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Review Article

ON NANO GENERALIZED - SEMI PRE CLOSED SETS AND NANO SEMI PRE – GENERALIZED CLOSED SETS IN NANO TOPOLOGICAL SPACES

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ABSTRACT
The objective of this paper is to introduce and investigate the Nano generalized – semi pre closed sets and Nano semi pre – generalized closed sets in a Nano topological spaces. Its basic properties are also analyzed.

Keywords: *Nano Semi Pre Closed Set, Nano Generalized – semi Pre Closed Set, Nano Semi Pre Generalized Closed Set, Nano Semi Pre Interior and Nano Semi Pre Closure*

INTRODUCTION
In 1970, Levine introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. This concept was found to be useful and many results in general topology were improved. Andrijevic (1986) introduced semi pre open sets and Dontchev (Lellis and Carmel) introduced generalized semi pre open sets in general topology.
The notion of Nano topology was introduced by Lellis (1963) which is defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and also defined Nano closed set, Nano interior and Nano closure.
In this paper, a new class of sets on Nano topological spaces called Nano generalized – semi pre closed sets and Nano semi pre – generalized closed sets are introduced and the relation of these new sets with the existing sets are discussed.
Preliminaries
Definition: 2.1 (Levine, 1963) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.
Let $\subseteq U$. Then,

- The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.

$L_R(X) = \bigcup\{R(x): R(x) \subseteq X, x \in U\}$
where $R(x)$ denotes the equivalence class determined by $x \in U$.

- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X) = \bigcup\{R(x): R(x) \cap X \neq \phi, x \in U\}$

- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not$-X$ with respect to R and is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$
Property: 2.2 (Levine, 1963) If (U, R) is an approximation space and $X, Y \subseteq U$, then
1) $L_R(X) \subseteq X \subseteq U_R(X)$
2) $L_R(\phi) = U_R(\phi) = \phi$
3) $L_R(U) = U_R(U) = U$
4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
11) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition: 2.3 (Levine, 1963) Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.2 $\tau_R(X)$ satisfies the following axioms:
i) U and $\phi \in \tau_R(X)$.
ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
iii) The intersection of all elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X, $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$.
Remark: 2.4 (Levine, 1963) If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.
Definition: 2.5 (Levine, 1963) If $(U, \tau_R(X))$ is a Nano topological space with respect X where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by $NInt(A)$. N$Int(A)$ is the largest Nano open subset of A.
- The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by $NCl(A)$. N$Cl(A)$ is the smallest Nano closed set containing A.

Definition: 2.6 (Levine, 1963) Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

- Nano semi open if $A \subseteq NCl(NInt(A))$
- Nano pre open if $A \subseteq NInt(NCl(A))$
- Nano α open if $A \subseteq NInt[NCl(NInt(A))]$
- Nano semi pre open if $A \subseteq NCl[NInt(NCl(A))]$

$NSO(U,X), NPO(U,X), \tau_R^{\alpha}(X)$ and $NSPO(U,X)$ respectively denote the families of all Nano semi open, Nano pre open, Nano α open and Nano semi pre open subsets of U.
Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$, A is said to be Nano semi closed, Nano pre closed, Nano α closed and Nano semi pre closed if its complement is respectively Nano semi open, Nano pre open, Nano α open and Nano semi pre open.
Definition: 2.7 (Bhuvaneswari and Mythili, 2014**)** A subset A of $(U, \tau_R(X))$ is called a Nano generalized closed set (briefly Ng-closed) if $NCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition: 2.8 (Bhuvaneswari and Thanga, 2014**)** Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

- Nano generalized α closed set if $N\alpha Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano α open in U.
- Nano α generalized closed set if $N\alpha Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in U.

Forms of Nano Generalized - Semi Pre Closed Sets and Nano Semi Pre – Generalized Closed Sets

Throughout this paper $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$. R is an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R.

In this section, we define and study the forms of Nano generalized – semi pre closed sets and Nano semi pre – generalized closed sets.

