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Research Article

TRANSACTION EXPOSURE MODEL IN A NEWSVENDOR FRAMEWORK WITH NORMALLY DISTRIBUTED EXCHANGE RATE ERROR AND ISOELASTIC DEMAND

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ABSTRACT
In a global supply chain consisting of one retailer and one manufacturer, both from different countries, when there is a time lag between the payments made while placing the order and the time when the order is realized, risk in the form of the exchange rate fluctuation affects the optimal pricing and order quantity decisions. We elaborate the effect of exchange rate fluctuation under normal distribution when the retailer or manufacturer undertakes to share the exchange rate risk and the demand error is modelled in the multiplicative form in the news vendor framework. We also have compared the exchange rate effect with the generalized beta distribution error in the model given in Arcelus *et al.*, (2013). This is accomplished by numerical example using maple software through nonlinear optimization to test the sensitivity of the model and to compare the two scenarios of the retailer and manufacturer.

Keywords: *Transaction Exposure, Exchange Rate Error, Newsvendor Problem, Optimal Pricing and Quantity, Normal Distribution, Generalized Beta Distribution*

INTRODUCTION
Suppose two different countries having different currencies are into a business. When the exchange rate between the two currencies gets an exposure to unexpected changes, there exists a financial risk and this risk is known as foreign exchange risk (or exchange rate risk).
A transaction exposure arises only when there exists a time lag between the time of the financial obligation has been incurred and the time its due to be settled. This is because of the purchase price to buyer/ retailer on the settlement day may differ from that when it was incurred, if the debt is denominated in the manufacturer currency.
Arcelus *et al.*, (2013) have developed a mathematical model in news vendor framework to find optimum ordering and pricing policies for retailer/manufacturer, when the foreign exchange rate between the two countries doing the business, faces transaction exposure. The complete derivation of optimum policies and expected profit of the foreign exchange model for multiplicative demand error is given in Patel and Gor (2015a), Our main contribution in this paper is to explain the effect of normal distribution in the exchange rate error under the iso-elastic demand with multiplicative error in news vendor setting. The effect of normal error is also compared with the generalized beta distribution error in the exchange rate.

Literature Review
This paper fundamentally follows the model of Arcelus *et al.*, (2013). Cases of transaction exposure when a firm has an accounts receivable or payable denominated in a foreign currency has been reported in Goel (2012).
The nature of global trade is that either the buyer or the seller has to bear what is commonly known in international finance as transaction exposure, Eitemann *et al.*, (2010), Shubita *et al.*, (2011). The very important newsvendor framework introduced by Petruzzi and Dada (1999) and the price dependent demand forms in the additive and multiplicative error structures by Mills (1958) and Karlin and Carr (1962) have been used. The derivation of the expected profit and optimal policies, when demand is multiplicative are given in Patel and Gor (2015a), and for linear demand error in Patel and Gor (2015b). We have also developed more general hybrid demand model of foreign exchange transaction exposure in Patel and Gor (2015c).

Transaction Exposure Model

Suppose a retailer wants to order q units from a foreign manufacturer of a certain product. The retailer does not know the demand (D) of the product, which is uncertain. But it partly depends upon the price (p) and partly random.

In this paper we take the price dependent demand with multiplicative error which can be defined as $D(p,\in) = g(p)\in$, where $\in$ is the multiplicative error in the demand and it follows some distribution (say $f(\in)$) with mean μ in some interval [A,B] and g(p) is the deterministic demand. [Generally $g(p)$ is taken as decreasing isoelastic function of p say, $g(p) = ap^{-b}$ in multiplicative demand error case with the restrictions a, b >1].

Let the exchange rate be 'r ' in the retailer currency when the order is placed. Let w be the cost of one unit of the product in the manufacturer currency. If the buyer pays immediately then he has to pay wr per unit of the product in his currency.

Suppose there is a time lag between the order is placed and the amount is paid for the product when it is acquired by the retailer. Thus there exists transaction exposure risk, since the exchange rate (r) may get fluctuate.

So, the buyer has to pay more or less according to the existing rate at the time of the arrival of the product. Generally, the fluctuation in the exchange rate r is very small and random. We model the future exchange rate (FER) as FER= current exchange rate + fluctuation in the exchange rate. The fluctuation in the exchange rate is always some percentage of r, hence we can take the FER= $r + r\in_r = r(1+\in_r)$. Where, $\in_r$ is also a random variable together with the random variable D.

