

ON AXIALLY SYMMETRIC COSMOLOGICAL MODEL WITH STRANGE QUARK MATTER ATTACHED TO STRING CLOUD IN GENERAL RELATIVITY

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ABSTRACT

We have investigated a simple axially symmetric inflationary cosmological universe with string cloud universe containing strange quark matter in general relativity. To get anisotropy and inflationary universe. The physical and kinematical behaviors are discussed. It is observed that most of the models admit initial singularity.

Keywords: *Quark Matter, String Cloud, Inflationary Universe*

INTRODUCTION

Several aspects of strange quark matter have been investigated by many researchers. The origin of our Universe is one of the greatest cosmological mysteries even today. The exact physical situation at early stage of the formation of our Universe is still a subject of study. The concept of string theory was developed to describe events of the early stage of the evolution of the Universe. The general relativistic treatment of strings was initiated by Adhav *et al.*, (2009) and Stachel (1980) and Letelier (1983). The gravitational effects of cosmic strings have been extensively discussed by Vilenkin (1981); Gott (1985) in general relativity.

Here we suppose that strange quark matters are attached to the string cloud. It is plausible to attach strange quark matter to the string cloud, because, one of such transitions during the phase transitions of the universe could be Quark Gluon Plasma (QGP) hadron gas (called quark-hadron phase transition) when cosmic temperature was $T \approx 200$ Mev. Itoh (1970), Bodmer (1971), Witten (1984) have formed two ways for creation of strange quark matter. One is the quark-hadron phase transition in the early Universe and another in the conversion of neutron stars into strange ones at ultrahigh density. In strong interaction theories it is supposed that breaking of physical vacuum takes place inside hadrons to form quark bag model.

As a result vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system. Alcock *et al.*, (1986) and Haensel *et al.*, (1986) examined that if the hypothesis of the quark matter is true, then some of neutron stars could actually be strange stars built entirely of strange matter. Cheng *et al.*, (1998) studied models of strange quark matter attached to the string cloud in the spherical symmetric space-time admitting conformal motion discussing strange star properties. Also Yavuz *et al.*, (2005), Yilmaz (2005) and Yilmaz (2006) studied 5-D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain walls.

The strange quark matter is modelled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are through as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density B_c (called as the bag constant). In the framework of this model, the quark matter is composed of massless u and d quarks, massive s quarks and electrons. Mak and Harko (2004) discussed charged strange quark matter in the spherically symmetric space-time admitting

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conformal motion. Kangujam and Kojam (2014) has discussed Bianchi type-III string cloud and strange quark matter. Adhav *et al.*, (2008) has discussed string cloud and domain walls with quark matter in n-dimensional Kaluza - Klien cosmological model in general relativity and strange quark matter attached to string cloud in Bianchi type-III space time in general relativity. Khadekar and Wanjari (2009) has confirmed their work with quark matter which is attached to the topological defects in general relativity. Katore and Shaikh (2012) has obtained cosmological model with strange quark matter attached to cosmic strings for axially symmetric space-time in general relativity. Recently, Khadekar and Rupali (2012) and Khadekar and Rajani (2012) has discussed geometry of quark and strange quark matter in higher dimensional general relativity. Sahoo and Misra (2014) has discussed about higher dimensional cosmological model with quark and strange quark matter. Rao and Sireesha (2013) discussed Bianchi Type II, VIII and IX cosmological models with strange quark matter attached to string cloud in B-D and general theory of gravitation.

Axially symmetric space time representing material distribution was obtained by Marder (1958). Axially symmetric space time play an important role in the study of universe on a scale in which anisotropy and in homogeneity is not ignored (Kilinc, 1994).

He showed that axially symmetric cosmological model have made significant contributions in understanding some essential features of the universe such as the formation of galaxies during the early stage of the evolution.

Axially symmetric space time with wet dark fluid in biometric theory is also discussed (Jain *et al.*, 2012). Energy-Momentum localization in Marders axially symmetric space-time has been studied (Aygun *et al.*, 2007). Recently, many authors (Sahoo *et al.*, 2014; Adhav *et al.*, 2007; Reddy and Naidu, 2012; Reddy *et al.*, 2008; Reddy *et al.*, 2008) have obtained an axially symmetric cosmological model with perfect fluid in general relativity and in f(R, T) gravity.

