International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2015 Vol. 5 (1) January-March, pp. 19-24/Kaur et al.

Research Article

BULK VISCOUS BIANCHI TYPE I COSMOLOGICAL MODELS WITH VARYING COSMOLOGICAL CONSTANT INSELF CREATION THEORY OF GRAVITATION

Harpreet Kaur[1], *Reena Behal[2], D. P. Shukla[3]
[1]*Sant Baba Bhag Singh Institute of Engineering & Technology, Department of Applied Sciences, Khiala, Padhiana, Jalandhar- 144030, Punjab, India*
[2]*Lovely Professional University, Jalandhar,Delhi G.T.Road [NH-1]Phagwara-144411, Punjab, India*
[3]*Government Model Science College, Rewa, M.P. India*
**Author for Correspondence*

ABSTRACT
In this paper we have we investigate Bulk viscous Bianchi type-I cosmological model with varying cosmological constant lambda in self-creation theory of gravitation by taking $H(t) = \frac{a}{t}$. The physical behavior of the models has also been discussed.

Keywords: *Bianchi type-I, Hubble Constant, Barbers scalar function* (t), *Cosmological Constant.*

INTRODUCTION

Still interesting area for researchers is origin of Universe. We find that for intertial properties of matter general theory of gravitation is not satisfactorily applicable. Number of researchers including Machs principle tried to generalize the general theory of gravitation features which are lacking in original theory. There fore

Barber [1] produced two continuous self-creation theories by modifying the BransDicke theory and general relativity. The modified theories create the universe out of self-contained gravitational and matter fields. The theory substantially accommodated Machs principle so Baber modified it.As he coupled the scalar field with the energy momentum tensor. Baber modified Einstein field equations in his self-creation theory.

These modified theories create the universe out of self-contained gravitational and matter fields. This second theory is a modification of general relativity to a variable G-theory and predicts local effects, and secondly is an adaptation of general relativity to include continuous creation. In this theory the Newtonian gravitational parameter G is not a constant but a function of time parameter t. Further, the scalar field does not gravitate directly, but divide the matter tensor, acting as a reciprocal gravitational constant. Which does not happens with Einstein theory of gravitation. This theory is capable of verify behavior of degenerate matter and photons. The theory predicts the same precession of the perihelia of the planets as general relativity 'and in that respect agrees with observation to within 1 %.

Pimentel [2], Soleng [3, 4], Singh [5], Reddy [6, 7, 8], Reddy et al. [9, 10] and Maharaj and Beesham [11] have investigated various aspects of Barber's self-creation theories. Reddy and Venkateshwarlu [12], Venkateshwarlu and Reddy [13], Shanti and Rao [14], Sanyasiraju and Rao [15], Shri and Singh [16,17], Mohanty et al. [18], Pradhan and Vishwakarma [19,20], Sahu and Panigrahi [21], Venkateshwarlu and Kumar [22] are some of the researchers who have studied various aspects of cosmological models in self-creation theory. Recently, Singh and Kumar [23] have studied LRS Bianchi type-Il models with perfect fluid in Barber's second self-creation theory of gravitation by using a special law of variation for Hubble parameter.

In this paper we investigate Bulk viscous Bianchi type-I cosmological model with varying cosmological constant lambda in self-creation theory of gravitation by taking $H(t) = \frac{a}{t}$ .The physical behaviour of the models has also been discussed.

International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2015 Vol. 5 (1) January-March, pp. 19-24/Kaur et al.
Research Article

THE METRIC AND FIELD EQUATIONS

We consider the Bianchi type-I metric in the form

$$ds^2 = - dt^2 + A^2(t)\, dx^2 + B^2(t)\, dy^2 + C^2(t)\, dz^2 \quad \ldots(1)$$

We assume that the cosmic matter with bulk viscosity with the energy- momentum tensor is

$$T_{ij} = (\rho + \bar{p})\upsilon_i \upsilon_j + \bar{p}\, g_{ij} \quad \ldots.(2)$$

Where ρ and $\bar{p}$ are the energy density, effective pressure of viscosity is isotropic pressure and υ_i is the four-velocity vector of the fluid satisfying the relation

$$\upsilon_i \upsilon^i = -1 \quad \ldots.(3)$$

$$\bar{p} = p - \varepsilon\theta \quad \ldots.(4)$$

The Einstein's field equation in Barbers second self-creation theory are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\phi^{-1} T_{ij} + \Lambda(t) g_{ij} \quad \ldots.(5)$$

and $$\varphi;_k^k = \frac{8\pi}{3}\lambda T \quad \ldots.(6)$$

where $\phi;_k^k$ is the invariant D[1]Alembertian and the contracted tensor T is trace of energy momentum tensor that describing all non-gravitational and non-scalar field matter and energy, it is Barbers scalar function of 't'.Here λ is a coupling constant to be determined by experiments.

