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***Research Article***

# TIME EVOLUTION OF MATTER AND DARK ENERGY OF THE UNIVERSE IN THE FRAMEWORK OF BRANS-DICKE THEORY

***Sudipto Roy**
*Department of Physics, St. Xavier's College, Kolkata,*
*30 Mother Teresa Sarani (Park Street), Kolkata – 700016, West Bengal, India*
**Author for Correspondence: roy.sudipto1@gmail.com*

**ABSTRACT**

In the framework of generalized Brans-Dicke theory, the time variation of the matter content of the universe has been determined by formulating a new model regarding the conversion of matter (both dark and baryonic) into dark energy. A new function $f(t) = \rho a^3/\rho_0 a_0^3$, which is proportional to the matter content of the universe, has been defined and its functional form has been determined from the field equations. For this purpose, empirical expressions of scale factor and scalar field parameter $(\varphi)$have been used. This scale factor has been chosen to generate signature flip of the deceleration parameter with time. The value of $f(t)$ is found to decrease with time, indicating a conversion of matter into dark energy. The time variations of the densities of matter and dark energy have been determined. The nature of variations of gravitational constant and deceleration parameter, with respect to the matter content of universe, has been explored. The dependence of Brans-Dicke dimensionless parameter $\omega(\varphi)$ upon the scalar field $\varphi$ has been determined. This parameter remains constant, at a negative value close to unity, over a long range of $\varphi$. It becomes less negative for higher rate of transformation of matter into dark energy. It may serve as an indicator of the rate at which dark energy is being produced from matter. Without taking recourse to any specific mechanism of interaction between matter and scalar field, the present model enables one to correlate the change of matter content with the change of deceleration parameter and gravitational constant.

***Keywords:*** *Accelerated Cosmic Expansion, Density of Matter and Dark Energy, Conversion of Matter into Dark Energy, Brans-Dicke Theory, Gravitational Constant, Cosmology*

## INTRODUCTION

It has been established beyond doubt by many recent astrophysical observations that the universe is expanding with acceleration (Bennet *et al*., 2003; Riess *et al*., 1998; Perlmutter *et al*., 1999). The entity which is regarded as responsible for causing this acceleration is known as dark energy. It generates an effect in the universe which is opposite to that produced by gravitational attraction. It has been found in several studies that approximately seventy percent of the total matter-energy content of the universe is constituted by dark energy. It has not yet been possible to determine its true nature. A widely known parameter of the general theory of relativity, the cosmological constant $(\lambda)$, is one of the most suitable candidates acting as the source for this repulsive gravitational effect and it fits the observational data reasonably well, although it has its own limitations (Sahni and Starobinsky, 2000). A large number of possible candidates for this dark energy component has already been proposed and their behaviour have been studied extensively (Sahni and Starobinsky, 2000; Padmanabhan, 2003). It is worth mentioning that this accelerated expansion is a very recent phenomenon and it follows a phase of decelerated expansion. This is important for the successful nucleosynthesis and also for the structure formation of the universe. As per observational findings, beyond a certain value of the redshift (z) (i.e. z > 1.5), the universe surely had a decelerated phase of expansion (Riess *et al.*, 2001). Thus, the evolution of the dark energy component has taken place in a manner such that its effect on the dynamics of the universe is dominant only during later stages of the matter dominated era. On the basis of a recent analysis of the data obtained from supernova, by Padmanabhan and Roy Choudhury (2003), it has been inferred that the sign of the deceleration parameter $(q)$ of the universe has surely changed from positive to negative, implying a transition of the expansion process from deceleration to acceleration.

Apart from the studies where the cosmological constant plays the role of dark energy, a large number of other models of dark energy are found in scientific literature with their own distinctive characteristics (Copeland *et al*., 2006; Martin, 2008). All these theoretical formulations have successfully accounted for cosmic acceleration. Out of these candidates, one of the most popular candidates is a scalar field with a positive potential which can generate an effective negative pressure if the potential term dominates over the kinetic term. This scalar field is often called the *quintessence scalar field*. One finds a large number of quintessence potentials in scientific literature and their behaviours have been studied extensively. One may go through a study by Sahni (2004) on this field in order to obtain detailed information in this regard. The origins of models of most of the quintessence potentials do not have a proper physical background or explanation.