We assume that $\in_r$ lies in [-a, a]. Here 0<a<1. The fluctuation $\in_r$ is unknown but its distribution is known say $\psi(\in_r)$. In this paper we consider truncated normal distribution for the exchange rate fluctuation $\in_r$ with support [a, b].

If the fluctuation $\in_r$ is positive buyer has to pay more and if it is negative seller will get less. So, the question arises here is that who will bare the exchange rate risk? Buyer/retailer OR seller/manufacturer?

In this paper we discuss the two situations under the multiplicative demand error. In each case the retailer's optimal policy is to determine the optimum order (q) and selling price (p) of the product so that his expected profit is maximum. At the same time we obtain the manufacturer's optimal policies as well.

Assumptions and Notations

The following assumptions are made in the foreign exchange transaction exposure model:

(i) The standard newsvendor problem assumptions apply.
(ii) The global supply chain consists of single retailer- single manufacturer.
(iii) The error in demand is multiplicative.
(iv) Only one of the two-retailer or manufacturer- bears the exchange rate risk.

The following notations are used in the paper:

q = order quantity
p = selling price per unit
D = demand of the product= no. of units required
$\in$ = demand error= randomness in the demand.
v = salvage value per unit
s = penalty cost per unit for shortage
c = cost of manufacturing per unit for manufacturer
w_r = purchase cost for retailer
$\in_r$ = exchange rate error
Π = profit function.

The Two Scenarios

Case-1: Retailer Bears the Exchange Rate Risk

In the case-1 we assume that the retailer bears the exchange rate risk and manufacturer does not. Thus, the manufacturer will get w per unit at any point of time and the buyer will have to pay according to the existing exchange rate. So, the buyer will be paying $wr\,(1+\in_r)$ per unit, on the settlement day or when the product is acquired by him. This amount in terms of manufacturer currency is $wr\,(1+\in_r)/r = w(1+\in_r) = w_r$ (say). Thus, w_r is the purchase cost to buyer in seller's currency.

Now, the buyer will choose the selling price p and the order quantity q so as to maximize his expected profit. The profit function for the retailer is given by,

$\Pi(p,q) =$ [revenue from q items] – [expenses for the q items]

$$\Pi(p,q) = \begin{cases} [pD + v(q-D)] - [qw_r] & if \;\; D \le q \;\; (overstocking) \\ [pq] - [s(D-q) + qw_r] & if \;\; D > q \;\;\;\; (shortage) \end{cases} \tag{1}$$

Note that all the parameters p, v, s, w_r are taken in manufacturer's currency and the salvage value v is taken as an income from the disposal of each of the q-D leftover.

Since the demand, $D(p,\in) = g(p)\in$, the retailer's profit function (1) for ordering q units and keeping selling price p is given by,

$$\Pi(p,q) = \begin{cases} [p(g(p)\in) + v\{q - (g(p)\in)\}] - [qw_r] & if \;\; D \le q \\ \quad [pq] - [s(\{g(p)\in\} - q) + qw_r] & if \;\; D > q \end{cases}$$

Put $g(p)=g$ and define $z = q\,/\,g(p) = q\,/\,g$ i.e. $q=g\,z$, for the multiplicative demand error.

Now $D \le q \Leftrightarrow g \in \le q \Leftrightarrow \in \le q\,/\,g \Leftrightarrow \in \le z$ and similarly $D > q \Leftrightarrow \in > z$.

$$\Rightarrow \Pi(z,p) = \begin{cases} pg\in + vg(z-\in) - gzw_r & if \;\; \in \le z \\ pgz - sg(\in - z) - gzw_r & if \;\; \in > z \end{cases} \tag{2}$$

The equation (2) describes the profit function for the retailer in the manufacturer currency. Note that the parameter q is replaced by z.

Now the retailer wants to find the optimal order quantity q = q* and optimal price $p = p^*$ to maximize his expected profit. In order to do this he must find optimal values of the price p and the parameter z, say p^* and z^* respectively which maximizes his expected profit so that he can determine the optimal order $q^*=z^*g(p^*)$.

The profit Π is a function of the random variable $\in$ with support [A, B]. Thus, the retailer's expected profit is given by,

$E\left[\Pi(z,p)\right] = \int_A^B \Pi(z,p) f(u)du$.   (Here we take $\in=u$ for simplicity in (2)).

Then we get,

$$E[\Pi(z,p)] = \int_A^z [pgu + vg(z-u) - gzw_r]\cdot f(u)\,du + \int_z^B [pgz - sg(u-z) - gzw_r]\cdot f(u)\,du \tag{3}$$

Define $\Lambda(z) = \int_A^z (z-u) f(u)du$ [expected leftovers] and

$\Phi(z) = \int_Z^B (u-z) f(u)du$ [expected shortages].