Definition: 3.1 If $(U, \tau_R(X))$ is a Nano topological space with respect X where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano semi pre closure of a set A is defined as the intersection of all Nano semi pre closed sets containing A and it is denoted by $NspCl(A)$. $NspCl(A)$ is the smallest Nano semi pre closed set containing A.
- The Nano semi pre interior of a set A is defined as the union of all Nano semi pre open subsets contained in A and it is denoted by $NspInt(A)$. $NspInt(A)$ is the largest Nano semi pre open subset of A.

Definition: 3.2 A subset A of $(U, \tau_R(X))$ is called a Nano generalized - semi pre closed set (briefly Ngsp-closed) if $NspCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition: 3.3 A subset A of $(U, \tau_R(X))$ is called a Nano semi pre - generalized closed set (briefly Nspg-closed) if $NspCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano semi open in $(U, \tau_R(X))$.

Theorem: 3.4 Every Nano closed set is a Nano semi pre closed set.

Proof: Let A be a Nano closed set. Then $NCl(A) = A$. We have to prove that $NInt[NCl(NInt(A))] \subseteq A$ which implies that A is a Nano semi pre closed set. $NInt[NCl(NInt(A))] = NInt(A) \subseteq A$. Hence A is a Nano semi pre closed set.

Remark: 3.5 The converse of the above theorem is not true which has been seen from the following example.

Example: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ which are open sets. Then $\{a\}, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}$ are Nano semi pre closed sets but are not Nano closed sets.

Theorem: 3.6 Every Nano closed set is a Nano generalized – semi pre closed set.

Proof: Let A be a Nano closed set of U and $A \subseteq V$, V is Nano open in U. Since A is Nano closed, $NCl(A) = A$, $A \subseteq V$,this implies $NCl(A) \subseteq V$**.** Also $NspCl(A) \subseteq NCl(A)$ which implies $NspCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in U.

Therefore A is a Nano generalized – semi pre closed set.

Theorem: 3.7 Every Nano closed set is a Nano semi pre - generalized closed set.

Proof: Let A be a Nano closed set of U and $A \subseteq V$, V is Nano open in U. Since every Nano open set is Nano semi open. Therefore V is Nano semi open in U. Here A is Nano closed, $NCl(A) = A$, $A \subseteq V$,this implies $NCl(A) \subseteq V$**.** Also $NspCl(A) \subseteq NCl(A)$ which implies $NspCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano semi open in U. Therefore A is a Nano semi pre - generalized closed set.

Remark: 3.8 The converse of the theorem 4.6 and 4.7 is not true. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ which are open sets. Then $\{a\}, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}$ are Nano generalized – semi pre closed sets and Nano semi pre – generalized closed sets but are not Nano closed sets.

Theorem: 3.9 Every Nano generalized closed set is a Nano generalized – semi pre closed set.

Proof: Let A be a Nano generalized closed set. Then $NCl(A) \subseteq V$ whenever $A \subseteq V$, V is Nano open in U. But $NspCl(A) \subseteq NCl(A)$, this implies $NspCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in U. Therefore A is a Nano generalized – semi pre closed set.

Theorem: 3.10 Every Nano generalized closed set is a Nano semi pre - generalized closed set.

Proof: Let A be a Nano generalized closed set.

Then $NCl(A) \subseteq V$ whenever $A \subseteq V$, V is Nano open in U. Since every Nano open set is Nano semi open set.

Therefore V is Nano semi open in U. Also $NspCl(A) \subseteq NCl(A)$,this implies $NspCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano semi open in U. Therefore A is a Nano semi pre - generalized closed set.

Remark: 3.11 The converse of the theorem 4.9 and 4.10 is not true. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ which are open sets. Then $\{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}$ are Nano generalized – semi pre closed sets and Nano semi pre – generalized closed sets but are not Nano generalized closed sets.

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Theorem: 3.12 The union of two Nano generalized – semi pre closed set in $(U, \tau_R(X))$ are also Nano generalized – semi pre closed set in $(U, \tau_R(X))$.