In order to interpret the effects produced by all these agents responsible for accelerated expansion, one must assume an interaction between the two components of matter-energy, namely, the cold dark matter and dark energy. This interaction may lead to a transfer of energy from one field to another. A number of models have been proposed where a transfer of energy takes place from the component of dark matter to the component of dark energy (Zimdahl, 2012; Reddy and Kumar, 2013), so that during the late time of evolution, the dark energy predominates over the ordinary matter and causes acceleration of the universe. However, most of these models are found to be based on interactions that have been chosen arbitrarily, without being supported by any physical theory. In order to explain the behavior of dark energy in governing the cosmic expansion, the scientific community has been searching for a model that describes facts in terms of an interaction between matter and the scalar field.

To avoid the discrepancies connected to the arbitrariness in the formulation of a particular quintessence field, one may take recourse to the scalar field theories. The non-minimally coupled scalar field theories are found to be much more capable of carrying out the possible role of an agent responsible for the late time acceleration. This is simply due to the presence of the scalar field in the purview of the theory and does not have to be introduced separately. The Brans-Dicke theory is considered to be the most natural choice as the scalar-tensor generalization of general relativity, due to its simplicity and a possible reduction to GR in some limit. For this reason, Brans–Dicke theory or its modified versions have been shown to generate the present cosmic acceleration (Banerjee and Pavon, 2001a; Brunier *et al*., 2005). It has also been shown that BD theory can potentially generate sufficient acceleration in the matter dominated era even without any help from an exotic Q - field (Banerjee and Pavon, 2001b). But one actually needs a theory which can account for a transition from a state of deceleration to acceleration. The dark energy and dark matter components are considered to be non-interacting in most of the models and are allowed to evolve independently.

Since these two components are of unknown nature, the interaction between them is expected to provide a relatively generalized framework for study. Zimdahl and Pavon (2004) have shown that the interaction between dark energy and dark matter can be very useful in solving the coincidence problem. Using these theoretical models, one may explore the possibilities of interactions between the Brans-Dicke scalar field and dark matter, leading to an inter-conversion between matter and dark energy. In a study conducted by Amendola (1999), the possibility of an inter-conversion of energy between the matter content of the universe and the non-minimally coupled scalar field was predicted.

It is observed in most of the studies in the framework of Brans-Dicke theory, that the accelerated expansion of the universe requires a very low value of $\omega$, typically of the order of unity (Das *et al*., 2014) and this value is negative. In one of such studies it was shown that, if the Brans-Dicke scalar field interacts with the dark matter, a generalized Brans-Dicke theory can perhaps serve the purpose of driving acceleration even with a high value of $\omega$ (Banerjee and Das, 2006). In all these studies, either Brans-Dicke theory is modified to meet the present requirement or, one chooses a quintessence scalar field to generate the required acceleration. It was shown by Barrow and Clifton (2006) and also found in the studies of Banerjee and Das (2006), that no additional potential was required to cause the signature flip (from positive to negative) of the deceleration parameter. To account for the observational findings in this regard, they used an interaction between the Brans-Dicke scalar field and the dark matter.

A generalized version of Brans-Dicke theory has been used in the present study. Bergman (1968) proposed this version and it was later expressed in a more useful form by Nordtvedt (1970). The Brans-Dicke parameter $\omega$ is regarded here as a function of the Brans-Dicke scalar field $\varphi$unlike the older version where it is treated as a constant. In the present study, no specific mechanism of interaction between matter and scalar field has been used.
This model is based on an assumption that the dark energy, which is widely regarded as responsible for the accelerated expansion of universe, has been created from the matter content of the universe, both dark and baryonic matter. To formulate this model, a new function $f(t) = \rho a^3/\rho_0 a_0^3$ has been defined. Its value is proportional to the matter content of the universe. Its functional form, expressed in terms of other cosmologically relevant parameters, has been derived from the Brans-Dicke field equations. For this purpose, empirical expressions the scale factor and the scalar field parameter has been used. According to its defining expression we must have $f(t) = 1$ at the present epoch ($t = t_0$). If matter is assumed to be conserved, $f(t)$ must satisfy this condition at all time. The value of the matter content of universe ($\rho a^3$) is found to decrease monotonically with time, clearly indicating its conversion into some other form, probably dark energy. Assuming the matter to be the only source of dark energy, the proportions of matter and dark energy have been determined. Using these expressions, the time dependence of the densities of matter and dark energy has been determined. Using the empirical scalar field expression, the Brans-Dicke parameter ($\omega$) has been expressed in terms of other relevant parameters. Time variation of gravitational constant has been studied. Dependence of deceleration parameter and gravitational constant on the matter content of the universe has been depicted graphically.