Here we study the model universe containing strange quark matter which is expanding, anisotropic, with a sign of dark energy that help in accelerated expansion of this universe. It is also seen that the model universe which we discuss contains both particles and strings, but ultimately will have fluid containing particles only. The model which we consider here is acceptable in view of the present observations of the Universe. The physical interpretations of the solutions obtained are also discussed.

Metric and Field Equations

We consider the uniform, anisotropic and axially symmetric space-time (Bhattacharya and Karade, 1993) as

$$ds^2 = dt^2 - A^2(t)[d\chi^2 + f^2(\chi)d\phi^2] - B^2(t)dz^2 \quad (1)$$

with the convention $x^1 = \chi, x^2 = \phi, x^3 = z$ and $x^4 = t$, A and B are functions of the proper time t along while f is a function of the coordinate χ alone.

For string cloud and energy momentum is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j \quad (2)$$

where ρ is the rest energy density for the cloud of string with particle attached to them, ρ_p

is the particle energy density and ρ_s is the string tension density such that

$$\rho = \rho_p + \rho_s \quad (3)$$

We assume quark are massless and non- interacting as in Bag model.

$$\text{So, quark pressure is } p_q = \frac{\rho_q}{3} \quad (4)$$

where ρ_q is quark energy density.

$$\text{Total energy density is } \rho = \rho_q + B_c \quad (5)$$

$$\text{and total pressure is } p = p_q - B_c \quad (6)$$

We know that string is free to vibrate. The vibration models of the string represent different types of particles because these models are seen as different masses or spins. Therefore, here we consider quarks instead of particles in the string cloud. So, we consider here quark matter energy density instead of particle energy density in the string cloud.

$$\text{From Eqs. (3) and (5), we obtain } \rho = \rho_q + \rho_s + B_c \quad (7)$$

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From Eqs. (3) and (7), we have the energy momentum tensor for strange quark matter attached to the string cloud [43] as follows :

$$T_{ij} = (\rho_q + \rho_s + B_c)u_i u_j - \rho_s x_i x_j \quad (8)$$

where u_i is the 4 velocity of the particles and x_i is the unit space-like vector representing the direction of the string .

$$\text{We also } u_i \text{ and } x_i \text{ with } u^i u_i = -x^i x_i = -1 \text{ and } u^i x_i = 0 \quad (9)$$

We have taken the direction of the string along the z-axis. The components of the energy momentum tensor are then

$$T_1^1 = T_2^2 = 0, T_3^3 = \rho_s, T_4^4 = \rho, \quad (10)$$

$$T_j^i = 0 \text{ for } i \neq j$$

where ρ and ρ_s are functions of t only.

Field Equations and their Solutions

$$\text{Einstein's field equation is } R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \quad (11)$$

The field equations of Eq.(11) for the metric of Eq.(1) can be written as follows:

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = 0 \quad (12)$$

$$2 \frac{A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{1}{A^2} \frac{f_{11}}{f} = \rho_s \quad (13)$$

$$2 \frac{A_4 B_4}{AB} + \frac{A_4^2}{A^2} - \frac{1}{A^2} \frac{f_{11}}{f} = \rho \quad (14)$$

Where the suffixes 1 and 4 after an unknown function denote partial differentiation with respect to χ and t respectively.

The functional dependence of the metric together with (13) and (14) imply

$$\left(\frac{f_{11}}{f} \right) = k^2, k^2 = \text{constant} \quad (15)$$

If $k=0$ the $f(\chi) = \text{constant}$, $0 < \chi < \infty$

This constant can be made equal to 1. Thus we shall have $f(\chi) = \chi$ resulting in the flat model of the universe (Hawking and Ellis, 1976).

Now the field Equations (12-14) becomes as follows

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = 0 \quad (16)$$

$$2 \frac{A_{44}}{A} + \frac{A_4^2}{A^2} = \rho_s \quad (17)$$

$$2 \frac{A_4 B_4}{AB} + \frac{A_4^2}{A^2} = \rho \quad (18)$$

Here, we also assume the relation between metric coefficients i.e., $A = B^n$, because of the fact that the field equations are highly nonlinear . Using this relation we get as solutions

$$A = \left[\left(\frac{n^2+n+1}{n+1} \right) (at + b) \right]^{\frac{n^2+n+1}{n^2+n}} \quad (19)$$

$$B = \left[\left(\frac{n^2+n+1}{n+1} \right) (at + b) \right]^{\frac{n^2+n+1}{n+1}} \quad (20)$$

where a, b are arbitrary constants.

