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Research Article

PERFECT FLUID BIANCHI TYPE-I MODEL WITH CONSTANT VALUE OF DECELERATION PARAMETER

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ABSTRACT
Bianchi type –I cosmological model of the universe with varying cosmological Term in general relativity are investigated by taking $\frac{-R\ddot{R}}{\dot{R}^2} = n$, where $0 < n < \frac{1}{2}$, R is scale factor. A new class of exact solution has been obtained.

Keywords: *Bianchi Type – I Model, Constant Deceleration Parameter, Variable Cosmological Term* Λ

INTRODUCTION

Recent observations (Singh and Beesham, 2009; Singh *et al.*, 2007; Chandel *et al.*, 2014; Yadav, 2013; Pradhan and Srivastava, 2007) indicate that the study of Bianchi type cosmological models create more interest at present. In earlier literature, the Bianchi type models have been investigated by many authors (Pradhan and Kumar, 2001; Mohanti and Mishra, 2001; Rao and Neelima, 2013; Singh and Baghel, 2009; Dwivedi and Tiwari, 2012) in different context. Bianchi type – I models are anisotropic generalization of FRW models with flat space sections. Singh *et al.*, (2008) have considered Bianchi type – I model with variable G and Λ term in general relativity. Tiwari (2010) generates Bianchi type – I cosmological model by taking $\Lambda \propto \frac{\ddot{1}}{R^m}$, where R is scale factor. These Bianchi type – I models have already been considered by number of researchers Saha (2000); Bali *et al.*, (2007); Brokar and Charjan (2013); Kumar and Singh (2007) have been studied Bianchi type models using different approach.

In the present article, we investigated a Bianchi type – I model by taking constant value n of deceleration parameter q, i.e. $q = \frac{-R\ddot{R}}{\dot{R}^2} = n$ where $0 < n < \frac{1}{2}$. It is remarkable to mention here that the sign of q indicated whether the model represents an accelerating or not. The positive sign of q corresponds to 'standard' decelerating model whereas the negative sign of q indicates an accelerating universe. The observation of SNe Ia and CMBR favours accelerating models $(q < 0)$, but q = 0 shows that every galaxy moves with constant speed.

Metric and Field Equation: We consider the Bianchi type – 1 spce - time in orthogonal form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{1}$$

where A , B and C are functions of time t. The energy momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p)v_i v_i + pg_{ij} \tag{2}$$

With equation of state

$$p = \omega\rho, \qquad 0 < \omega < 1 \tag{3}$$

where p and ρ are pressure and energy density respectively and v_i is the unit flow vector satisfying $v_i v^i$ = -1 Einstein's field equation (in gravitational units $8\pi G = c = 1$) with variable cosmological term Λ is given by

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$$R_{ij} - \frac{1}{2} R g_{ij} = - T_{ij} + \Lambda g_{ij} \tag{4}$$

Since, the vacuum has symmetry of the background, its energy momentum tensor has the form in $T_{ij}^{vac} = \Lambda g_{ij}$, where Λ is an invariant function of cosmic time t. We now assume that the law of conservation of energy $T_{j;i}^{i} = 0$ gives

$$\dot{\rho} + (\rho + p)\left(\frac{3\dot{R}}{R}\right) = 0 \tag{5}$$

Using equation (3) in (5) we obtain

$$\rho = \frac{k_3}{R^{3(1+\omega)}} \tag{6}$$

In co-moving coordinates, this corresponds to a perfect fluid with energy density $\rho_v = \Lambda$ and pressure $p_v = -\Lambda$ satisfying the equation of state $p_v = -\rho_v = -\Lambda$. Einstein's field equation (4) for the metric (1) leads to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p + \Lambda \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p + \Lambda \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -p + \Lambda \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \rho + \Lambda \tag{10}$$

Subtracting equation (8) from (9), we get

$$\left(\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC}\right) = 0$$

$$\Rightarrow \quad \ddot{A}BC - A\ddot{B}C + B\dot{A}\dot{C} - A\dot{B}\dot{C} = 0$$

Adding and subtracting $\dot{A}\dot{B}C$ in above equation

$$\ddot{A}BC - A\ddot{B}C + B\dot{A}\dot{C} - A\dot{B}\dot{C} + \dot{A}\dot{B}C - \dot{A}\dot{B}C = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(\dot{A}BC\right) - \frac{d}{dt}\left(A\dot{B}C\right) = 0 \tag{11}$$

Similarly subtracting ( ) and ( ) from ( ) and proceeding as above we get

$$\frac{d}{dt}\left(\dot{A}BC\right) - \frac{d}{dt}\left(AB\dot{C}\right) = 0 \tag{12}$$

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$$\frac{d}{dt}\left(A\dot{B}C\right)-\frac{d}{dt}\left(AB\dot{C}\right) = 0 \tag{13}$$

First integral of (11), (12) and (13) are

$$\left(\dot{A}BC\right)-\left(A\dot{B}C\right)=y_1 \tag{14}$$

$$\left(\dot{A}BC\right)-\left(AB\dot{C}\right)=y_2 \tag{15}$$

$$\left(A\dot{B}C\right)-\left(AB\dot{C}\right)=y_3 \tag{16}$$

Divided by ABC of equations (14), (15), (16) and using $ABC = R^3$ we have

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B} = \frac{y_1}{R^3} \tag{17}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C} = \frac{y_2}{R^3} \tag{18}$$

$$\frac{\dot{A}}{A}-\frac{\dot{C}}{C} = \frac{y_3}{R^3} \tag{19}$$

where y_1 , y_2 and y_3 are constants of integration.