***Field Equations and the Theoretical Model***
The action in the generalized Brans-Dicke theory (Brans and Dicke, 1961) is,
$$S=\int\left[\frac{\varphi R}{16\pi G}+\frac{\omega(\varphi)}{\varphi}\varphi_{,\mu}\varphi^{,\mu}+L_m\right]\sqrt{-g}\,d^4x \qquad (1)$$
Where, $R$ is the Ricci scalar, $L_m$ is the matter Lagrangian, $\varphi$ is the Brans-Dicke scalar field and $\omega$ is a dimensionless parameter which is considered to be a function of $\varphi$ here instead of being regarded as a constant. For a spatially flat Friedmann-Robertson-Walker space-time, the line element is given by,
$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2] \qquad (2)$$
Here $a$ is the scale factor of the universe. For a zero curvature Friedmann-Robertson-Walker space-time, variation of action (1) with respect to the metric tensor components yields the following field equations of the generalized Brans-Dicke theory (Banerjee and Ganguly, 2009).
$$3\left(\frac{\dot a}{a}\right)^2 = \frac{\rho}{\varphi} + \frac{\omega(\varphi)}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2 - 3\frac{\dot a}{a}\frac{\dot\varphi}{\varphi} \qquad (3)$$
$$2\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2 = -\frac{\omega(\varphi)}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2 - 2\frac{\dot a}{a}\frac{\dot\varphi}{\varphi} - \frac{\ddot\varphi}{\varphi} \qquad (4)$$
Combining (3) and (4) one gets,
$$2\frac{\ddot a}{a} + 4\left(\frac{\dot a}{a}\right)^2 = \frac{\rho}{\varphi} - 5\frac{\dot a}{a}\frac{\dot\varphi}{\varphi} - \frac{\ddot\varphi}{\varphi} \qquad (5)$$
From equation (3), the expression of $\omega(\varphi)$is obtained as,
$$\omega(\varphi) = 2\left[3\left(\frac{\dot a}{a}\right)^2 - \frac{\rho}{\varphi} + 3\frac{\dot a}{a}\frac{\dot\varphi}{\varphi}\right]\left(\frac{\dot\varphi}{\varphi}\right)^{-2} \qquad (6)$$
There are many theoretical studies where the content of matter (dark + baryonic) of the universe has been assumed to be conserved (Banerjee and Ganguly, 2009). This conservation of matter is mathematically expressed as,
$$\rho a^3 = \rho_0 a_0^3 = \rho_0 (\text{taking } a_0 = 1) \qquad (7)$$
There are some studies on Brans-Dicke theory of cosmology where one takes into account an interaction between matter and the scalar field. A possibility of an inter-conversion between dark energy and matter (both dark and baryonic matter) is taken into consideration in these studies. Keeping in mind this possibility, we propose the following relation for the density of matter ($\rho$).
$$\rho a^3 = f(t)\rho_0 a_0^3 = f(t)\rho_0 \ (\text{taking } a_0 = 1) \qquad (8)$$

In the present study we have not considered any theoretical model to explain or analyze the mechanism of interaction between matter and the scalar field. We have only considered a simple fact that the right hand side of equation (7) cannot be independent of time when one takes into account non-conservation of matter due to, probably, its generation from dark energy or its transformation into dark energy. We propose to incorporate a function of time $f(t)$ into equation (7) to get a new relation represented by equation (8). This function $f(t) = \frac{\rho a^3}{\rho_0 a_0^3}$, at any instant of time $t$ is the ratio of matter content of the universe at the time $t$ to the matter content at the present instant ($t = t_0$).

Thus, $f(t)$ can be regarded as proportional to the total content of matter (dark + baryonic) $M(t)$of the universe at the instant of time $t$. We have denoted this ratio by $R_1$where $R_1 \equiv f(t) = M(t)/M(t_0)$. We have defined a second ratio $R_2 = \frac{1}{f}\frac{df}{dt} = \frac{1}{M}\frac{dM}{dt}$ which represents fractional change of matter per unit time. If, at any instant, $R_2$ is negative, it indicates a loss of matter or a change of matter into some other form due to its interaction with the scalar field.

We have also defined a third ratio $R_3 = f - 1 = \frac{M(t) - M(t_0)}{M(t_0)}$ indicating a fractional change of matter content from its value at the present time.

