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Research Article

ROBERTSON-WALKER MODEL WITH VARIABLE GRAVITATIONAL CONSTANT- AND VARIABLE COSMOLOGICAL CONSTANT

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ABSTRACT
R-W cosmological model is investigated with time dependent gravitational constant G (t) and the cosmological constant Λ (t) by using a law of variation of Hubble parameter. Expressions for some important cosmological parameters have been obtained and physical behaviors of the model are discussed in detail.

Keywords: *R.W. Cosmological Model, Hubble Parameter and Varying G and Cosmological Constant*

INTRODUCTION

In recent years researchers have considered cosmological models with variable gravitational constant G and cosmological constant Λ **(**Dirac, 1937; Lau, 1985; Abdel-Rahman, 1990; Berman and Som, 1990; Abdul and Vishwakarma, 1997; Vishwakarma, 2000; Vishwakarma, 2005; Pradhan and Ostrod, 2006; Singh *et al.*, 2007) Different phenomenological models have been suggested to explain the evolution of different constants. However, we believe that there should be only one model to explain all these parameters if the underlying theory is correct. Moreover, whatever physical process is responsible for the evolution of one parameter should also be responsible for the evolution of the others. It means that the different parameters are coupled together somehow. It should therefore, be the evaluation of the universe itself which should explain the dynamics of all the parameters. In this chapter we investigate such a model from the Einstein's field equations which explains the variability of Λ and G.

The idea of gravitational constant G in the frame work of general relativity was first proposed by Dirac (1937). Lau (1985) working in the frame work of general relativity proposed modification linking the variation of G with that of Λ. The phenomenological Λ decay scenarios have considered by number of authors **(**Holye *et al.*, 1997; Olson and Jordan, 1987; Beeshan, 1990; Maia and Silva, 1994**,** Silvera and Waga, 1997). Some authors have argued for the dependence $\Lambda \sim t^{-2}$ keeping in mind the dimensional consideration in the spirit of quantum cosmology, Chen and Wu (1990) considered Λ varying as R^{-2} where R is a scale factor, Carvalho and Lima (1992) generalized it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson Walker metric H is the Hubble parameter and α and β are adjustable dimension less parameters on the basis of quantum field estimations in the curved expanding background. Schutzhold (2002a, 2002b) recently proposed vacuum density proportional to the Hubble parameter this leads to a vacuum energy density decaying as $\Lambda \approx m^3 H$, where $m \approx 150$ MeV is energy scale of chiral phase transmission of QCD. Borges and Carneiro (2005) have considered anisotropic and homogeneous flat space filled with matter and cosmological term proportional to H, obeying the equation to state of the vacuum. Recently we have studied Bianchi type-I cosmological models with time dependent G and Λ by considering different approach for Λ Tiwari (2008, 2009), Tiwari and Naven (2009).

In this paper a Robertson-Walker Model is considered with variable G and Λ our approach is similar to that of Dirac (1937) and Schutzhold (2002a, 2002b) for solving the Einstein field equations.

Model and Field Equations

We consider the R-W space time with flat space is given by the metric

$$ds^2 = -dt^2 + R^2(t)\,(dx^2 + dy^2 + dz^2) \qquad (1)$$

where R is a function of cosmic time t

The non-vanishing Christoffel symbols of the second kind are

$$\Gamma^4_{11} = R\dot{R}\ ,\ \Gamma^1_{14} = \frac{\dot{R}}{R}\ ,\ \Gamma^4_{22} = R\dot{R}\ ,\ \Gamma^4_{24} = \frac{\dot{R}}{R}\ ,\ \Gamma^4_{33} = R\dot{R}$$

$$\Gamma^3_{34} = \frac{\dot{R}}{R}$$

The Non-zero component of Ricci tensor R_{ij} are

$$R_{11} = -R\ddot{R} - 2\dot{R}^2\ ,\ R_{22} = -R\ddot{R} - 2\dot{R}^2\ ,\ R_{33} = -R\ddot{R} - 2\dot{R}^2$$

$$R_{44} = \frac{3\ddot{R}}{R}$$

The Ricci Scalar is

$$R = -6R\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right)$$

Matter components of the cosmic fluid consists of perfect fluid represented by energy-momentum tensor

$$T_i^{\,j} = (\rho + p)\upsilon_i\upsilon^j + pg_i^{\,j} \qquad (2)$$

with equation of state

$$p = (\omega - 1)\ \rho\ , \qquad 1 \le \omega \le 2 \qquad (3)$$

where p and ρ are the pressure and density, respectively, and υ_i is the unit flow vector satisfying $\upsilon_i\ \upsilon^j$ = -1

Einstein's field equations with time dependent gravitational constant G(t) and cosmological constant Λ(t) are

$$R^i_j - \frac{1}{2}Rg^{\,i}_{\,j} = -\left(8\pi G T^i_j - \Lambda g^{\,i}_{\,j}\right) \qquad (4)$$

where we choose the units such that c = 1 Einstein's equation (4) for the metric (1) leads to the following equations.

$$8\pi Gp - \Lambda \ = -\frac{-2\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} \qquad (5)$$

$$8\pi G\rho + \Lambda \ = \frac{3\dot{R}^2}{R^2} \qquad (6)$$

where the dot stands for ordinary differentiation with respect to time t.

Divergence of equation (4) leads to

$$\dot{\rho} + 3(\rho + p)H = 0 \qquad (7)$$

where H = $\frac{\dot{R}}{R} = \frac{\theta}{3}, \theta$ being the volume expansion scalar. When Λ is constant, we get the continuity equation of matter. In view of energy conservation, equation (7) shows that decaying vacuum term leads to matter production.