Therefore the uniform, anisotropic and axially symmetric space-time can be written as by Eq.(1)

$$ds^2 = dt^2 - \left[\left(\frac{n^2+n+1}{n+1} \right) (at + b) \right]^{2 \left[\frac{n^2+n+1}{n^2+n} \right]} [d\chi^2 + f^2(\chi) d\phi^2] - \left[\left(\frac{n^2+n+1}{n+1} \right) (at + b) \right]^{2 \left[\frac{n^2+n+1}{n+1} \right]} dz^2 \quad (21)$$

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Some Physical Properties

The model (21) represents uniform, anisotropic and axially symmetric cosmological model in Einstein's theory of gravitation. The model has no initial singularities at $t=0$

The string tension density for the model

$$\rho_s = 2 \frac{A_{44}}{A} + \frac{A_4^2}{A^2} = \left[\frac{n^4 + 3n^3 + 5n^2 + 4n + 2}{n^4 + 2n^3 + n^2} \right] \left[\frac{a}{at+b} \right]^2 \quad (22)$$

The rest energy density for the model is

$$\rho = 2 \frac{A_4 B_4}{AB} + \frac{A_4^2}{A^2} = \left[\frac{2+n}{n} \right] \left[\frac{n^2+n+1}{n^2+n} \right]^2 \left[\frac{a}{at+b} \right]^2 \quad (23)$$

The string particle density for the model is

$$\rho_p = \rho - \rho_s = \left[\left[\frac{2+n}{n} \right] \left[\frac{n^2+n+1}{n^2+n} \right]^2 - \left[\frac{n^4 + 3n^3 + 5n^2 + 4n + 2}{n^4 + 2n^3 + n^2} \right] \right] \left[\frac{a}{at+b} \right]^2 \quad (24)$$

The quark energy density for the model is

$$\rho_q = \rho - B_c = \left[\frac{n^4 + 3n^3 + 5n^2 + 4n + 2}{n^4 + 2n^3 + n^2} \right] \left[\frac{a}{at+b} \right]^2 - B_c \quad (25)$$

The quark pressure for the model is

$$p_q = \frac{\rho_q}{3} = \frac{1}{3} \left[\left[\frac{n^4 + 3n^3 + 5n^2 + 4n + 2}{n^4 + 2n^3 + n^2} \right] \left[\frac{a}{at+b} \right]^2 - B_c \right] \quad (26)$$

The spatial volume of the model is

$$V^3 = \sqrt{-g} = A^2 B = \left[\left(\frac{n^2+n+1}{n+1} \right) (at+b) \right]^{[2+n] \left[\frac{n^2+n+1}{n^2+n} \right]} \quad (27)$$

The scalar expansion θ for the model is

$$\theta = \frac{1}{3} u^i_{;i} = \left[\frac{2n+1}{3n} \right] \left[\frac{n^2+n+1}{n^2+n} \right] \left[\frac{a}{at+b} \right] \quad (28)$$

The shear scalar σ for the model is

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{1}{54} \left[\left[\frac{2n+1}{n} \right] \left[\frac{n^2+n+1}{n^2+n} \right] \left[\frac{a}{at+b} \right] \right]^2 \quad (29)$$

The deceleration parameter q for the model is

$$q = \frac{-3}{\theta^2} \left[\theta_{;i} u^i + \frac{1}{3\theta^2} \right] = - \left[\frac{1+3n-6n^2-8n^3}{1+3n+3n^2+2n^3} \right] \quad (30)$$

It may be observed that at initial moment, then $t=0$, the spatial volume will be zero while the string tension density, rest energy density, string particle density, quark energy density, quark pressure, scalar expansion and shear scalar diverge. It is also observed the model of Eq.(21) admits initial singularity.

For large values of t ($t \rightarrow \infty$) we observed that special volume increases and the universe has accelerated expansion. Also $\frac{\sigma^2}{\theta^2} \neq 0$ and hence the model Eq.(21) does not approach isotropy for large values of t . The decelerating parameter is negative. Hence, the model of Eq. (21) is inflationary.

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CONCLUSION

Axially symmetric cosmological model with string cloud universe containing strange quark matter are important in the study of early stage of evolution of the universe. Here we presented an axially symmetric inflationary model and possess initial singularity. It is also observed that the model does not approach isotropy for large t .

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