International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2016 Vol. 6 (1) January-March, pp. 88-94/Tripathi
Research Article

PERFECT FLUID LRS BIANCHI TYPE-I MODEL WITH CONSTANT VALUE OF DECELERATION PARAMETER

***Shailendra Kumar Tripathi**
Department of Mathematics, Government V. PG College Maihar, Satna (M.P.), India,
**Author for Correspondence*

ABSTRACT
Locally rotationally symmetric (LRS) Bianchi type –I cosmological model of the universe with varying cosmological Term in general relativity are investigated by taking $\frac{-R\ddot{R}}{\dot{R}^2} = n$, where $n \geq -1, n \neq 2$,R is scale factor. A new class of exact solution has been obtained.

Keywords: *LRS Bianchi Type – I Model, Constant Deceleration Parameter, Variable Cosmological Term* Λ

INTRODUCTION

Recent observations (Singh and Beesham, 2009; Singh *et al.*, 2007; Brokar and Charjan, 2010; Chandel *et al.*, 2014; Yadav, 2013) indicate that the study of Bianchi type cosmological models create more interest at present. In earlier literature, the Bianchi type models have been investigated by many authors Dwivedi and Tiwari, (2012); Bali *et al.*, (2007); Pradhan and Srivastava, (2006); Mohanti and Mishra, (2001); Singh and Baghel, (2009), in different context. Bianchi type – I models are anisotropic generalization of FRW models with flat space sections. Singh *et al.*, (2008) have considered Bianchi type – I model with variable G and Λ term in general relativity. Tiwari (2014) generates LRS Bianchi type – I cosmological model by taking $\Lambda \propto \frac{\ddot{R}}{\dot{R}}$, where R is scale factor. These LRS Bianchi type – I models have already been considered by number of researchers Pradhan and Kumar (2001), Rao and Neelima (2013), Akarsu and Kilinc (2009) and others have been studied Bianchi type models using different approach.

In the present article, we investigated a Locally Rotationally Symmetric Bianchi type – I model by taking constant value n of deceleration parameter q, i.e. $q = \frac{-R\ddot{R}}{\dot{R}^2} = n, n \geq -1 and \neq 2$. It is remarkable to mention here that the sign of q indicated whether the model represents an accelerating universe or not. The positive sign of q corresponds to 'standard' decelerating model whereas the negative sign of q indicates an accelerating universe. The observation of SNe Ia and CMBR favours accelerating models $(q < 0)$, but q = 0 shows that every galaxy moves with constant speed.

Metric and Field Equations

We consider the LRS Bianchi type – 1 space time in orthogonal form represented by the line element

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2) \quad (1)$$

Where, A and B are functions of time t. The energy momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p)v_i v_i + pg_{ij} \quad (2)$$

With equation of state $p = \omega\rho$, $\quad 0 < \omega < 1$ (3)

Where, p and ρ are pressure and energy density respectively and v_i is the unit flow vector satisfying $v_i v^i$ = -1

Einstein's field equation (in gravitational units $8\pi G = c = 1$) with variable cosmological term Λ is given by

$$R_{ij} - \frac{1}{2} Rg_{ij} = -T_{ij} + \Lambda g_{ij} \tag{4}$$

Since, the vacuum has symmetry of the background, its energy momentum tensor has the form $T_{ij}^{vac} = \Lambda g_{ij}$, where Λ is an invariant function of cosmic time t. We now assume that the law of conservation of energy $T^{i}_{j;i} = o$ gives

$$\dot{\rho} + (\rho + p)\left(\frac{3\dot{R}}{R}\right) = 0 \tag{5}$$

Using equation (3) in (5) we obtain

$$\rho = \frac{k_3}{R^{3(1+\omega)}} \tag{6}$$

In co-moving coordinates, this corresponds to a perfect fluid with energy density $\rho_v = \Lambda$ and pressure $p_v = -\Lambda$ satisfying the equation of state $p_v = -\rho_v = -\Lambda$. Einstein's field equation (4) for the metric (1) leads to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -p + \Lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p + \Lambda \tag{8}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \rho + \Lambda \tag{9}$$

