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Research Article

MHD VISCOELASTIC BOUNDARY LAYER FLOW AND HEAT TRANSFER PAST A CONVECTIVELY HEATED RADIATING STRETCHING/SHRINKING SHEET WITH TEMPERATURE DEPENDENT HEAT SOURCE/SINK, EMBEDDED IN A SATURATED POROUS MEDIA

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ABSTRACT
The present work analyzes the convective heat transport in a steady boundary layer viscoelastic fluid flow and heat transfer over a stretching/shrinking sheet, in a saturated porous media. Three cases were considered here. That is (i) The sheet with prescribed surface temperature. (ii) The sheet with prescribed surface heat flux. (iii) Convective heating. The governing boundary value problem, which is in the form of nonlinear partial differential equations are transformed into nonlinear ordinary differential equations, using a suitable similarity transformation and are solved numerically using Runge Kutta fourth order method with shooting technique. The effects of Viscoelastic parameter (k_1), Chandrashekar number (Q), thermal radiation parameter (Nr), Prandtl number (Pr), wall temperature parameter (s), heat source/sink parameter (α), Biot number (B_i), porous parameter k_2 on flow and heat transfer characteristics are depicted graphically.

Keywords: *Boundary Layer Flow; Runge-Kutta Fourth Order Method; Biot Number; Porous Parameter; Similarity Transformation*

INTRODUCTION
Many fluids such as blood, dyes, yoghurt, ketchup, shampoo, paint, mud, clay coatings, polymer melts, certain oils and greases etc, have complicated relations between stresses and strains. Such fluids do not obey the Newton's law of viscosity and are usually called non-Newtonian fluids. The flows of such fluids occur in a wide range of practical applications and have key importance in polymer devolatisation, bubble columns, fermentation, composite processing, boiling, plastic foam processing, bubble absorption and many others. The effect of convective heating for flow past porous media plays a major role in agricultural engineering and petroleum industry in extracting petroleum from crude. Therefore, non-Newtonian fluids have attracted the attention of a large variety of researchers including the interests of experimentalists and theoreticians like engineers, modelers, physicists, computer scientists and mathematicians.
However, as these fluids are in themselves varied in nature, the constitutive equations which govern them are many taking account of the variations of rheological properties. The model and hence, the arising equations, are much more complicated and of higher order than the well known Navier—Stokes equations. The adherence boundary conditions are insufficient for the determinacy of unique solution. This issue has been discussed in the excellent and fundamental studies (Rajagopal, 1995; Rajagopal and Kaloni, 1989; Rajagopal, 1984; Rajagopal and Gupta, 1984; Szeri and Rajagopal, 1985). Now the literature on the non-Newtonian fluids is extensive. Some recent contributions in this direction are made in the investigations (Tan and Masuoka, 2005; Fetecau and Fetecau, 2005, 2006; Hayatv and Kara, 2006; Khan *et al.*, 2006; Hayat *et al.*, 2007). Furthermore, the boundary layer flow caused by a moving

continuous solid surface occurs in several engineering processes. Specifically such flows encounter in aerodynamic extrusion of plastic sheets, wire drawing, glass fiber and paper production, cooling of an infinite metallic plate and polymer processing (Cortell, 2007). Sakiadis (1961) attempted the first problem regarding boundary layer viscous flow over a moving surface having constant velocity.

Later this problem i.e Sakiadis (1961) studied extensively through various aspects. Very recent investigations relevant to this problem have been made in (Rajagopal *et al.*, 1984; Cortell, 2007, 2006; Ariel *et al.*, 2006; Sajid *et al.*, 2007).

In order to verify these aspects, a thorough review on the boundary conditions, existence and uniqueness of the solution have been provided by Dunn and Fosdick (1974) and Girault and Scott (1999). On the other hand, Abel and Veena (1998) investigated a viscoelastic fluid flow and heat transfer in a porous medium over a stretching sheet and observed that the dimensionless surface temperature profiles increases with an increase in viscoelastic parameter $k1$, however, later, Abel *et al.*, (2001) studied the effect of heat transfer on MHD viscoelastic fluid over a stretching surface and an important finding was that the effect of viscoelasticity is to decrease dimensionless surface temperature profiles in that flow. Furthermore, Char (1994) studied MHD flow of a viscoelastic fluid over a stretching sheet, however, only the thermal diffusion is considered in the energy equation. Sarma and Rao (1998) analysed the effects of work done due to deformation in that equation.

Mathematical Formulation of the Problem

Consider a steady, laminar free convective flow of an incompressible and electrically conducting visco-elastic fluid over continuously moving stretching surface embedded in a porous medium. Two equal and opposite forces are introduced along the x-axis so that sheet is stretched with a speed proportional to the distance from the origin. The resulting motion of the otherwise quiescent fluid is thus caused solely by the moving surface. A uniform magnetic field of strength B_0 is imposed along y-axis. This flow satisfies the rheological equation of state derived by Beard and Walters (1964).

The steady two-dimensional boundary layer equations for this in usual notation are,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_o\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma B_o^2 u}{\rho}-\frac{\gamma}{k'}u \qquad (2)$$

Here x and y are respectively the directions along and perpendicular to the surface, u, v are the velocity components along x & y directions respectively. k_o is viscoelasticity, k' is permeability of porous media, γ is kinematic viscosity, σ electrical conductivity, ρ -density of fluid B_0 is strength of magnetic field.