Then the expected profit of the retailer as a function of z and p is given by

$$E\left[\Pi(z,p)\right] = [(g\mu)(p-w_r)] - g[(w_r - v)\Lambda + (p+s-w_r)\Phi] \tag{4}$$

as derived in Patel and Gor(2015)-a. Where $\mu = \int_A^B uf(u)du$ in the equation (4) and it gives the expected value of the randomness u in the demand D.

We use Whitin's method (1955) to maximize the expected profit function. In this method first we keep *p* fixed in (4) and use the second order optimality conditions $\frac{\partial E}{\partial z}=0$ and $\frac{\partial^2 E}{\partial z^2}<0$ to find the optimum value of *z** as a function of *p*. Then we substitute the value of *z** in the expected profit (4) so that it becomes a function of single variable *p* and hence, the optimal *p** can also be obtained. The authors have already derived the optimal policies given below in Patel and Gor (2015a).

$$z^*=F^{-1}\left(\frac{p+s-w_r}{p+s-v}\right). \text{ Where } F(z)=\int_A^z f(u)\,du \text{ is the CDF.} \quad (5)$$

This *z** gives the optimum solution for maximum profit as a function of *p*. Now substitute this *z** in $E[\Pi(z,p)]$ and obtain optimum *p** using the second order optimality criteria. Hence, the retailer's optimal order *q =q** is given by,

$$q^*=g(p^*)z^*=g(p^*)F^{-1}\left(\frac{p^*+s-w_r}{p^*+s-v}\right) \quad (6)$$

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller)]× no. of units sold, $\Pi_m=(w-c)q^*$. (7)

Case-2: Manufacturer Bears the Exchange Rate Risk

In the case-2 we assume that the manufacturer bears the exchange rate risk and retailer does not. Thus, the retailer pays *w* per unit in manufacturer's currency at any point of time and the manufacturer will get according to the existing exchange rate.

So, the manufacturer will be getting $wr/(r(1+\in_r))=w_m$ per unit on the settlement day in his currency. Now, the retailer's profit function, his expected profit and optimal policies to get maximum expected profit can be obtained by replacing *wr* by w in case-1. So, we get the retailer's profit as,

$$\Pi(p,q)=\begin{cases}[pD+v(q-D)]-[qw] & if\ D\le q\ (overstocking)\\ [pq]-[s(D-q)+qw] & if\ D>q\quad (shortage)\end{cases} \quad (8)$$

And his expected profit as,

$$E\left[\Pi(z,p)\right]=[(g\mu)(p-w)]-g[(w-v)\Lambda+(p+s-w)\Phi] \quad (9)$$

The optimal value of z is given by $z^*=F^{-1}\left(\frac{p+s-w}{p+s-v}\right)$ and hence the optimum order quantity is,

$$q^*=g(p^*)z^*=g(p^*)F^{-1}\left(\frac{p^*+s-w}{p^*+s-v}\right). \quad (10)$$

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller)]× no. of units sold. $\Pi_m=(w_m-c)q^*$. (11)

Sensitivity Analysis

We assume isoelastic demand with multiplicative demand error *u* which follows the normal distribution $f(u)$ with support [A, B]. Then, we obtain the optimum policies and maximum expected profit of the retailer and manufacturer using MAPLE software when anyone of them bears the exchange rate risk.

We compute the optimum values by using normal distribution $\psi(\in_r)$ in the exchange rate error $\in_r$ with support [-0.1, 0.1].

In case-1 and case-2 we also compare it with the policies obtained by Arcelus *et al.*, (2013), for the generalized beta distribution in the exchange rate error for each of the case positive, negative and symmetrical beta distribution. We have also tested the sensitivity of the model by comparing uniform and

beta distribution in the exchange rate error in Patel and Gor (2016)-a and b, for additive and multiplicative demand errors.
Recall the general normal probability density function is given by,

$f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty<x<\infty$ with mean μ and standard deviation σ.

The truncated normal distribution for the error support (a,b) is given by,

$\psi(x)=\frac{\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\Phi(b')-\Phi(a')}, a<x<b$. Where $\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy$ is the CDF of the standard normal density function and $a'=\frac{a-\mu}{\sigma}, b'=\frac{b-\mu}{\sigma}$.