One may assume that the process of conversion of matter into dark energy started in the past at the time of $t = \gamma t_0$ with $\gamma < 1$. Hence, $M(\gamma t_0) = M(t_0) R_1(\gamma t_0)$ is the total matter content of the universe, at $t = \gamma t_0$, when no dark energy existed. Thus, $M(\gamma t_0)$ is the total content of matter and dark energy at all time. Assuming matter to be the only source of dark energy, the proportion of dark energy in the universe at any time $t$ is given by the following ratio ($R_4$).

$$R_4 = \frac{M(\gamma t_0) - M(t)}{M(\gamma t_0)} = \frac{f(\gamma t_0) - f(t)}{f(\gamma t_0)} \quad \text{with } \gamma < 1 \tag{9}$$

Thus ($R_4 \times 100$)is the percentage of dark energy present in the universe. Nearly 70% of the total matter-energy of the universe is dark energy at the present time (Das and Mamon, 2014). For a proper choice of $\gamma$ and $k$ (to be defined later), we must have $R_4(t_0) \times 100 = 70$ approximately.

The proportion of matter (dark + baryonic) in the universe is therefore given by,

$$R_5 = 1 - R_4 = 1 - \frac{M(\gamma t_0) - M(t)}{M(\gamma t_0)} = \frac{M(t)}{M(\gamma t_0)} = \frac{f(t)}{f(\gamma t_0)} \tag{10}$$

Thus, ($R_5 \times 100$)is the percentage of matter (ordinary+dark) present in the universe.

The purpose of the present study is to determine a functional form of $f(t)$ to explore the time dependence of the ratios$R_1, R_2, R_3, R_4 and R_5$.

Using these parameters, the density of dark energy can be expressed as,

$$\rho_d = (R_4/R_5)\rho \tag{11}$$

To formulate the expression of $f(t)$ we have used the following relation which is based on equation (8).

$$f(t) = a^3 \frac{\rho}{\rho_0} \tag{12}$$

Here, the density of matter ($\rho$) can be obtained from equation (5). For this purpose one needs to choose some suitable functional form of the Brans-Dicke scalar field $\varphi$. In the present study we have chosen an empirical forms of $\varphi$, following some recent studies in this regard (Banerjee and Ganguly, 2009; Roy *et al*., 2013; Roy, 2016). The proposed ansatz for $\varphi$ is expressed as,

$$\varphi = \varphi_0 \left(\frac{a}{a_0}\right)^k = \varphi_0 a^k \tag{13}$$

Here $k$ is a constant which determines the rapidity with which the parameter $\varphi \left(\equiv \frac{1}{G}\right)$ changes with time.

Combining equation (13) with equation (5), one gets the following expression of the density of matter of the universe ($\rho$).

$$\rho = \varphi H^2[k^2 + (4 - q)k + (4 - 2q)] \tag{14}$$

Using equation (14), $\rho_0$ can be written as,

$$\rho_0 = \varphi_0 {H_0}^2[k^2 + (4 - q_0)k + (4 - 2q_0)] \tag{15}$$

Substituting from equations (14) and (15) into equation (12) we get,

$$f(t) = a^3 \frac{\varphi H^2[k^2+(4-q)k+(4-2q)]}{\varphi_0 {H_0}^2[k^2+(4-q_0)k+(4-2q_0)]} \quad (16)$$

In our derivation of the equations (14) we have used the standard expressions of Hubble parameter ($H$) and deceleration parameter ($q$), which are, $H = \dot{a}/a$ and $q = -\ddot{a}a/\dot{a}^2$ respectively. In the expression of $f(t)$, in the equation (16), the parameters $\varphi$, $H$ and $q$ are all functions of time. Their time evolution depends upon the time variation of the scale factor from which they are calculated. To calculate $f(t)$using equation (16), we have used an empirical scale factor.

This scale factor has been chosen in order to satisfy a recent observation regarding the deceleration parameter $q(\equiv -\ddot{a}a/\dot{a}^2)$. According to this observation the universe had a state of decelerated expansion before the present phase of acceleration began (Banerjee and Ganguly, 2009; Banerjee and Das, 2006; Das and Mamon, 2014). Thus, the deceleration parameter had a positive value before reaching the present stage of negative values. The functional form of our chosen scale factor is such that the deceleration parameter, calculated from it, shows a change of sign as a function of time. This scale factor is,

$$a = a_0 Exp\left[-\alpha t_0^\beta\right] Exp[\alpha t^\beta] \quad (17)$$

Here, $\alpha$ and $\beta$ are constants with both $\alpha, \beta > 0$ to make sure that the scale factor increases with time. The scalar field parameter ($\varphi$), Hubble parameter ($H$) and deceleration parameter ($q$), based on this scale factor are given respectively by the following equations,

$$\varphi = \varphi_0 \left(\frac{a}{a_0}\right)^k = \varphi_0 Exp\left[-k\alpha t_0^\beta\right] Exp[k\alpha t^\beta] \quad (18)$$

$$H = \dot{a}/a = \alpha\beta t^{(\beta-1)} \quad (19)$$

$$q = -\ddot{a}a/\dot{a}^2 = -1 + \frac{1-\beta}{\alpha\beta} t^{-\beta} \quad (20)$$