Where dots on A, B denote the ordinary differentiation with respect to t. Eliminating p and Λ from equation (7) and (8), we obtain $\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0$ which on integration gives

$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{K}{R^3}$ where, K is a constant, $R^3 = AB^2$ and volume expansion scalar is given by $H = \frac{\dot{R}}{R} = \frac{\theta}{3}$,. Equation (6), (7), (8) and (9) can also be written in terms of Hubble parameter H, deceleration parameter q, shear σ and scale factor R as

$$H^2(2n-1) - \sigma^2 = p - \Lambda \tag{10}$$

$$3H^2 - \sigma^2 = \rho + \Lambda \tag{11}$$

Where $\sigma^2 = \frac{K_1{}^2}{3R^6}$

from equation (11), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{3\rho}{\theta^2} - \frac{3\Lambda}{\theta^2}$$

which implies that for $8\pi G = 1, \Lambda \geq 0$

$$0 < \frac{\sigma^2}{\theta^2} < 1/3, 0 < \frac{\rho}{\theta^2} < \frac{1}{3} \quad (12)$$

Therefore, the presence of positive Λ lowers the upper limit of anisotropy. From Eqns. (3), (10) and (11), we obtain $\frac{\Lambda}{H^2} = 2 - n + \frac{(\omega-1)\rho}{2H^2}$ (13)

For stiff matter, From Equation (13) it is clear that $\Lambda \sim H^2$, which follows from the model of Singh *et at.*,(2008). Also we can determine the present value of the cosmological constant.

If $n < 2 + \frac{(\omega-1)\rho}{2H^2}$, $\Lambda > 0$ whereas $\Lambda < 0$ $n > 2 + \frac{(\omega-1)\rho}{2H^2}$.

To solve the systems (3) and (5) --- (8) completely consider

$$\frac{-R\ddot{R}}{\dot{R}^2} = n$$

Which lead to a differential equation $\frac{\ddot{R}}{\dot{R}} + n\frac{\dot{R}}{R} = 0$ (14)

A partial list of cosmological model in which the deceleration parameter q is constant, Maharaj and Naidoo (1993), Pradhan *et al.*, (2010) & Akarsu and Kilinc (2009), but our assumption is different from these model because they have been solved Einstein's field equations by applying a law of variation for Hubble's parameter and observed that this law yields a constant value of deceleration parameter q which proposed by Berman (1983) whereas we considered a constant value of deceleration parameter.

To determine the scale factor integrating equation (14), we get

$$R = (k_1 t + k_2)^{1/n+1} \quad (15)$$

Where k_1 and k_2 are constants of integration. For this solution metric (1) assume the form

$$ds^2 = -dt^2 + (k_1 t + k_2)^{2/3(n+1)}\left[\exp\left\{\frac{4k(n+1)}{3k_1(n-2)}(k_1 t + k_2)^{n-2/n+1}\right\}dx^2 + \exp\left\{\frac{-2k(n+1)}{3k_1(n-2)}(k_1 t + k_2)^{n-2/n+1}\right\}(dy^2 + dz^2)\right] \quad (16)$$

Cosmology for Decelerating Universe (q = n = 1)

In this case differential equation (14), becomes $\frac{\ddot{R}}{\dot{R}} + \frac{\dot{R}}{R} = 0$ (17)

Which gives the value of average scale factor $R = (k_1 t + k_2)^{1/2}$ (18)

Where k_1 and k_2 are constants of integration. For this solution line element (1) becomes

$$ds^2 = -dt^2 + (k_1 t + k_2)^{1/3}\left[\exp\left\{\frac{8k}{3k_1}(k_1 t + k_2)^{-1/2}\right\}dx^2 + \exp\left\{\frac{-4k}{3k_1}(k_1 t + k_2)^{-1/2}\right\}(dy^2 + dz^2)\right] \quad (19)$$