In deriving these equations, it is assumed, that in addition to the usual boundary layer approximations that the contribution due to the normal stress and shear stress are of the same order of magnitude inside a boundary layer.

The boundary conditions applicable to the flow problem are,

$$\begin{aligned} &u=bx, v=0,\ at\ \ y=0\\ &u\ \to 0, as\quad y\to\infty \end{aligned} \qquad (3)$$

b<0 (Shrinking sheet), b>0 (stetching sheet)

Equations (1) and (2) admit self-similar solution of the form,

$$u=b\,x\,f',\quad v=-\sqrt{b\gamma}\ f, Where\ \eta=\sqrt{\frac{b}{\gamma}}\,y \qquad (4)$$

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Where, prime denotes the derivative with respect to η. Clearly u & v satisfy the equation (1) identically. Substituting these new variables in equation (2), we obtain,

$$f'^2 - f\,f'' = f''' - k_1\left\{2f'\,f''' - f\,f^{IV} - f''^2\right\} - (Q + k_2)f' \quad (5)$$

Where, $k_1 = \dfrac{k_0 b}{\gamma}$, $Q = \dfrac{\sigma B_0^2}{b\rho}, k_2 = \dfrac{\gamma}{k'b}$, are the viscoelastic parameter Chandrashekar number and porous parameter respectively.

Similarly, boundary condition (3) takes the form

$$\begin{aligned} & f'(0)=1, \quad f(0)=0 \quad at \quad \eta=0 \\ & f'(\eta)\to 0, \quad f''(\eta)\to 0 \quad as \quad \eta\to\infty \end{aligned} \quad (6)$$

Heat Transfer Analysis

The energy equation in the presence of radiation and internal heat generation / absorption for two-dimensional flow is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = K\frac{\partial^2 T}{\partial y^2} + \frac{\lambda}{\rho C_p}(T - T_\infty) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (7)$$

Where is K thermal conductivity, λ is heat source/sink, q_r is radiative heat flux.

By using Rosseland approximation, the radiative heat flux is given by

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \quad (8)$$

Where σ^* and K^* are respectively, the Stephan-Boltzman constant and the mean absorption coefficient. We assume the differences within the flow are such that T^4 can be expressed as a linear function of temperature. Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms thus,

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (9)$$

The boundary conditions are

$$T = T_w = T_\infty + A\left(\frac{x}{l}\right)^s \text{ at y=0 and } T \to T_\infty, as, y \to \infty \quad \text{PST Case} \quad (10)$$

$$-K\frac{\partial T}{\partial y} = Q_w = D\left(\frac{x}{l}\right)^s \text{ at y=0 and } T \to T_\infty, as, y \to \infty \quad \text{PHF Case} \quad (11)$$

$$-K\frac{\partial T}{\partial y} = h(T_f - T) \text{ at,y=0and} T \to T_\infty, as, y \to \infty \quad \text{Convective Case} \quad (12)$$

The fluid temperature which is characterized by T_f, heat transfer coefficient h

And s is wall temperature parameter.

The similarity transformations are

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad where \quad T_w - T_\infty = A\left(\frac{x}{l}\right)^s \quad \text{PST Case} \quad (13)$$

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad T_w - T_\infty = \frac{D}{k}\sqrt{\frac{\gamma}{b}}\left(\frac{x}{l}\right)^s \quad \text{PHF Case} \quad (14)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \qquad \text{Convective Case} \qquad (15)$$

PST and Convective Case

Now using equations (13), and (15) equation (7) becomes

$$(1+ Nr)\theta'' + \Pr f\,\theta' - \Pr(f' - \alpha)\theta = 0 \qquad (16)$$

Where, $\Pr = \dfrac{\mu C_p}{K}$, $\quad Nr = \dfrac{16\sigma^* T_\infty^3}{3K^* K_\infty}$, $\quad \alpha = \dfrac{\lambda}{b\rho C_p}$

are the Prandtl Number, Radiation parameter and heat source / Sink Parameter respectively.

PHF Case

Now using equations (14), equation (7) becomes

$$(1+ Nr) g'' + \Pr f\, g' - \Pr(f' - \alpha) g = 0 \qquad \ldots\ldots\ldots\ldots\ldots.. \ (17)$$

The boundary condition takes the form:

$$\theta(0) = 1, \qquad \theta(\eta) \to 0 \qquad as \quad \eta \to \infty \qquad \text{PST Case}$$

$$\theta'(0) = -1, \qquad \theta(\eta) \to 0 \qquad as \quad \eta \to \infty \qquad \text{PHF Case}$$

$$\theta'(0) = -B_i\left(1 - \theta(0)\right), \quad \theta(\eta) \to 0, as \ \eta \to \infty \qquad \text{Convective Case}$$

……………………………..(18)

Numerical Solution

Because of the non-linearity and couplings between the momentum and the thermal boundary layer equations, exact solutions do not seem feasible for complete set of equations (16), (17) and (18), therefore, solution must be obtained numerically. In order to solve them, we employ most efficient shooting technique with fourth order Runge-Kutta integration scheme.

