this particular case, the field equations (10 -12) reduce to

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} = 0 \qquad (19)$$

Equation (19) being highly non-linear, we use the condition that the scalar expansion θ is proportional to shear scalar σ of the space-time so that

$$A = B^m \qquad (20)$$

Where, m>1 is a constant (Collins *et al.,* 1980)

Now, using Equation (20) in (19) we get the metric coefficients as

$$A = [(m + 1)(C_1 t + C_2)]^{\frac{m}{(m+1)}}, B = [(m + 1)(C_1 t + C_2)]^{\frac{1}{(m+1)}} \quad (21)$$

Where, C_1 and C_2 are constants of integration. By a suitable choice of integration constants (i.e., $C_1=1, C_2=0$) the metric(1) with the help of (21) can, now, be written as

$$ds^2 = dt^2 - (Mt)^{\frac{2m}{M}}dx^2 - (Mt)^{\frac{2}{M}}(dy^2 + dz^2) \quad (22) \qquad \text{where } M = m + 1$$

Equation (22) represents Bianchi type-I rotating cosmological model in f(R,T) gravity with the following physical and kinematical parameters which are important in the discussion of cosmological models.

Spatial volume in the model is

$$V = ((m + 1)t)^{\frac{m+2}{m+1}} \qquad (23)$$

Hubble's parameter is

$$H = \left(\frac{m+2}{3(m+1)}\right)\left(\frac{1}{t}\right) \qquad (24)$$

The scalar expansion is

$$\theta = \left(\frac{m+2}{m+1}\right)\left(\frac{1}{t}\right) \qquad (25)$$

The shear scalar is

$$\sigma^2 = \frac{1}{3}\left[\frac{m-1}{m+1}\right]^2 \frac{1}{t^2} \qquad (26)$$

The anisotropy parameter is

$$A_\alpha = \frac{4}{3} \qquad (27)$$

Also $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{1}{3}\left[\frac{m-1}{m+2}\right]^2 \neq 0 \qquad (28)$

The deceleration parameter $q = -\frac{a\ddot{a}}{(\dot{a})^2} = \frac{(1-m-m^2)}{(m+2)} \qquad (29)$

The pressure and density in the model is

$$\rho = 3p = \frac{3}{(8\pi)}\left[\frac{(2m+1)}{(m+1)^2}\right]\frac{1}{t^2} \qquad (30)$$

The above results reveal that the deceleration parameter q in this case, depends on the values of m, i.e., the universe accelerates or decelerates according as q is negative or positive respectively.

Conclusion

Here we have investigated LRS Bianchi type-II cosmological model in f(R,T) gravity. In particular, we have radiating, anisotropic models in the modified f(R,T) gravity. It is well known that anisotropic models represent cosmos in its early stage of evolution and isotropic FRW models represent present day universe. It is observed that all the above models in f(R,T) gravity have stability and have no initial singularity.

It may also be noted that even though the early universe in f(R,T) theory of gravity decelerates in the standard way, it will accelerate in finite time thus, establishing with the present day accelerated expansion of the universe. This is possible by cosmic re-collapse of the universe in finite future as the universe in turns inflates, decelerates and then accelerates (Nojiri and Odintsov, 2003).

Research Article

REFERENCES

Adhav KS (2012). LRS Bianchi type-I cosmological model in f(R,T) theory of gravity. *Astrophysics and Space Science* **339** 365-369.

Bennett CL *et al.*, (2003). First-year Wilkinson Microwave Anisotropy (WMAP) observations: Foreground emission. *The Astrophysical Journal Supplement Series* **148**(1) 97-117.

Brans CH and Dicke RH (1961). *Physical Review* **24** 925.

Caroll SM, Duwuri V, Trodden M and Turner MS (2004). Is cosmic speed-up due to new gravitational physics. *Physical Review D* **70** 043528.

Chiba T, Smith W and Erichcek AL (2007). Solar System constraints to general f(R) gravity. *Physical Review D* **75** 124014.

Collins CB, Glass EN and Wilkinson DA (1980). Exact spatially homogeneous cosmologies. *General Relativity and Gravitation* **12** 805.

Harko T, Lobo FSN, Nojiri S and Odintsov SD (2011). Extended f(R,Lm) gravity with generalized scalar field and kinetic term dependences. *Physical Review D, Particles and Fields* **84** 024020.

Nojiri S and Odintsov SD (2003a). Modified gravity with negative and positive powers of curvature: Unification of inflation and cosmic acceleration. *Physical Review D* **68** 123512.

Nojiri S and Odintsov SD (2003b). Quantum de Sitter cosmology and phantom matter. *Physics Letters B* **562**.

Nojiri S and Odintsov SD (2004). Dark Energy and Modified Gravities. *Physics Letters A* **19** 627.

Nojiri S and Odintsov SD (2007). Introduction to Modified Gravity and Gravitational Alternative for Dark Energy. *International Journal of Geometric Methods in Modern Physics* **4** 115-146.

Perlmutter S *et al.*, (1999). Measurements ofomega and mangold from 42 high – redshift supernova. *The Astrophysical Journal* **517**.

Reddy DRK and Santhi Kumar R (2013). Some anisotropic cosmological models in a modified theory of gravitation. *Astrophysics and Space Science* **344** 253-257.

Reddy DRK, Naidu RL and Satyannarayana B (2013). Bianchi Type-III Dark Energy Model in a Saez-Ballester Scalar-Tensor. Theory. *International Journal of Theoretical Physics* **51** 2857-2862.

Reddy DRK, Santhi Kumar R and Naidu RL (2013). Bianchi type-III Dark Energy Model in f(R, T) Gravity. *Astrophysics and Space Science* **342** 249.

Reiss A *et al.*, (1998). Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal* **116** 1009

Saez D and Ballester VJ (1986). A simple coupling with cosmological implications**.** *Physics Letters A* **113** 467.

Satyannaryana B and Santhi Kumar R (2016). Anisotropic Bianchi Type-I Cosmological Model in Modified Theory of Gravitation. *International Journal of Computer and Mathematical Sciences* **5**(2) 2347-8527.