For the metric (19), Hubble parameter H, expansion scalar θ, shear scalar σ are given by

$$H = \frac{k_1}{2}(k_1 t + k_2)^{-1} \quad (20)$$

$$\theta = \frac{3k_1}{2}(k_1 t + k_2)^{-1} \quad (21)$$

$$\sigma = \frac{k}{\sqrt{3}\left((k_1 t + k_2)^{3/2}\right)} \quad (22)$$

The spatial volume V, cosmological energy density ρ, pressure p and cosmological constant Λ are given by

$$V = (k_1 t + k_2)^{3/2} \quad (23)$$

$$\frac{p}{\omega} = \rho = \frac{k_3}{(k_1 t + k_2)^{\frac{3(\omega+1)}{2}}} \quad (24)$$

$$\Lambda = \frac{3k_1^2}{4(k_1 t + k_2)^2} - \left[\frac{k^2}{3(k_1 t + k_2)^3} + \frac{k_3}{(k_1 t + k_2)^{\frac{3(\omega+1)}{2}}}\right] \quad (25)$$

In this model, we observe that the spatial volume $V \to 0$ as $t \to \frac{-k_2}{k_1}$ and expansion scalar $\theta \to \infty$ as $t \to \frac{-k_2}{k_1}$ which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factor also vanish at $t = \frac{-k_2}{k_1}$ and hence the model has a point type singularity at the initial epoch. The cosmological energy density ρ, pressure p, shear scalar σ and cosmological constant Λ all are infinite at $t = \frac{-k_2}{k_1}$.

With t increases the expansion scalar and shear scalar decrease but spatial volume increases. As t increases all the parameters ρ, p, θ, Λ, p_v and critical density $\rho_c = \frac{-3k_1^2}{4(k_1 t + k_2)^2}$ decrease and tend to zero asymptotically.

Therefore, the model essentially gives an empty universe for large value of t. The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that the model approaches isotropy for the large value of t.

Cosmology when Every Galaxy Moves Constant Speed (q = n = 0)

In this case, average scale factor is given by

$$R = (k_1 t + k_2)^{1/2} \quad (26)$$

And for this solution line element (1) take the form (Where k_1 and k_2 are constants)

$$ds^2 = -dt^2 + (k_1 t + k_2)^2 \left[\exp\left\{ \frac{-2k}{3k_1} (k_1 t + k_2)^{-4} \right\} dx^2 + \exp\left\{ \frac{k}{3k_1} (k_1 t + k_2)^{-4} \right\} (dy^2 + dz^2) \right] \qquad (27)$$

For the model (27), Hubble parameter H, expansion scalar θ, shear scalar σ are given by

$$H = \frac{k_1}{(k_1 t + k_2)} \qquad (28)$$

$$\theta = \frac{3k_1}{(k_1 t + k_2)} \qquad (29)$$

$$\sigma = \frac{k}{\sqrt{3}\left((k_1 t + k_2)^3\right)} \qquad (30)$$

The spatial volume V, cosmological energy density ρ, pressure p and cosmological constant Λ are given by

$$V = (k_1 t + k_2)^3 \qquad (31)$$

$$\frac{p}{\omega} = \rho = \frac{k_3}{(k_1 t + k_2)^{3(\omega+1)}} \qquad (32)$$

$$\Lambda = \frac{k_1^2}{(k_1 t + k_2)^2} - \left[\frac{k^2}{3(k_1 t + k_2)^6} - \frac{k_3}{(k_1 t + k_2)^{3(\omega+1)}} \right] \qquad (33)$$

In this case, we observe that the spatial volume $V \to 0$ as $t \to \frac{-k_2}{k_1}$ and expansion scalar $\theta \to \infty$ as $t \to \frac{-k_2}{k_1}$ which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factor also vanish at $t = \frac{-k_2}{k_1}$ and hence the model has a point type singularity at the initial epoch. The cosmological energy density ρ, pressure p, shear scalar σ and cosmological constant Λ all are infinite at $t = \frac{-k_2}{k_1}$.