Selection of an appropriate finite value of η_∞ is most important aspect in this method. To select η_∞, we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f''(0)$ and $\theta'(0)$. The solution process is repeated with another larger (or smaller, as the case may be) value of η_∞. The values of $f''(0)$ and $\theta'(0)$ compared to their respective previous values, if they agreed to about six significant digits, the last value of η_∞ used was considered the appropriate value for that particular set of parameters; otherwise the procedure was repeated until further changes in η_∞ did not lead to any more change in the values of $f''(0)$ and $\theta'(0)$. The initial step size employed was h=0.01. The convergence criterion largely depends on fairly good guesses of the initial conditions in the shooting technique.

RESULTS AND DISCUSSION

The present study considers the flow of viscoelastic incompressible electrically conducting fluid flow past a stretching/shrinking sheet in presence of Magnetic field, uniform heat source/sink, Convective heat transfer, and porous media. The aim of the following discussion is to bring about the effect of magnetic field, viscoelasticity, permeability of porous media, heat source /sink, radiative heat transfer, and convective heat transfer over stretching/shrinking sheet on flow and heat transfer characteristics.

In Figure (1) it is noticed that the effect of Chandrashekar number Q is to accelerate motion in case of shrinking sheet.

This is due to the fact that the presence of viscoelasticity contributes to stored energy by obstructing energy loss, as one is aware of the fact that in viscoelastic fluid flows, a fixed amount of energy is stored up in the material as stored energy. Because of this the resistive force due to magnetic field is overcome, resulting in enhancement in magnitude of velocity. Where as in case of stretching sheet the effect of Q is to retard flow velocity within the boundary layer.

Figure 2 is a graph concerns to the effect of viscoelastic parameter k1 on horizontal flow velocity for both Stretching/shrinking sheet.

Here this Figure 2 depicts that for an increase in viscoelastic parameter k1 results in decrease of velocity in boundary layer in case of shrinking sheet. This result is consistent with the fact that the introduction of tensile stress due to visco-elasticity cause transverse contraction of the boundary and hence velocity decreases. Whereas for stretching sheet the opposite effect is noticed.

From Figure 3 it is noticed that the effect of porous parameter k_2 is to decrease the horizontal velocity profile, in case of stretching sheet.

This is due to the increase of porous parameter k_2 leading to the enhanced deceleration of the flow. An opposite trend is noticed for shrinking sheet.

The effect of Prandtl Number(Pr) is analyzed in view of Figure 4, for both PST as well as PHF cases. This figure illustrates that increase in Prandtl Number (Pr) results in decrease of temperature distribution in thermal boundary layer region, which obviously a means for decrease of boundary layer thickness. Decrease of boundary layer thickness results slow rate of thermal diffusion. It is also noticed that wall temperature distribution is at unity in case of PST, whereas in PHF case it is other than unity, due to adiabatic boundary condition.

The effect of Chandrashekar number Q, on heat transfer is depicted in Figure 5 in case of PST and PHF respectively. Here it is noticed that the contribution of transverse magnetic field, is to thicken thermal boundary layer.

This is due to the fact the applied transverse magnetic field produces a body force, in the form of Lorentz force, which enhances temperature distribution in flow region. The enhancement in temperature distribution in flow region is because of resistance offered by Lorentz force on flow velocity.

Figure 6 shows the effect of viscoelastic parameter K1 on temperature profile, and it is noticed that Temperature profile increases with the increase of viscoelastic parameter K1, in both PST and PHF cases.

An increase in temperature distribution due to the presence of elastic elements may be attributed to the fact that when a viscoelastic fluid is in flow, a certain amount of energy is stored up in the material as strain energy, which is responsible for enhancement of temperature distribution in thermal boundary layer region.

Figure 7 reveals the influence of radiation parameter Nr on temperature profile, where in it produces a significant increase in the thickness of thermal boundary layer, resulting in enhancement of temperature in thermal boundary layer region in both PST and PHF cases. The prominent effect of Nr is to enhance heat transfer, therefore Nr should be kept at minimum value to facilitate the cooling process of polymer extrudate in polymer industry.

The influence of wall temperature parameter s for both PST as well as PHF cases on temperature distribution is depicted in Figure 8.

Numerical solutions are sought in the range of values of s as mentioned follows, i.e $-2.0 \leq s \leq 2.0$ and $-2.0 \leq s \leq 2.0$ for PST and PHF cases.

Here, we notice that as the value of s is incremented from negative values to positive values, temperature distribution decreases in thermal boundary layer.

The effect of heat source/sink parameter α on temperature profile within the boundary layer is depicted in Figure 9. In this figure it is noticed that the direction of heat transfer depends on temperature difference $(T_w - T_\infty)$ and dimensionless rate of heat transfer $\theta'(0)$.

To interpret the heat transfer result physically, we discuss the result of positive α and negative α separately. For positive α, we have a heat source in the boundary layer when $T_w < T_\infty$ and heat sink when $T_w > T_\infty$. Physically, these correspond, respectively, recombination and dissociation within the boundary layer. For the case of cooled wall ($T_w < T_\infty$), there is heat transfer from the fluid to the wall even without a heat source. The presence of heat source $(\alpha > 0)$ will further increase the heat flow to the wall.