Here we find that, for $0 < \beta < 1$ $and$ $\alpha > 0$ we have,

$q > 0$ $for$ $t = 0$ and $q \to -1$ $as$ $t \to \infty$

It clearly means that the chosen scale factor generates an expression of deceleration parameters which changes sign from positive to negative as time goes on. The values of constant parameters $(\alpha, \beta)$ have been determined from the following conditions.

1st Condition: $H = H_0$ at $t = t_0$ (21)

2nd Condition: $q = q_0$ at $t = t_0$ (22)

Combining the equations (21) and (22) with the equations (19) and (20) respectively, one obtains $\alpha = 6.39 \times 10^{-10}$ and $\beta = 5.37 \times 10^{-01}$

The values of different cosmological parameters used in the present study are,

$H_0 = 2.33 \times 10^{-18} sec^{-1}$, $t_0 = 4.415 \times 10^{17} sec$, $\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} m^{-3} Kgs^2$

$\rho_0 = 2.831 \times 10^{-27} Kg/m^3$ and $q_0 = -0.55$

To determine the value of $f(t)$ from the equations (16), one must use the expressions of (18), (19) and (20) and also the values of the constants $\alpha$ and $\beta$.

The function $f(t)$ is defined by the relation $\rho a^3 = f(t)\rho_0 a_0^3$. According to this relation, the value of $f(t)$is always positive and, $f(t) = 1$ at $t = t_0$(taking $a_0 = 1$). The functional form $f(t)$ in equation (16) ensures that $f(t) = 1$ at $t = t_0$. The values of $k$ for which $f(t)$ is positive over the entire range of study (say, from $t = 0.5t_0$ to $t = 1.5t_0$) is given below.

$k < (k_-)_{min}$ or $k > (k_+)_{max}$ over the entire range of study. Here,

$(k_-)_{min} = (q-2)_{min}$ over the range from $t = 0.5t_0$ to $t = 1.5t_0$ (23)

$(k_+)_{max} = (q-2)_{max}$ over the range from $t = 0.5t_0$ to $t = 1.5t_0$ (24)

For our range of study, i.e. from $t = 0.5t_0$to $t = 1.5t_0$, we find,

$(k_-)_{min} = -2.64$ and $(k_+)_{max} = -2.33$

Thus we have a lower and an upper range of permissible values for $k$ which are $k < (k_-)_{min}$ or $k > (k_+)_{max}$ respectively. The upper range, $k > (k_+)_{max}$, includes both positive and negative values of $k$ and the lower range,$k < (k_-)_{min}$, has only negative values. According to equation (13), the parameter $\varphi$

is a decreasing function of time for negative values of $k$, causing the gravitational constant($G = \frac{1}{\varphi}$)to increase with time. Therefore, we find that the upper range of $k$ allows $G$ to be both increasing and decreasing function of time, although the lower range of $k$ causes $G$ to be an increasing function of time. To choose between these two ranges of $k$, we have to determine the values of $\omega_0$ at different values of $k$ and compare them with those obtained from other studies. Using equations (3) and (13), we can write the following expression (equation25) regarding $\omega$ for this model.

$$\omega = \frac{2}{k^2}\left[3(1+k) - \frac{\rho}{\phi H^2}\right] \tag{25}$$

Using equations (25) we get the following expression of $\omega_0$ (the value of $\omega$ at the present epoch).

$$\omega_0 = \frac{2}{k^2}\left[3(1+k) - \frac{\rho_0}{\phi_0 {H_0}^2}\right] \tag{26}$$

According to several studies on Brans-Dicke theory $\omega_0$ has a negative value close to $-1$(Das and Mamon, 2014).