The four parameter beta density function is given by,
$f(y/a,b,\alpha,\beta)=\frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}}$, where $a\le y\le b, \alpha,\beta>0$. And by taking $y=a+(b-a)x$ we get its transformation in the standard beta distribution as $f(x/0,1,\alpha,\beta)=\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$,

Where, $0\le x\le 1, \alpha,\beta>0$ and $B(\alpha,\beta)=\int_0^1 t^{1-\alpha}(1-t)^{1-\beta}dt$ is the beta function.

We assume the following parameter values:
Demand support = [A, B]=[0.7,1.1]

Mean Demand = $\mu=\frac{A+B}{2}=0.9$

Linear Demand $g(p)=ap^{-b}, a=500,000,000, b=2.5$
v =Salvage value = 10
s=Penalty cost = 5
c= cost of manufacturing per unit for manufacturer= 20
r=current exchange rate = 45

The following computation is done through Maple software using non linear optimization techniques. We have assumed the mean μ=0.0001 for the exchange rate error in the interval [-0.1, 0.1] and the support of the exchange rate covers 6 standard deviations of the normal distribution and hence, $\sigma=\frac{0.1-(-0.1)}{6}=\frac{2}{6}=\frac{1}{3}\approx 0.33$.

We compute the optimal price p*, optimum order quantity q*, optimum profit of the retailer and manufacturer in different cases.
Also the variation in percentage for each quantity is shown w.r.t the values of the quantity using beta distribution.

CONCLUSION

We elaborate normally distributed exchange rate fluctuation when the retailer or manufacturer undertakes to share the exchange rate risk and the demand error is modelled in the multiplicative form in the news vendor framework. We also have compared the exchange rate effect with the generalized beta distribution error which is evident from the tables given below.

Case-1 Retailer Bears the Risk

Distribution	Parameters of the Dist.	p*	q*	Seller's Selling Price w*	Optimum Exp. Profit of Buyer	Optimum exp. Profit of Seller
Beta	α=1 , β=3	55.04	20702	32.82	423229	265586
Normal		58.01	18083	32.87	390442	232895
Variation%		5.396076	12.65095	0.152346	7.74687	12.30901
		↑	↓	↑	↓	↓
Beta	α=3 , β=1	60.97	15919	32.92	361846	205696
Normal		58.01	18083	32.87	390442	232895
Variation%		4.854847	13.59382	0.151883	7.902809	13.22291
		↓	↑	↓	↑	↑
Beta	α=1 , β=1	58.01	18088	32.87	390501	232953
Normal		58.01	18083	32.87	390442	232895
Variation%		0	0.027643	0	0.015109	0.024898
			↓		↓	↓
Beta	α=2 , β=5	55.47	20297	32.83	418165	260544
Normal		58.01	18083	32.87	390442	232895
Variation%		4.579052	10.90802	0.12184	6.62968	10.61203
		↑	↓	↑	↓	↓
Beta	α=5 , β=2	60.55	16206	32.91	365718	209306
Normal		58.01	18083	32.87	390442	232895
Variation%		4.19488	11.58213	0.121544	6.7604	11.2701
		↓	↑	↓	↑	↑

Case-2 Manufacturer Bears the Risk

Distribution	Parameters of the Dist.	p*	q*	Buyer's Purchase Cost w_r*	Optimum Exp. Profit of Buyer	Optimum Exp. Profit of Seller
Beta	α=1 , β=3	55.04	20702	32.82	423299	265586
Normal		58.01	18084	32.87	390443	232895
Variation%		5.3960756	12.646121	0.1523461	7.7618894	12.309007
		↑	↓	↑	↓	↓
Beta	α=3 , β=1	60.97	15919	32.92	361846	205696
Normal		58.01	18084	32.87	390443	232895
Variation%		4.8548466	13.600101	0.1518834	7.9030858	13.222911
		↓	↑	↓	↑	↓
Beta	α=1 , β=1	58.01	18088	32.87	390501	232953
Normal		58.01	18084	32.87	390443	232895
Variation%		0	0.0221141	0	0.0148527	0.0248977
			↓		↓	↓
Beta	α=2 , β=5	55.47	20297	32.83	418165	260544
Normal		58.01	18084	32.87	390443	232895
Variation%		4.5790517	10.903089	0.1218398	6.6294405	10.612027
		↑	↓	↑	↓	↓
Beta	α=5 , β=2	60.55	16206	32.91	365718	209306
Normal		58.01	18084	32.87	390443	232895
Variation%		4.1948803	11.588301	0.1215436	6.7606735	11.270102
		↓	↑	↓	↑	↑

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