When α is negative, this indicates a heat source for $T_w > T_\infty$ and a heat sink for $T_w < T_\infty$. This corresponds to combustion and an endothermic chemical reaction. For the case of heated wall ($T_w > T_\infty$), the presence of a heat source $(\alpha < 0)$ creates a layer of hot fluid adjacent to the surface and therefore, the heat from the wall decreases. For cooled wall case ($T_w < T_\infty$), the presence of heat sink $(\alpha < 0)$ blankets the surface with a layer of cool fluid and therefore, heat flow in to the surface decreases.

The effect of Biot number Bi on temperature profile is depicted in figure 10. Here, it is noticed that an increase in biot number Bi results in increase in rate of heat transfer in thermal boundary layer region. Further it is noticed that there is increase in thickness of thermal boundary layer.

Concluding Remarks

The governing boundary layer equations of flow and heat transfer for a steady, flow of an incompressible and electrically conducting visco-elastic fluid over continuously moving stretching surface with combined effect of thermal radiation and convective heating is analyzed.

The governing boundary value problem, which is in the form of nonlinear partial differential equations are converted into nonlinear ordinary differential equations and are solved numerically using Runge-Kutta fourth order method with shooting technique.

here we investigate the influences of porosity parameter, Prandtl number, heat source / sink thermal radiation parameter, Biot number, on the dimensionless velocity, and temperature distribution,.

The important results the present investigation is

- The effect of visco-elastic parameter is to decrease the horizontal velocity profile in the boundary layer.

Numerical results of the transformed boundary layer equation have been obtained. In this chapter, we investigate the influences of permeable parameter, porosity parameter, Grash of number, Prandtl number, heat source / sink parameter on the dimensionless velocity, temperature distribution, Skin friction coefficient and the rate of heat transfer coefficient on horizontal velocity profile as well as temperature distribution.

The important results of our investigation are

For Stretching Sheet

- Horizontal velocity profile decreases with the increase of distance from the boundary
- The effect of visco-elastic parameter is to decrease the horizontal velocity profile in the boundary layer in presence / absence of porous medium.
- The increase of porosity parameter leads to the enhanced deceleration of the flow and hence the velocity decreases.

For Shrinking Sheet

- An opposite trend is noticed for shrinking sheet

Heat Transfer Effect, Both for PST and PHF

- In the presence of heat source ($\alpha. > 0$) in the thermal boundary layer, generates the energy, which causes the temperature of the fluid to increase.

On the other hand in the presence of heat absorption (α <0) effects caused reductions in the fluid temperature, which resulted in decrease in the fluid velocity.

- The effect of Prandtl number is to decrease the thermal boundary layer thickness.
- Increase in biot number Bi results in enhancement of rate of heat transfer in thermal boundary layer region.

Further it is noticed that there is increase in thickness of thermal boundary layer.

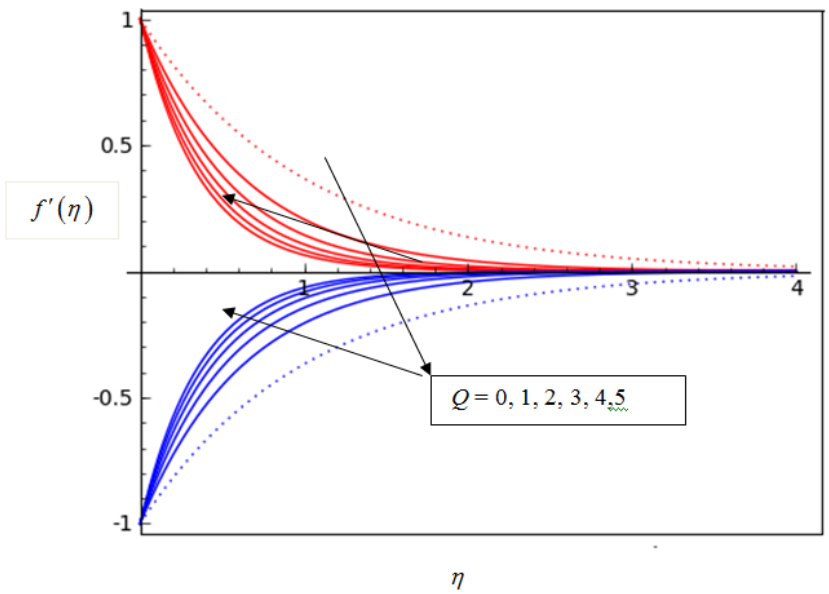

Figure 1: Plot of Axial Velocity $f'(\eta)$ Versus η for Different Values of Chandrasekhar Number Q with K_1 = 0.2, and k_2=0.1