To have $\omega_0 < 0$, the condition to be satisfied by $k$ is given by,

$$k < \frac{\rho_0}{3\,\phi_0 {H_0}^2} - 1 \quad \text{or,} \quad k < -0.9884 \tag{27}$$

For the entire lower range of $k$ values and for a part of its upper range, the above condition is satisfied. The gravitational constant ($G$), which is reciprocal of the Brans-Dicke scalar field parameter ($\varphi$) is given by,

$$G = \frac{1}{\varphi} = \frac{(a/a_0)^{-k}}{\varphi_0} = \frac{1}{\varphi_0} Exp\left[k\alpha t_0^{\beta}\right] Exp[-k\alpha t^{\beta}]$$

Thus, $$\frac{G}{G_0} = Exp\left[k\alpha t_0^{\beta}\right] Exp[-k\alpha t^{\beta}] \tag{28}$$

An experimentally measurable parameter $\frac{\dot{G}}{G}$ is given by,

$$\frac{\dot{G}}{G} = \frac{1}{G}\frac{dG}{dt} = -k\frac{\dot{a}}{a} = -kH = -k\alpha\beta t^{(\beta-1)} \tag{29}$$

Using equation (29) we get,

$$\left(\frac{\dot{G}}{G}\right)_{t=t_0} = -kH_0 = -k\alpha\beta {t_0}^{(\beta-1)} \tag{30}$$

The value of $k$ should be so chosen that $\left(\frac{\dot{G}}{G}\right)_{t=t_0} < 4 \times 10^{-10}\ Yr^{-1}$ (Weinberg, 1972).

***Graphical Interpretation of Theoretical Findings***

Figure 1 show the variation of the Brans-Dicke parameter ($\omega$) as a function of the cosmological red shift parameter (z). The curves for $k = -1, +20$ correspond to negative and positive values of the upper range of $k$. The curve for $k = -4.5$ correspond to the lower range of $k$. It is evident that this parameter can be both positive and negative for the upper range of $k$ values but this parameter is negative for the lower range of $k$ values.

Positive values of $\omega$ are contrary to the findings of other studies in this regard (Das and Mamon, 2014). This observation is in favour of using the lower ranges of $k$ values.

According to equation (28), the gravitational constant increases with time for negative values of $k$. Thus, for the entire lower range of $k$ values, $G$ increases with time. Only for the negative values of the upper range, which constitute a small part of this range, $G$ increases with time. There are experimental observations and theoretical models where $G$ has been shown to be increasing with time (Pradhan *et al*., 2015).

Keeping in mind the above observations, regarding $\omega$ and $G$, we have used the values of $k$ from its lower range, in the present study.

Figure 2 shows the plots of density of matter ($\rho_m$), dark energy ($\rho_d$) and total density ($\rho_t$) as functions of time. Equations (11), (14) have been used for these plots. With $k = -4$ and $\gamma = 0.5$, one obtains approximately correct values from these curves for $t = t_0$.

Figure 3 shows the plots of $\rho_d/\rho_m$ as a function of time for three different values of $k$ with $\gamma = 0.4$. This ratio is found to increase with time and its rate of increase gets enhanced for more negative values of $k$.
Figure 4 shows the plots of $\rho_d/\rho_m$ as a function of time for three different values of $\gamma$ with $k = -3$. This ratio is found to increase with time and its value, at any instant, is larger for smaller values of $\gamma$. The parameter $\gamma$ indicates the time when the matter to dark energy conversion started. The smaller its value, the earlier it started and larger would be the value of $\rho_d/\rho_m$ at the present time ($t = t_0$).

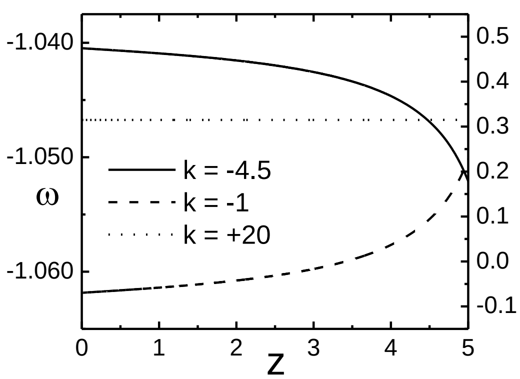


**Figure 1: Plot of the $\omega$ as a function of $z$, for different values of $k$ in its lower and upper ranges, shown respectively along the left and right vertical axes**