For the metric (1) and energy momentums tensor (2) is co-moving system of coordinates the field equations (5) and (6) lead to the following equations,where the dot (.) stands for ordinary differentiation with respect to t.

$$8\pi\varphi^{-1}\bar{p} - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} \quad \ldots.(7)$$

$$8\pi\varphi^{-1}\bar{p} - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} \quad \ldots.(8)$$

$$8\pi\varphi^{-1}\bar{p} - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} \quad \ldots.(9)$$

$$8\pi\phi^{-1}\rho + \Lambda = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} \quad \ldots.(10)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi\eta}{3}(\rho - 3p) \quad \ldots.(11)$$

SOLUTION OF THE FIELD EQUATIONS

Equations (7) – (11) supply five equation is seven unknown function of time A, B, C, ρ, p, ϕ *and* Λ. In order to have the solution, we require two more condition. For this purpose, we assume two conditions.

$p = w\rho$….(12)

From equations (7) to (9), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \ldots.(13)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \ldots.(14)$$

International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2015 Vol. 5 (1) January-March, pp. 19-24/Kaur et al.
Research Article

We get

$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{K_1}{ABC}$….(15)

$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{K_2}{ABC}$….(16)

Where K_1 and K_2 are constants.

Let R be the average scale factor and V spatial volume of Bianchi type-I universe i.e.

$$V = R^3 = \sqrt{-g} = ABC \quad ….(17)$$

The expansion scalar θ , Hubble parameter H, shear σ and deceleration parameter q are given by

$$\theta = 3H = \frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \quad ….(18)$$

$$\sigma = \frac{k}{\sqrt{3}\,R^3}, \quad k=\sqrt{k_1^{\,2}+k_1k_2+k_2^2} \quad ….(19)$$

$$q = -1-\frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2} \quad ….(20)$$

$$H=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)….(21)$$

Put value of equation (21) in equation (15) and (16), we get

$\frac{\dot{A}}{A}=\frac{\dot{R}}{R}+\frac{2K_1+K_2}{3R^3}$….(22)

$\frac{\dot{B}}{B}=\frac{\dot{R}}{R}+\frac{K_2-K_1}{3R^3}$….(23)

$\frac{\dot{C}}{C}=\frac{\dot{R}}{R}-\frac{2K_2+K_1}{3R^3}$….(24)

We assume H as

$H(t)=\frac{a}{t}$….(25)

Where 'a' is constant

The deceleration parameter is

$$q = -1-\frac{\dot{H}}{H^2} = -1+\frac{1}{a}….(26)$$

We observe when a = 1,q=0;a< ,q< and when a> ,q>

$R = t^a e^{t_0}$=Mt^a….(27)

Where $M = e^{t_0}$ is constant

Put value of R in equations (22),-(24) we get

$A = b\,t^a\left[exp\left(\left[\frac{2K_1+K_2}{3(1-3a)M^3}\right]\frac{1}{t^{3a-1}}\right)\right]$….(28)

$B = ct^a\left[exp\left(\left[\frac{K_2-K_1}{3(1-3a)M^3}\right]\frac{1}{t^{3a-1}}\right)\right]$….(29)

$C = dt^a\left[exp\left(-\left[\frac{2K_2+K_1}{3(1-3a)M^3}\right]\frac{1}{t^{3a-1}}\right)\right]$….(30)

Where b,c,d are constants of integrations

Metric given by equation (1) is

$ds^2 = -dt^2+ b^2t^{2a}\left[exp\left(\left[\frac{2K_1+K_2}{3(1-3a)M^3}\right]\frac{2}{t^{3a-1}}\right)\right]dx^2+ c^2t^{2a}\left[exp\left(\left[\frac{K_2-K_1}{3(1-3a)M^3}\right]\frac{2}{t^{3a-1}}\right)\right]dy^2+t^{2a}\left[exp\left(-\left[\frac{2K_2+K_1}{3(1-3a)M^3}\right]\frac{2}{t^{3a-1}}\right)\right]dz^2$ ….(31)

International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2015 Vol. 5 (1) January-March, pp. 19-24/Kaur et al.
Research Article

GEOMETRICAL AND PHYSICAL PARAMETERS SIGNIFICANCE

The components of Shear tensor for model (31) are

$\sigma_1^1 = \frac{\dot{A}}{A} - \frac{\dot{R}}{R} = \frac{2K_1+K_2}{3R^3}$....(32)