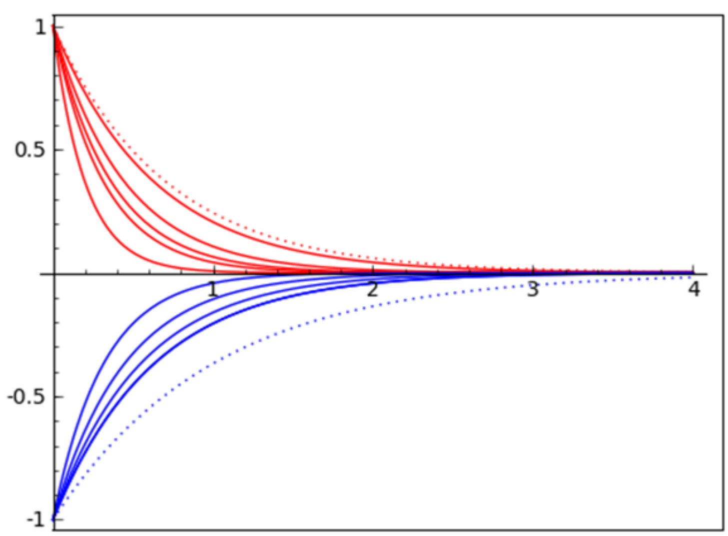

Figure 2: Plot of Axial Velocity Versus η for Different Values of Viscoelastic Parameter K_1 with Q =1, and k_2=0.1

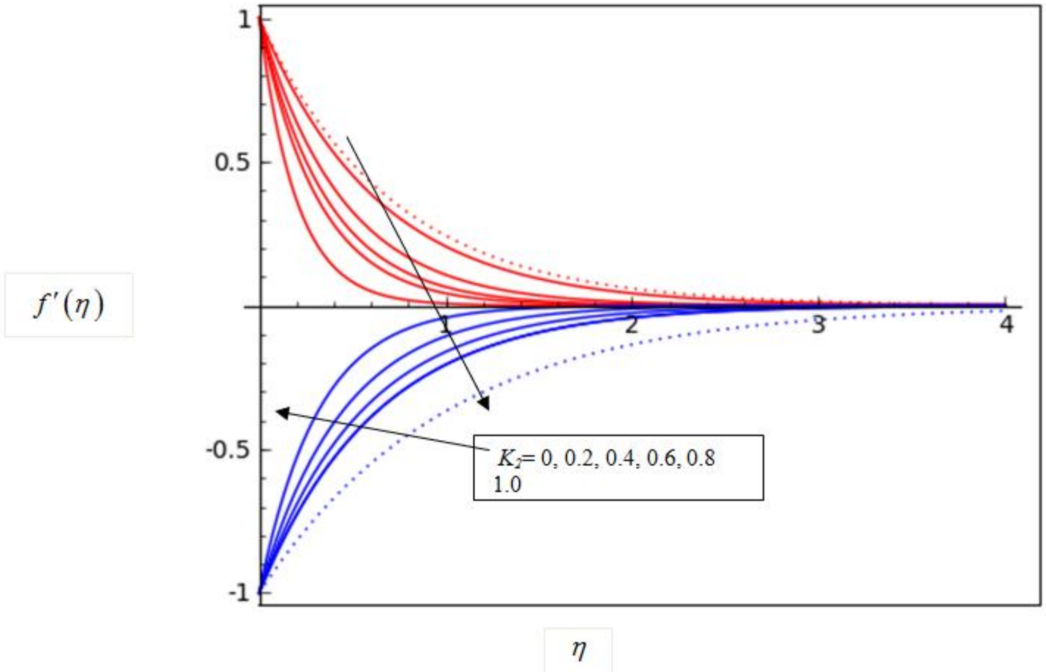

Figure 3: Plot of Axial Velocity $f'(\eta)$ Versus η for Different Values of Porous Parameter K_2 with Q =1, and k_1=0.1