With t increases the expansion scalar and shear scalar decrease but spatial volume increases. As t increases all the parameters ρ, p, θ, Λ, p_v and critical density $\rho_c = \frac{k_1^2}{(k_1 t + k_2)^2}$ decrease and tend to zero asymptotically.

Therefore, the model essentially gives an empty universe for large value of t .The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that the model approaches isotropy for the large value of t.

Cosmology for Accelerating Universe (q = n = -1)

In this case, average scale factor is by $R = e^{(k_1 t + k_2)}$ and metric (1) assume the form

$$ds^2 = -dt^2 + e^{2(k_1 t + k_2)} \left[\exp\left\{ \frac{-4k}{9k_1} e^{-3(k_1 t + k_2)} \right\} dx^2 + \exp\left\{ \frac{2k}{9k_1} e^{-3(k_1 t + k_2)} \right\} (dy^2 + dz^2) \right] \qquad (34)$$

For the metric (34), Hubble parameter H, expansion scalar θ, shear scalar σ are given by

$$H = k_1 \tag{35}$$

$$\theta = 3k_1 \tag{36}$$

$$\sigma = \frac{k}{\sqrt{3}e^{3(k_1 t + k_2)}} \tag{37}$$

The spatial volume V, cosmological energy density ρ, pressure p and cosmological constant Λ are given by

$$V = e^{3(k_1 t + k_2)} \tag{38}$$

$$\rho = \frac{k}{e^{3(1+\omega)(k_1 t + k_2)}} \tag{39}$$

$$\Lambda = 3k_1^2 - \frac{k^2}{3e^{6(k_1 t + k_2)}} - \frac{k_3}{e^{3(1+\omega)(k_1 t + k_2)}} \tag{40}$$

In this case, the model has no initial singularity. The expansion in the model starts at $t = \frac{-k_2}{k_1}$ with the spatial volume, scale factor, pressure, energy density and other cosmological parameters all finite. As t increases energy density, pressure p, shear scalar and cosmological term are decreases whereas spatial valume V and average scale factor R increases. The expansion scalar is constant throughout the expansion, thus the model represents uniform expansion. At $t = \frac{-k_2}{k_1}$, all parameters remain non zero constants. The ratio $\frac{\sigma}{\theta} \neq 0$ as $t \to \frac{-k_2}{k_1}$, therefore the model maintained anisotropy throughout the expansion.

Conclusion

In this paper we have investigated a new class of LRS Bianchi type – I space – time by using the value $n \geq -1$ ($n \neq 2$) of deceleration parameter q in which cosmological constant Λ varies with cosmic time t. We would like to mention have that Tiwari and Jha (2009), Tiwari (2010) have also derived the LRS Bianchi type – I model with variable cosmological term Λ using different approach but our derived result differ from these models in the sense that both have been solved Einstein's field equations by using law of variation of scale factor with variable cosmological term Λ whereas we have presented a class of solution by taking $\frac{-R\ddot{R}}{\dot{R}^2} = n$. Expressions for some important cosmological parameters have been obtained and physical behavior of the models are discussed in detail, clearly the model represent shearing, non-rotating and expanding models with a big-bang start. The models have point type singularity at the initial epoch and approach isotropy for $n \neq 0$ and $n < 3$ whereas for $n = 0$ model has no singularity also anisotropy in universe is maintained throughout, with $n \geq 3$. $n \geq 3$

Finally the solutions presented here are new and useful for a better understanding of the evolution of the universe in the LRS Bianchi type-I universe with value of $q = n(\geq -1, \neq 2)$. More cosmological models may be analyzed using this technique, which may lead to interesting and different behavior of the universe.