Again integrating equations (17) - (19) we have three relations

$$\frac{\text{A}}{\text{B}} = \delta_1 \exp\left[y_1\int\frac{dt}{R^3}\right] \tag{20}$$

$$\frac{\text{B}}{\text{C}} = \delta_2 \exp\left[y_2\int\frac{dt}{R^3}\right] \tag{21}$$

$$\frac{\text{A}}{\text{C}} = \delta_3 \exp\left[y_3\int\frac{dt}{R^3}\right] \tag{22}$$

where δ_1 , δ_2 δ_3 y_1 , y_2 and y_3 are constants of integration. From equations (20) – (22) the metric function be explicity written as

$$A(t) = m_1\text{R}\exp\left[\frac{(y_1+y_3)}{3}\int\frac{dt}{R^3}\right] \tag{23}$$

$$B(t) = m_2\text{R}\exp\left[\frac{(y_2-y_1)}{3}\int\frac{dt}{R^3}\right] \tag{24}$$

$$C(t) = m_3\text{R}\exp\left[\frac{(-y_2-y_3)}{3}\int\frac{dt}{R^3}\right] \tag{25}$$

where $m_1 = (\delta_1\delta_3)^{\frac{1}{3}}$

$m_2 = \left(\delta_1^{-1}\delta_2\right)^{\frac{1}{3}}$

$m_3 = (\delta_2\delta_3)^{\frac{-1}{3}}$ and $m_1 m_2 m_3 = 1$

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Where dots on A, B, C and elsewhere denote the ordinary differentiation with respect to t. Hubble parameter H, volume expansion θ , shear scalar σ and deceleration parameter q are given by

$$H = \frac{\dot{R}}{R} = \frac{\theta}{3},$$

$$\sigma^2 = \frac{k}{3R^6} \quad , \qquad q = -\frac{R\ddot{R}}{\dot{R}^2}$$

Equation (6), (7), (8) and (9) can also be written in terms of Hubble parameter H, deceleration parameter q, shear σ and scale factor R as

$$H^2(2n-1) - \sigma^2 = \text{p - } \Lambda \tag{26}$$

$$3H^2 - \sigma^2 = \rho + \Lambda \tag{27}$$

from equation (14), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{3\rho}{\theta^2} - \frac{3\Lambda}{\theta^2}$$

which implies that for $8\pi G = 1, \Lambda \geq 0$

$$0 < \frac{\sigma^2}{\theta^2} < 1/3, 0 < \frac{\rho}{\theta^2} < \frac{1}{3}$$

therefore the presence of positive Λ lowers the upper limit of anisotropy. From equation (13) and (14) we obtain

$$\dot{\theta} = \frac{\ddot{R}}{R} + \frac{p}{1-2n} + \frac{\sigma^2}{1-2n} - \frac{\Lambda}{1-2n} \tag{28}$$

It means that the rate of volume expansion increases during time evolution and presence of positive Λ is to halt this increase whereas the negative Λ promotes it. Also from Eqns. (3), (16) and (17), we obtain

$$\frac{\Lambda}{H^2} = 2 - n - \frac{(1-\omega)\rho}{2H^2} \tag{29}$$

Using the above equation, we can determine the present value of the cosmological constant Λ.

If $2 > n + \frac{(1-\omega)\rho}{2H^2}$, $\Lambda > 0$ whereas $\Lambda < 0$ for $2 < n + \frac{(1-\omega)\rho}{2H^2}$.

Also from equation (16) (for stiff fluid) , we observe that $\Lambda \sim H^2$, which follows from the model of Singh et.al. (2008). To solve the systems (3) and (5) --- (8) completely consider

$$\frac{-R\ddot{R}}{\dot{R}^2} = n, \; 0 < n < \frac{1}{2} \tag{30}$$

Which lead to a differential equation $\frac{\ddot{R}}{\dot{R}} + n\frac{\dot{R}}{R} = 0$ (31)

A partial list of cosmological model in which the deceleration parameter q is constant, Maharaj and Naidoo (1993); Pradhan *et al.*, (2010); Akarsu and Kilinc (2009); Kumar and Singh (2007), but our assumption is different from these model because they have been solved Einstein's field equations by applying a law of variation for Hubble's parameter and observed that this law yields a constant value of