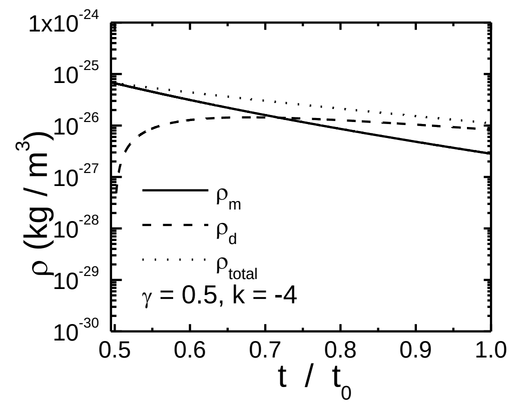


**Figure 2: Plot of $\rho_m(\equiv \rho)$, $\rho_d$ and $\rho_{total}$ as functions of time. The conversion of matter into dark energy began at $t = \gamma t_0$**

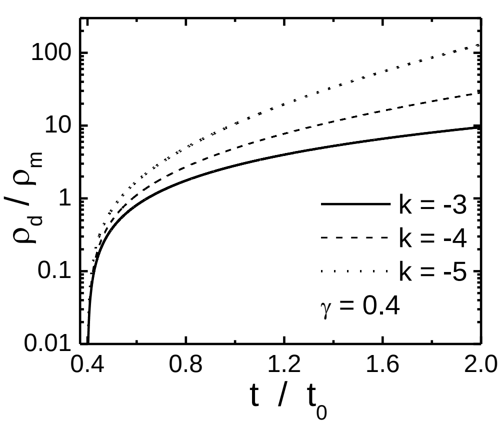


**Figure 3: Plot of $\rho_d/\rho_m$ as a function of time for three different values of $k$ with $\gamma = 0.4$**

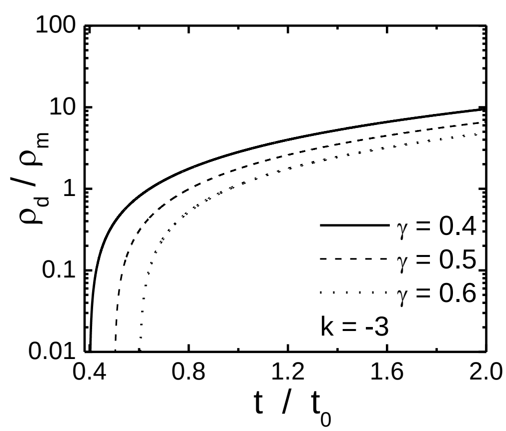


**Figure 4: Plot of $\rho_d/\rho_m$ as a function of time for three different values of $\gamma$ with $k = -3$**

Figure 5: Plot of $\omega(\varphi)$ as a function of $\varphi$ for three different values of $k$. This parameter remains constant at a negative value over a long range of $\varphi$. This constant value becomes less negative for more negative values of $k$. This behavior indicates that, the faster the transition from matter to energy, the negative value of$\omega$ would be closer to zero.
In Figure 6 we have the plot of $\rho a^3$ (a measure of the matter content of the universe) as a function of time for three different values of $k$. This matter content decreases at a gradually slower rate. The rate of depletion of matter content gets enhanced for more negative values of $k$.

Figure 7 shows a plot of deceleration parameter ($q$) as a function of matter content ($\rho a^3$) for three different values of $k$. As the matter content decreases, the universe gradually makes a transition from a phase of deceleration to acceleration. The amount of matter content at the time when $q = 0$ is found to be greater for more negative values of $k$.

Figure 8 shows a plot of $G/G_0$ as a function of matter content ($\rho a^3$) for three different values of $k$. The gravitational constant increases with a fall of matter content and its rate of increase gets enhanced for higher values of $k$.

These plots show that the rate at which matter is converted into dark energy depends on the value of the parameter $k$. This rate is enhanced with more negative values of this parameter. According to equation (28), the gravitational constant increases faster with more negative values of $k$. Thus, it is established that faster the production of dark energy from matter, faster will be changes in deceleration parameter and gravitational constant.