$\sigma_2^2 = \frac{\dot{B}}{B} - \frac{\dot{R}}{R} = \frac{K_2-K_1}{3R^3}$....(33)

$\sigma_3^3 = \frac{\dot{C}}{C} - \frac{\dot{R}}{R} = \frac{2K_2+K_1}{3R^3}$....(34)

$\sigma_4^4 = 0$ ....(35)

Thus

$\sigma = \frac{K_1^2+K_2^2+K_1K_2}{3R^6} = \frac{K^2}{3R^6} = \frac{K^2}{3t^{6a}e^{6t_0}}$....(36)

Where $K^2 = K_1^2 + K_2^2 + K_1K_2$

Equation (7) – (11) can be written in terms of H, σ and q as

$$8\pi\varphi^{-1}\bar{p} = H^2 (2q-1) - \sigma^2 + \Lambda \quad ....(37)$$

$$8\pi\phi^{-1}\rho = 3H^2 - \sigma^2 - \Lambda \quad ....(38)$$

$$\ddot{\varphi} + \dot{\varphi}3H = \frac{8\pi\lambda}{3}\left(\rho - 3\bar{p}\right) .....(39)$$

Conservation of matter from equation (2) as

$$8\pi\left[\dot{\rho} + 3(\rho + \bar{p})\frac{\dot{R}}{R} - \rho\varphi^{-1}\dot{\varphi}\right] + \dot{\lambda}\varphi = 0....(40)$$

Using equation

Put value of R from equation (27), to find deceleration parameter q, shear tensor, scalar expansion θ and Hubble's Constant H from equations (26),(36),(18) and (25) .

$$q = -1 + \frac{1}{a} ....(41)$$

$$\sigma = \frac{K^2}{3t^{6a}e^{6t_0}} ....(42)$$

$$\theta = 3H = \frac{3a}{t} \quad ....(43)$$

$$H(t) = \frac{a}{t} ....(44)$$

On adding equation (37) and (38), we get

$$8\pi\varphi^{-1}\left(\bar{p}+\rho\right) = 8\pi\varphi^{-1}\left(p - \varepsilon\theta + \rho\right) = 2H^2 (q+1) - 2\sigma^2 \quad ....(45)$$

From equation (45) using (12) ,(41)-(44) we get

$$\rho = \left[\frac{a}{t^2} - \frac{K^2}{3t^{6a}e^{6t_0}}\right]\frac{\varphi(t)}{4\pi(1+w)} - \frac{3a\varepsilon}{t(1+w)} ....(46)$$

$$p = \left[\frac{a}{t^2} - \frac{K^2}{3t^{6a}e^{6t_0}}\right]\frac{w\varphi(t)}{4\pi(1+w)} - \frac{3aw\varepsilon}{t(1+w)} ....(47)$$

$$\Lambda = -8\pi\varphi^{-1} + \frac{3a^2}{t^2} + \frac{1}{3t^{6a}e^{6t_0}}[2(K_1^2 + K_2^2 + K_1K_2)] ....(48)$$

Put values of ρ ,p, Λ in equation (39) we get

$$\ddot{\varphi} + \frac{3a}{t}\dot{\varphi} + \varphi(t)\left(\frac{\alpha_1}{t^2} - \frac{\beta_1}{t^{6a}}\right) = \frac{\alpha_2 - \beta_2}{(1+w)t} ....(49)$$

Where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants given by

$\alpha_1 = \frac{2a\lambda(3w-1)}{3(1+w)}$, $\alpha_2 = 3a\varepsilon(1-3w)$, $\beta_1 = \frac{2\lambda(3w-1)K^2}{9(1+w)e^{6t_0}}$ $\beta_{2=24\pi\lambda(1+w)a\varepsilon}$

Thus in equation (49) we assume coefficient of φ as zero, we get

$\beta_1 = \alpha_1 t^{6a-2}$

Equation (3.49) becomes

$$\ddot{\varphi} + \frac{3a}{t}\dot{\varphi} = \frac{\alpha_2 - \beta_2}{(1+w)t} = \frac{\alpha}{t} \quad \text{....(50)}$$

Where $\alpha = \dfrac{\alpha_2 - \beta_2}{(1+w)}$

$$\varphi(t) = \frac{\alpha t}{3a} + \frac{C_1 t^{1-3a}}{1-3a} + C_2 \text{....(51)}$$

RESULTS AND DISCUSSION

In the model we observe that Barbers scalar function $\varphi(t)$ is in the physical quantities like ρ, Λ, p.As we are aware that in Einstein Theory G is constant whereas Barbers scalar function $\varphi(t)$ is introduced in the field equations in order to have G as function of "t".The effect of $\varphi(t)$ on universe results the model of universe to yield more than three spatial dimensions.