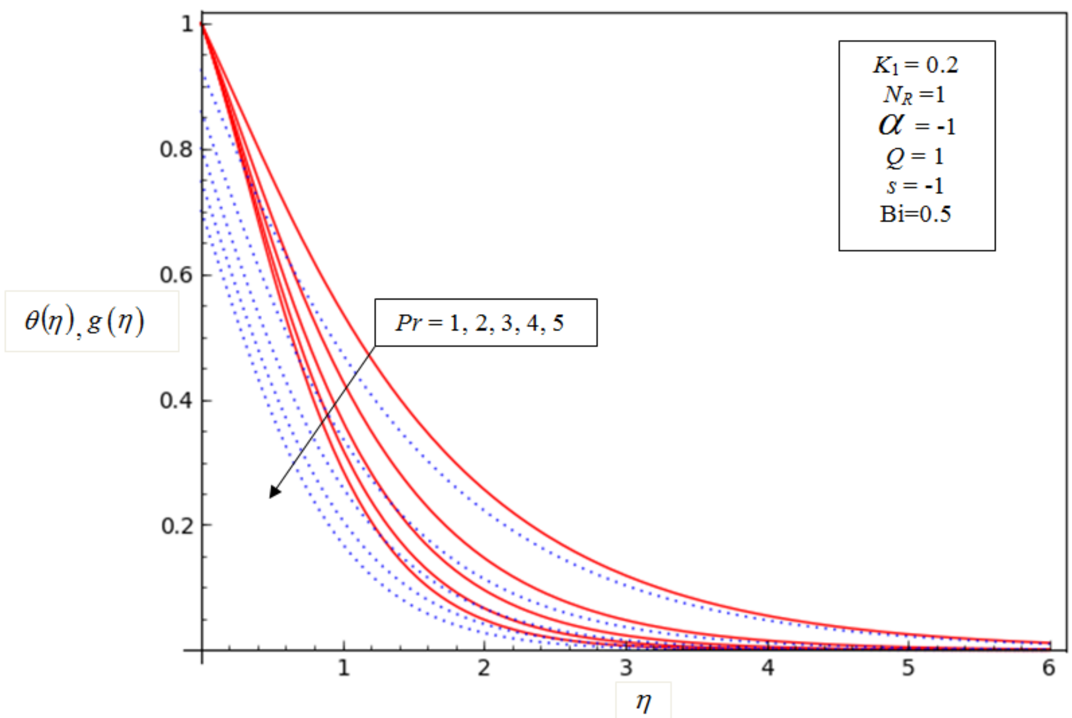

Figure 4: Variation of the Non-Dimensional Temperature θ with η the Transformation Co-Ordinate Normal to the Surface for Different Values of Prandtl Number *Pr* for the cases PST and PHF

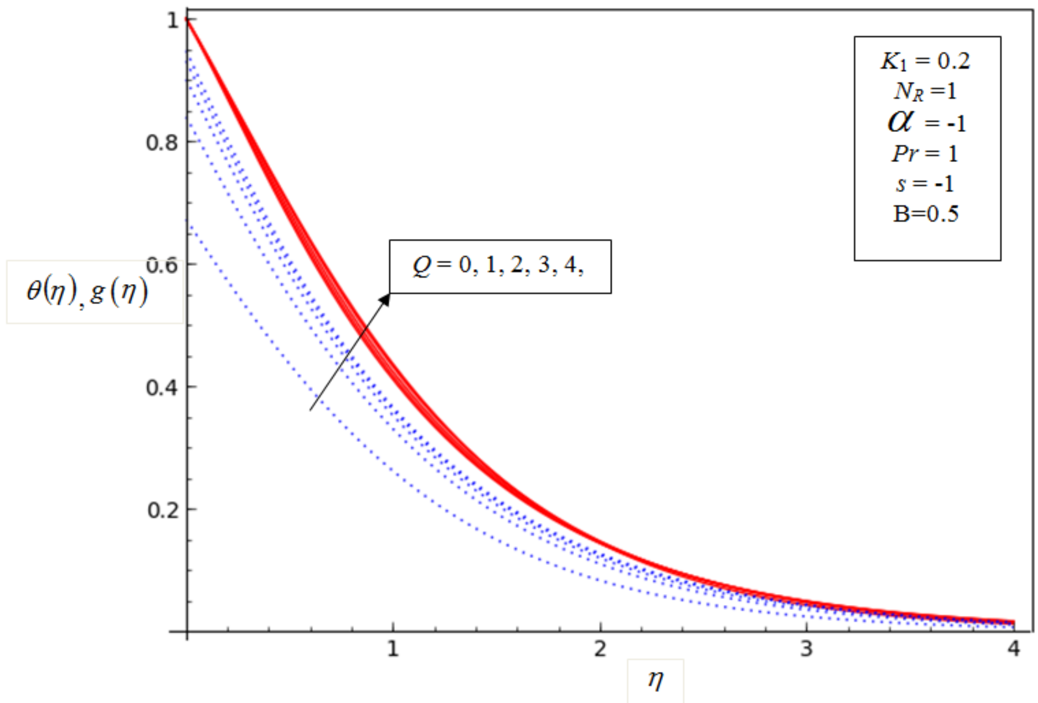

Figure 5: Variation of the Non-Dimensional Temperature θ with η the Transformation Co-Ordinate Normal to the Surface for Different Values of Chandrasekhar Number *Q* for the Cases PST and PHF

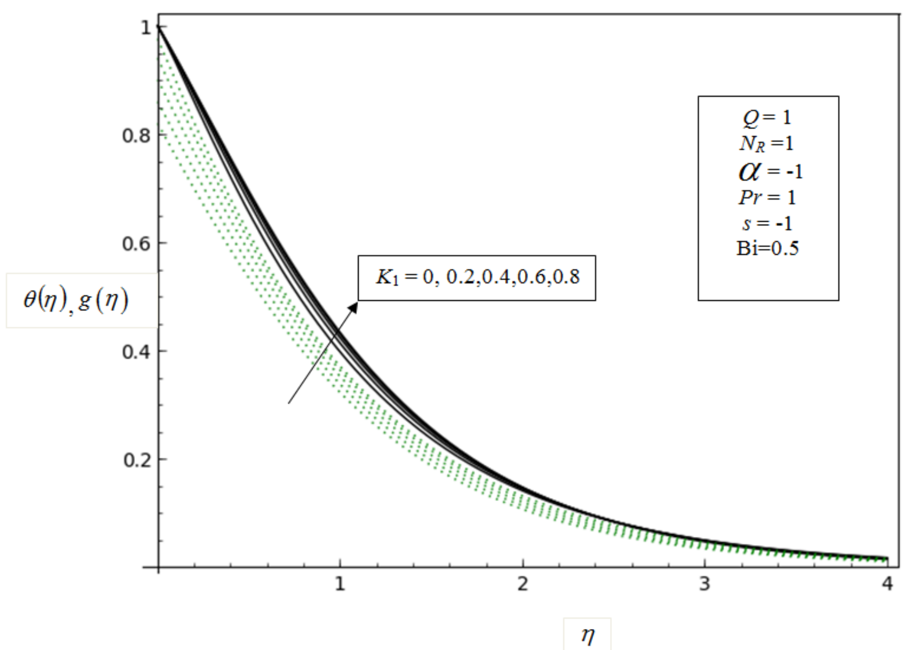

Figure 6: Variation of the Non-Dimensional Temperature θ with η the Transformation Co-Ordinate Normal to the Surface for Different Values of Viscoelastic Parameter K_1 for the Cases PST and PHF