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deceleration parameter q which proposed by Berman (1983) whereas we considered a constant value of deceleration parameter. To determine the scale factor integrating equation (18), we get

$$R = (k_1 t + k_2)^{1/n+1} \tag{32}$$

For this solution, metric functions can be written as

$$A(t) = m_1 (k_1 t + k_2)^{1/n+1} \exp\left[\left\{\frac{(y_1 + y_3)(n+1)}{3k_1(n-2)}\right\}(k_1 t + k_2)^{\{(n-2)/(n+1)\}}\right]$$

$$B(t) = m_1 (k_1 t + k_2)^{1/n+1} \exp\left[\left\{\frac{(-y_1 + y_2)(n+1)}{3k_1(n-2)}\right\}(k_1 t + k_2)^{\{(n-2)/(n+1)\}}\right]$$

$$C(t) = m_1 (k_1 t + k_2)^{1/n+1} \exp\left[\left\{\frac{(-y_2 - y_3)(n+1)}{3k_1(n-2)}\right\}(k_1 t + k_2)^{\{(n-2)/(n+1)\}}\right]$$

Where k_1 and k_2 are constants of integration. For this solution metric (1) assume the for

$$ds^2 = -dt^2 + (k_1 t + k_2)^{2/(n+1)} \left[\begin{array}{l} \exp\left\{2\frac{(y_1 + y_3)(n+1)}{3k_1(n-2)}(k_1 t + k_2)^{n-2/n+1}\right\}dx^2 + \\ \exp\left\{2\frac{(-y_1 + y_2)(n+1)}{3k_1(n-2)}(k_1 t + k_2)^{n-2/n+1}\right\}dy^2 \\ + \exp\left\{2\frac{(-y_2 - y_3)(n+1)}{3k_1(n-2)}(k_1 t + k_2)^{n-2/n+1}\right\}dz^2 \end{array}\right] \tag{33}$$

DISCUSSION

For the model (20), Hubble parameter H, expansion scalar θ, shear scalar σ are given by

$$H = \frac{k_1}{(n+1)}(k_1 t + k_2)^{-1} \tag{34}$$

$$\theta = \frac{3k_1}{(n+1)}(k_1 t + k_2)^{-1} \tag{35}$$

$$\sigma = \frac{k}{\sqrt{3}\left((k_1 t + k_2)^{3/(n+1)}\right)} \tag{36}$$

The spatial volume V, cosmological energy density ρ, pressure p and cosmological constant Λ are given by

$$V = (k_1 t + k_2)^{3/(n+1)} \tag{37}$$

$$\frac{p}{\omega} = \rho = \frac{k_3}{(k_1 t + k_2)^{3(\omega+1)/(n+1)}} \tag{38}$$

$$\Lambda = \frac{3k_1^2}{(n+1)^2 (k_1 t + k_2)^2} - \frac{k^2}{3(k_1 t + k_2)^{\frac{6}{n+1}}} - \frac{k_3}{(k_1 t + k_2)^{\frac{3(\omega+1)}{n+1}}} \tag{39}$$

In this model, we observe that the spatial volume $V \to 0$ as $t \to \frac{-k_2}{k_1}$ and expansion scalar $\theta \to \infty$ as $t \to \frac{-k_2}{k_1}$ which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factor also vanish at $t = \frac{-k_2}{k_1}$ and hence the model has a point type singularity at the initial epoch. The cosmological energy density ρ, pressure p , shear scalar σ and cosmological constant Λ all are infinite at $t = \frac{-k_2}{k_1}$.

With t increases the expansion scalar and shear scalar decrease but spatial volume increases. As t increases all the parameters ρ, p, θ , Λ, p_v and critical density $\rho_c = \frac{-k_1^2}{(n+1)^2 (k_1 t + k_2)^2}$ decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large value of t. ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that the model approaches isotropy for the large value of t.

Conclusion

In this paper we have investigated a new class of Bianchi type – I space – time by using the value n, ($0 < n < \frac{1}{2}$) of deceleration parameter q in which cosmological constant Λ varies with cosmic time t.

We would like to mention have that Tiwari (2010); Tiwari (2014) & Tiwari and Jha (2010) have also derived the Bianchi type – I, LRS Bianchi type -I models with variable cosmological term Λ using different approach but our derived result differ from these models in the sense that both have been solved Einstein's field equations by using law of variation of scale factor with variable cosmological term Λ whereas we have presented a class of solution by taking $\frac{-R\ddot{R}}{\dot{R}^2} = n$. Expressions for some important cosmological parameters have been obtained and physical behaviour of the models are discussed in detail, clearly the model represent shearing, non-rotating and expanding models with a big-bang start. The models have point type singularity at the initial epoch and approach isotropy for large value of cosmic time t.

Finally the solutions presented here are new and useful for a better understanding of the evolution of the universe in the LRS Bianchi type-I universe with value of $q = n\left(0 < n < \frac{1}{2}\right)$. More cosmological models may be analyzed using this technique, which may lead to interesting and different behaviour of the universe.

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