Our model (31) describes geometrical and physical features in more than three spatial dimensions. Initially Barbers scalar function $\varphi(t)$ is constant and increase with increase of time and diverges to infinity at universe final stage. Keeping in view of nature of $\varphi(t)$, it is observed lambda is infinity in earlier universe and becomes positive later epoch of time. Although Barbers scalar function $\varphi(t)$ is present in ρ, p ie density and pressure of fluid initially diverges to infinity and gradually decrease with increase with time and when $t \to \infty$ they become constant. The cosmological model (31) due to proposed variation law for Hubble Parameters is dominated by decelerating phase when a>0 and when a<0 it accelerating. In the model (31) we observe that the spatial volume V is zero at t=0 and expansion scalar θ is infinite at t=0 which shows that the universe starts evolving with zero volume and infinite rate of expansion at t = 0. . As t $\to\infty$ the spatial volume V becomes infinitely large. All the parameters ρ, p, σ, H_1, H_2, H_3 tend to zero when $t\to\infty$. Therefore the model essentially gives an empty universe for large time "t". Nature of density, pressure and lambda is not effected by presences of bulk viscosity as it is taken to be constant. Model is quasi–isotropic i.e. $\dfrac{\sigma}{\theta} = 0$.

CONCLUSION

In this chapter we have we investigate Bulk viscous Bianchi type-I cosmological model with varying cosmological constant lambda in self-creation theory of gravitation by taking $H(t) = \frac{a}{t}$ .The physical behavior of the models has also been discussed. It is observed that the effect of $\varphi(t)$ on universe results the model of universe to yield more than three spatial dimensions. Therefore the model essentially gives an empty universe for large time "t". Nature of density, pressure and lambda is not effected by presences of bulk viscosity as it is taken to be constant. Model is quasi–isotropic i.e. $\dfrac{\sigma}{\theta} = 0$.

REFERENCES

1. Barber, G.A.: Gen. Relativ. ravit. 14, 117 (1982).
2. Pimentel, L.: Astrophys. Space Sci. 116,395 (1985).
3. Soleng, H.H.: Astrophys. Space Sci. 138, 19 (1987a).
4. Soleng, H.H.: Astrophys. Space Sci. 139, 13 (1987b).
5. Singh, T.: J. Math. Phys. 27, 1074 (1986).
6. Reddy, D.R.K.: Astrophys. Space Sci. 132,401 (1987a).
7. Reddy, D.R.K.: Astrophys. Space Sci. 133, 189 (1987c).
8. Reddy, D.R.K.: Astrophys. Space Sci. 133,389 (I987b).
9. Reddy, D.R.K., Venkateshwarlu, R.: Astrophys. Space Sci. 155, 135 (1989)
10. Reddy, D.R.K., Avadhanlu, M.B., Venkateshwarlu, R.: Astrophys. Space Sci. 134, 20 I (1987)
11. Maharaj, *S.D.*, Beesham, A.: Astrophys. Space Sci. 140, 33 (1988).
12. Reddy, D.R.K., Avadhanlu, M.B., Venkateshwarlu, R.: Astrophys. Space Sci. 141, 181 (1988).
13. Venkateshwarlu, R., Reddy, D.R.K.: Astrophys. Space Sci. 168, 193 (1990).
14. Shanti, K., Rao, V.U.M.: Astrophys. Space Sci. 179, 147 (1991).
15. Sanyasiraju, Y.V.S.S., Rao, V.U.M.: Astrophys. Space Sci. 189, 39 (1992).
16. Shri, R., Singh, C.P.: Astrophys. Space Sci. 257,123 (I 998a).
17. Shri, R., Singh, C.P.: Astrophys. Space Sci. 257, 287 (I 998b).
18. Mohanty, G., Mishra, B., Das, R.: Bull. Inst. Math. Acad. Sinica (ROC) 28, 43 (2000).
19. Pradhan, A., Vishwakarma, A.K.: Indian J. Pure Appl. 33, 1239 (2002b).
20. Pradhan, A., Vishwakarma, A.K.: Int. J. Mod. Phys. D 11, 1195 (2002a).
21. Sahu, R.C., Panigrahi, U.K.: Astrophys. Space Sci. 288, 601 (2003).
22. Venkateshwarlu, R., Kumar, P.K.: Astrophys. Space Sci. 301, 73 (2006).
23. Singh, C.P., Kumar, *S.:* Astrophys. Space Sci. 310, 31 (2007).