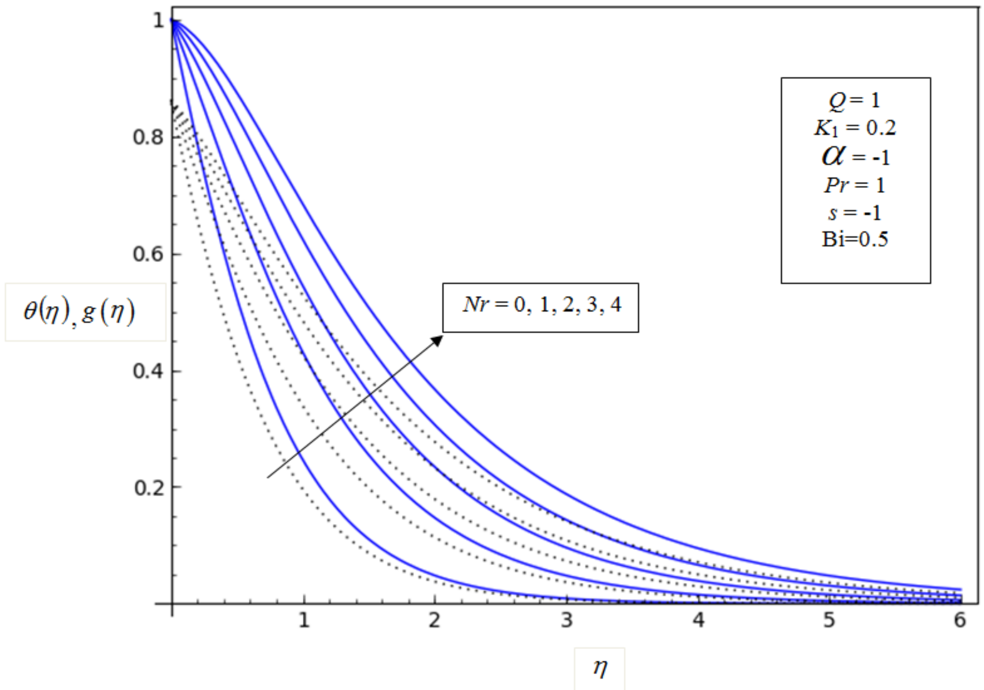

Figure 7: Variation of the Non-Dimensional Temperature θ with η the Transformation Co-Ordinate Normal to the Surface for Different Values of Radiation Parameter *Nr* for the Cases PST and PHF

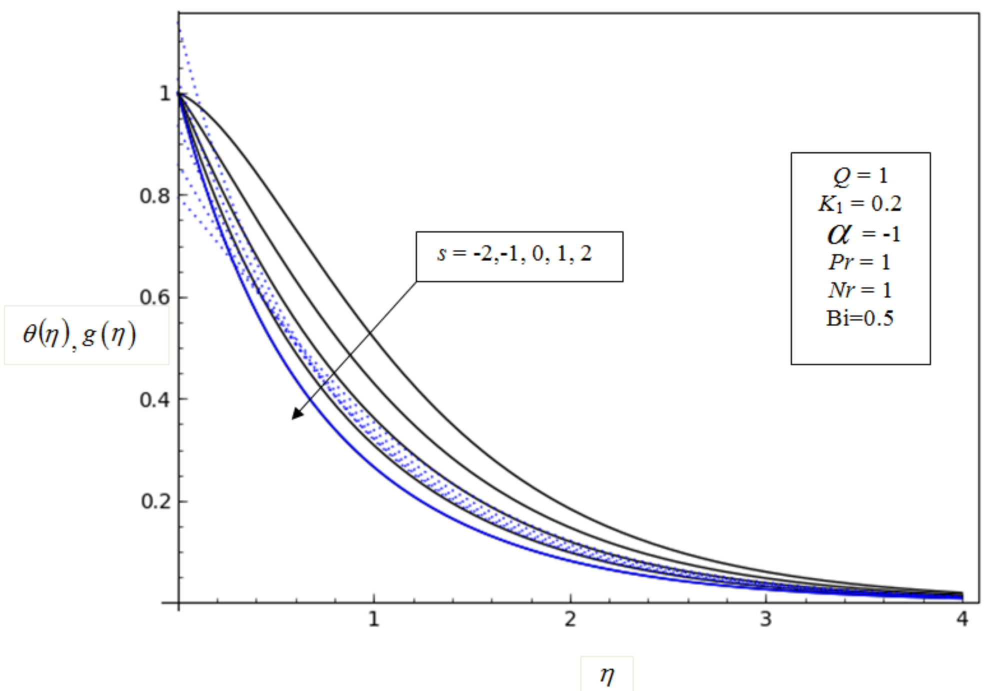

Figure 8: Variation of the Non-Dimensional Temperature θ with η the Transformation Co-Ordinate Normal to the Surface for Different Values of Wall Temperature Parameter *s* for the Cases PST and PHF