ACKNOWLEDGEMENTS

The author is grateful to the referee for his valuable comments

REFERENCE

Akarsu O and Kilinc CB (2009). LRS Bianchi type – I models with anisotropic dark energy and constant deceleration parameter. *General Relativity and Gravitation* **42** 119-140. arXiv:0807.4867v2[qr-gc].

Bali R, Pareek UK and Pradhan A (2007). Bianchi type – I massive string magnetized barotropic perfect fluid cosmological model in general relativity. *Chinese Physics Letters* **24**(8) 2455.

Berman MS (1983). Special law of variation for Hubble parameter. *Nuovo Cimento B* Italy **74** 182- 6.

Brokar MS and Charjan SS (2013). Bianchi type – I magnetized cosmological model in biometric theory of gravitation. *Applications and Applied Mathematics* **5**(10) 1660-1671.

Chandel S, Singh MK and Ram S (2014). Bianchi type – VI_0 cosmological models with isotropic dark energy. *Electronic Journal of Theoretical Physics* **11**(30) 101 - 108.

Dwivedi UK and Tiwari RK (2012). Bianchi type – V cosmological models with time varying G and Λ. *International Journal of Physics and Mathematical Sciences* **2**(2) 1 - 7.

Maharaj SD and Naidoo R (1993). Solutions to the field equations and the deceleration parameter. *Astrophysics and Space Science* **208** 261 - 276.

Mohanti G and Mishra B (2001). LRS Bianchi type – I cosmological models with perfect fluid. *Bulgarian Journal of Physics* **28**(3/4) 185 – 192.

Pradhan A and Kumar A (2001). LRS Bianchi type – I cosmological universe models with time varying cosmological term Λ. *International Journal of Modern Physics* **10**(3) 291-298.

Pradhan A and Srivastava SK (2007). Titled Bianchi type – V bulk viscous cosmological models in general relativity. *Romanian Reports in Physics* **59**(3) 749 -761.

Pradhan A, Amirhashchi H and Saha B (2010). Bianchi type – I anisotropic dark energy models with constant deceleration parameter. *International Journal of Theoretical Physics* **50** 2923-2938. arXiv:1010.1121v1[gr-qc].

Rao VUM and Neelima D (2013). LRS Bianchi type – I dark energy cosmological models in general scalar tensor theory of gravitation. *ISRN Astronomy and Astrophysics* **2013** Article ID 174741, 6.

Singh CP and Beesham A (2009). Locally – rotationally – symmetric Bianchi type – V cosmology with heat flow. *Pramana Journal of Physics* **73**(4) 793 - 798.

Singh JP and Baghel PS (2009). Bianchi type – V Bulk Viscouc Cosmological Models with Time Dependent Term. *Electronic Journal of Theoretical Physics* **22** 85 - 96.

Singh JP, Pradhan A and Singh AK (2008). Bianchi type – I cosmological models with variable G and Λ - term in general relativity. *Astrophysics and Space Science* **314** 83-88. arXiv:0705.0459v2[gr-qc].

Singh JP, Tiwari RK and Shukla P (2007). Bianchi type – III cosmological models with gravitational constant G and the cosmological constant Λ. *Chinese Physics Letters* **24**(12) 3325 - 3327.

Tiwari RK (2010). Bianchi type – I cosmological models with perfect fluid in general relativity. *Research in Astronomy and Astrophysics* **10**(4) 291 -300.

Tiwari RK (2014). LRS Bianchi type – I model with time dependent cosmological term Λ. *African Review of Physics* **9**(0021) 159 -163.

Tiwari RK and Jha NK (2010). Locally rotationally symmetric Bianchi type – I model with time varying . *Chinese Physics Letters* **26**(10) 109804 1-4.

Yadav AK (2013). Bianchi type – V matter filled universe with varying Λ in general relativity. *Electronic Journal of Theoretical Physics* **10**(28) 169 - 182.