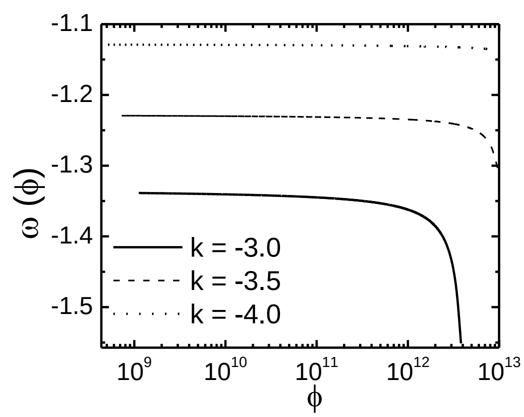


**Figure 5: Plot of $\omega(\varphi)$ as a function of $\varphi$ for three different values of $k$**

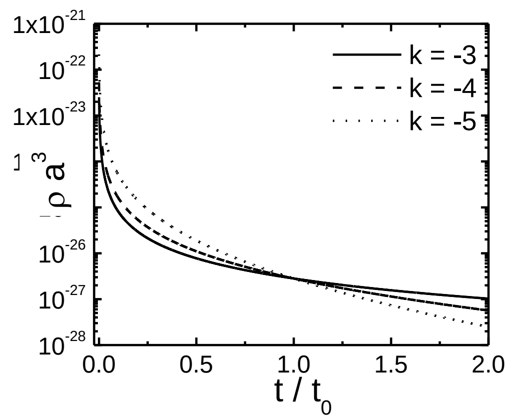


**Figure 6: Plot of $\rho a^3$ as a function of time for three different values of $k$**

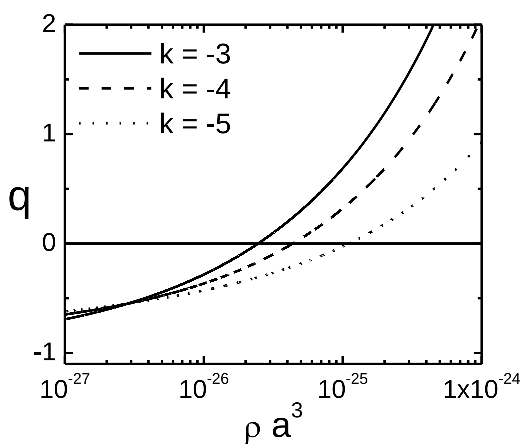


**Figure 7: Plot of $q$ as a function of matter content ($\rho a^3$) for three different values of $k$**

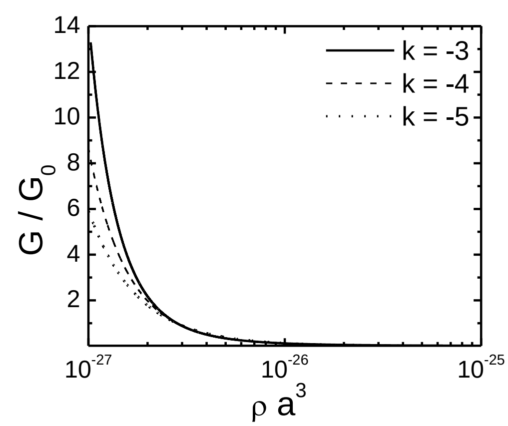


**Figure 8: Plot of $G/G_0$ as a function of matter content ($\rho a^3$) for three different values of $k$**

***Conclusion***

In the present study we have determined the nature of depletion of matter content of the universe with time, considering a model for its conversion into dark energy which is responsible for the accelerated expansion of the universe. It is very clearly found that as the rate of conversion of matter (both dark and baryonic) into dark energy increases, the rate of increase of gravitational constant increases along with the rate of change of the deceleration parameter.

With the rise in the rate of matter to energy conversion, the deceleration parameter makes a quicker transition from positive to negative values, indicating a faster journey towards the phase of acceleration from decelerated expansion, implying the enhancement of an interaction whose effect is opposite to that of the gravitational force upon matter. The present study also shows that the ratio of dark energy to matter contents of the universe depends on two parameters, $\gamma$and $k$, where the first one determines the time at which dark energy started forming from matter and the second parameter indicates the speed at which the transformation of matter took place.

For a faster transformation, the value of $\omega$ becomes less negative and thus, $\omega$ serves as an indicator of this rate. The present study has shown graphically the variation of the matter content of the universe, and various other relevant parameters, as functions of time. The variation of different parameters as functions of the matter content has been explored. All these analyses have shown clearly that there exists a relation between the transformation of matter into dark energy and the time variation of different parameters, such as gravitational constant and deceleration parameter that characterize the process of expansion of the universe. The theoretical model used for this study can be improved by using a different empirical expression for the scale factor and also by assuming a relationship between the function $f(t)$ and the scale factor. One may also choose a different expression regarding the relation between the scalar field parameter ($\varphi$) and the scale factor. Instead of using an empirical expression of the scale factor, one may determine the scale factor by solving the Brans-Dicke field equations, using suitable forms of $\varphi$ and $f(t)$.

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