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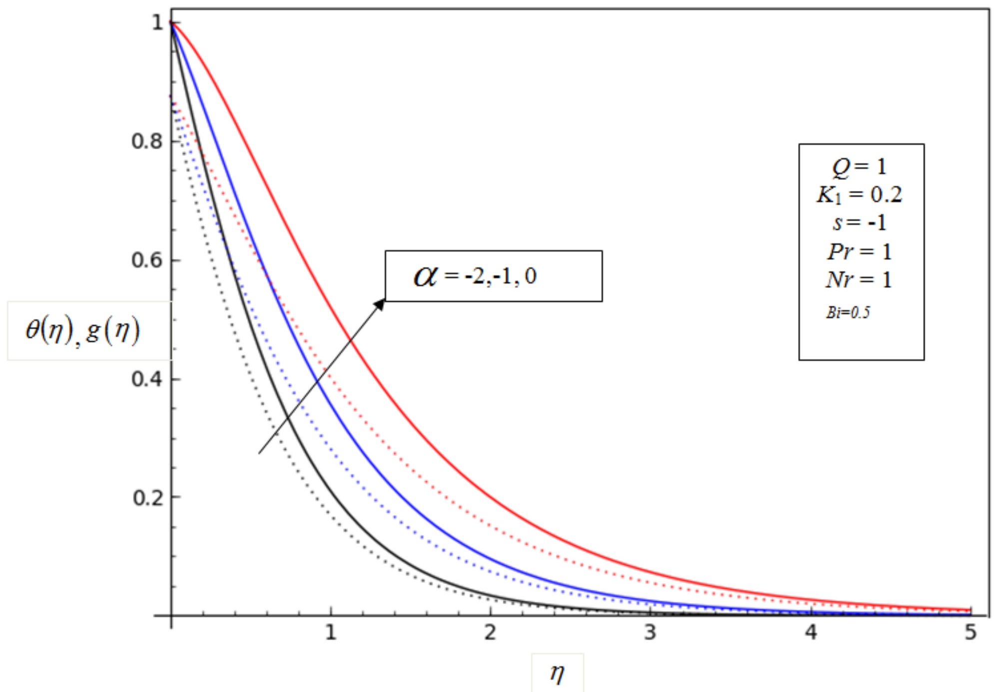

Figure 9: Variation of the Non-Dimensional Temperature θ with η the Transformation Co-Ordinate Normal to the Surface for Different Values of Heat Source Parameter N_I for the Cases PST and PHF

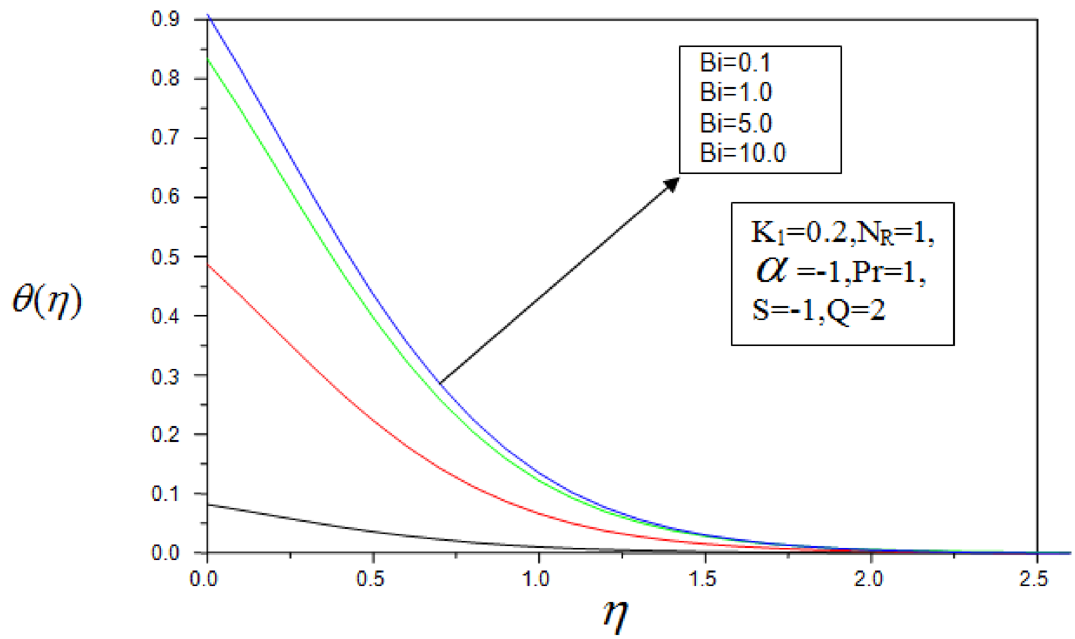

Figure 10: Variation of the Non-Dimensional Temperature θ with η the Transformation Co-Ordinate Normal to the Surface for Different Values of Biot Number Bi for the Cases PST and PHF

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