Equations (5) and (6) can be rewritten in terms of Hubble parameter H, deceleration parameter q as.

$$8\pi Gp - \Lambda = H^2\ (2q-1) \qquad (8)$$

$$8\pi G\rho + \Lambda = 3H^2 \qquad (9)$$

where q = - $\dfrac{\ddot{R}}{RH^2}$

Solution of the Field Equation

We have the three independent equations (3), (5) and (6) connecting five unknown variables (R, ρ, p, Λ and G). Thus two more relation connecting these variables areneeded to solve these equation's. According to the Large Number Hypothesis (LNH) Dirac (1937) gravitational constant G varies linearly with Hubble parameter i.e. G decreases with the age of the universe. Chen and Wu (1990) considered as $\Lambda \propto R^{-2}$, later on Schutzhold (2002a, 2002b) argued in favor of a new term of the type $\Lambda \propto H$. Thus the phenomenological approach to investigate the cosmological constant is generalized to include a term proportional to H on time dependent Λ, i.e.

G (t) = bH (10)

and Λ (t) = a H (11)

where b and a are positive constants.

From equations (8), (9) and (11) eliminating ρ and p we obtain

$$2\dot{H} + 3\omega H^2 - a\,\omega\, H = 0 \tag{12}$$

Integrating (12), we get \

$$H = \frac{a}{3\left(1 - e^{\frac{-\omega t}{2}}\right)} \tag{13}$$

where the integration constant is related to the choice of origin of time. Form equation (12) we obtain the scale factor

$$R = m\left(e^{\frac{a\omega t}{2}} - 1\right)^{\frac{2}{3\omega}} \tag{14}$$

where m is a constant of integration

For this solution metric (1) assumes the form

$$ds^2 = - dt^2 + m^2\left(e^{\frac{a\omega t}{2}} - 1\right)^{4/3\omega} (dx^2 + dy^2 + dz^2) \tag{15}$$

For the model (15) the physical and geometrical parameters can be easily obtained. The expressions for the spatial volume energy density ρ, pressure p, cosmological constant Λ (t) gravitational constant G (t) are respectively given by

$$V = m^3\left(e^{\frac{a\omega t}{2}} - 1\right)^{2/\omega} \tag{16}$$

$$\rho = \frac{a}{8\pi b}(e^{\frac{a\varpi t}{2}} - 2 = \frac{a}{8\pi b\left(\frac{R^{\frac{3\omega}{2}}}{m} - 2\right)} \tag{17}$$

$$p = \frac{a(\omega - 1)}{8\pi b\left(e^{\frac{a\omega t}{2}} - 1\right)} = \frac{a(\omega - 1)}{8\pi b\left(\frac{R^{\frac{3\omega}{2}}}{m} - 2\right)} \tag{18}$$

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$$\Lambda = \frac{a^2}{3\left(1-e^{\frac{-a\omega t}{2}}\right)} = \frac{a^2}{3} - \frac{R_m^{\frac{3\varpi}{2}} - 1}{R_m^{\frac{3\varpi}{2}} - 2} \tag{19}$$

$$G = \frac{ab}{3\left(1-e^{\frac{-a\omega t}{2}}\right)} = \frac{ab}{3}\frac{\left(1-mR^{\frac{-3\omega}{2}}\right)}{\left(1-2mR^{\frac{-3\omega}{2}}\right)} \tag{20}$$

Expansion scalar θ and shere σ are given by

$$\theta = \frac{a}{1-e^{\frac{a\omega t}{2}}} \tag{21}$$

$$\sigma = \frac{1}{\sqrt{3m^3}}\frac{k}{\left(e^{\frac{a\omega t}{2}}-1\right)^{2/\omega}} \tag{22}$$

Critical density ρ_c, vacuum density ρ_u and the density parameter Ω are given as.

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{a}{8\pi b\,(1-e^{\frac{-a\omega t}{2}}} \tag{23}$$

$$\rho_v = \frac{\Lambda}{8\pi G} = \frac{a}{8ab} \tag{24}$$

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2} = \frac{1}{e^{\frac{a\omega t}{2}}} \tag{25}$$

The deceleration parameter q for the model is

$$q = -1 + \frac{3\omega}{2}e^{\frac{-a\omega t}{2}} \tag{26}$$

Thus for the model (15) we observe that the spatial volume is zero at t = 0 and expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume initially at infinite rate of expansion. The scale factor R = 0 initially hence the space time exhibits point type singularity at initial epoch. The cosmological energy density, shear scalar, diverges at the initial singularity. As t increases the scale factor R and spatial volume increases but the expansion scalar decreases. Thus the rate of expansion slows down with increasing time. The gravitational constants G and the cosmological constant Λ are zero initially, gradually decrease and both tend to a genuine constant at late times Vishwakarama (2005).

CONCLUSION

We have studied the R-W cosmological model with time dependent gravitational constant G (t) and the cosmological constant Λ (t) by using a law of variation of Hubble parameter i.e. a cosmological term that scales as $\Lambda \propto$ H and gravitational constant G varies as $G \propto$ H where H is the Hubble's parameter. Expressions for some important cosmological parameters have been obtained and physical behaviors of the model are discussed in detail. The model has point type singularity at initial epoch. The cosmological term tends asymptotically to a genuine cosmological constant and the model tend to a de sitter universe it is interesting that proposed variation laws provides an alternative approach to obtain exact solution of Einstein's field Equation.

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