Proof: Let A and B be two Nano generalized – semi pre closed set in $(U, \tau_R(X))$**.** Let V be a Nano open set in U such that $A \subseteq V$ and $B \subseteq V$. Then we have $A \cup B \subseteq V$**,** since A and B are Nano generalized – semi pre closed set in $(U, \tau_R(X))$, $NspCl(A) \subseteq V$ and $NspCl(B) \subseteq V$. Now $NspCl(A \cup B) \subset NspCl(A) \cup NspCl(B)$, this implies $NspCl(A \cup B) \subseteq V$ whenever $A \cup B \subseteq V$ and V is Nano open in $(U, \tau_R(X))$. Thus $A \cup B$ is a Nano generalized – semi pre closed set in $(U, \tau_R(X))$.

Remark: 3.13 The intersection of two Nano generalized – semi pre closed set in $(U, \tau_R(X))$ are also Nano generalized – semi pre closed set in $(U, \tau_R(X))$ as seen from the following example.

Example: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ which are open sets. Then the Nano generalized – semi pre closed sets are $\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}$. Here $\{b, c, d\} \cap \{a, c, d\} = \{c, d\}$ is also a Nano generalized - semi pre closed sets.

Theorem: 3.14 The union of two Nano semi pre - generalized closed set in $(U, \tau_R(X))$ are also Nano semi pre - generalized closed set in $(U, \tau_R(X))$.

Proof: Let A and B be two Nano semi pre - generalized closed set in $(U, \tau_R(X))$**.** Let V be a Nano semi open set in U such that $A \subseteq V$ and $B \subseteq V$. Then we have $A \cup B \subseteq V$**,** since A and B are Nano semi pre - generalized closed set in $(U, \tau_R(X))$, $NspCl(A) \subseteq V$ and $NspCl(B) \subseteq V$. Now $NspCl(A \cup B) \subset NspCl(A) \cup NspCl(B)$, this implies $NspCl(A \cup B) \subseteq V$ whenever $A \cup B \subseteq V$ and V is Nano semi open in $(U, \tau_R(X))$. Thus $A \cup B$ is a Nano semi pre - generalized closed set in $(U, \tau_R(X))$.

Remark: 3.15 The intersection of two Nano semi pre - generalized closed set in $(U, \tau_R(X))$ are also Nano semi pre - generalized closed set in $(U, \tau_R(X))$ as seen from the following example.

Example: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ which are open sets. Then the Nano semi pre - generalized closed sets are $\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}$. Here $\{a, c\} \cap \{c, d\} = \{c\}$ is also a Nano semi pre - generalized closed sets.

Theorem: 3.16 If A is Nano α generalized closed set in $(U, \tau_R(X))$ then it is Nano generalized – semi pre closed set.

Proof: Let A be a Nano α generalized closed set and $A \subseteq V$, V is Nano open in U. Then we have $N\alpha Cl(A) \subseteq V$. But $NspCl(A) \subseteq N\alpha Cl(A)$, this implies $NspCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in U. Therefore A is a Nano generalized – semi pre closed set.

Theorem: 3.17 Let A be a Nano generalized – semi pre closed set in $(U, \tau_R(X))$. If $A \subseteq B \subseteq NspCl(A)$, then B is also a Nano generalized – semi pre closed set in $(U, \tau_R(X))$.

Proof: Let V be a Nano open set of a Nano generalized – semi pre closed subset of $\tau_R(X)$ such that $B \subseteq V$. Since $\subseteq B$, we have $A \subseteq V$. As A is a Nano generalized – semi pre closed set, $NspCl(A) \subseteq V$**.** Given $\subseteq NspCl(A)$, we have $NspCl(B) \subseteq NspCl(A)$**,** Since $NspCl(B) \subseteq NspCl(A)$ and $NspCl(A) \subseteq V$, we have $NspCl(B) \subseteq V$ whenever $B \subseteq V$ and V is Nano open. Hence B is also a Nano generalized – semi pre closed set.

Note: 3.18 Evey Nano generalized – semi pre closed set is Nano semi pre - generalized closed set.

Note: 3.19 Every Nano semi pre closed set is Nano generalized – semi pre closed set and Nano semi pre - generalized closed set.

Conclusion

In this paper, some of the properties of Nano generalized – semi pre closed sets and Nano semi pre – generalized closed sets are discussed. This shall be extended in the future research with some applications.

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