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Review Article

ON NANO REGULAR GENERALIZED STAR b-CLOSED SET IN NANO TOPOLOGICAL SPACES

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ABSTRACT
The purpose of this paper is to introduce a new class of nano generalized closed sets namely, Nano regular generalized star b-closed sets in Nano topological spaces and some of its basic properties are analyzed.

Keywords: *Nano rg*-closed set, Nano rg*-open set and nano rg*b-closed set*

INTRODUCTION
In 1970, Levine introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Ahmed and Mohd (2009) studied the class of generalized b-closed sets. Sindhu and Indirani (2013) introduced the concept of regular generalized star b-closed sets in topological spaces.
The notion of Nano topology was introduced by Lellis which was defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and also defined Nano closed set, Nano interior and Nano closure.
The aim of this paper is to continue the study of nano generalized closed sets in nano topological space. In particular, we introduce a new class of nano sets on Nano topological spaces called Nano regular generalized star b-closed sets and obtain their characteristics with counter examples.

Preliminaries
Definition 2.1 [8]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space.
Let $X \subseteq U$. Then,

- The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.
$L_R(X) = \bigcup\{R(x): R(x) \subseteq X, x \in U\}$
where $R(x)$ denotes the equivalence class determined by $x \in U$.
- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
$U_R(X) = \bigcup\{R(x): R(x) \cap X \neq \phi, x \in U\}$
- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not$-X$ with respect to R and is denoted by $B_R(X)$.
$B_R(X) = U_R(X) - L_R(X)$

Property 2.2 [8]: If (U, R) is an approximation space and $X, Y \subseteq U$, then
1) $L_R(X) \subseteq X \subseteq U_R(X)$
2) $L_R(\phi) = U_R(\phi) = \phi$
3) $L_R(U) = U_R(U) = U$
4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

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9) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
11) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3 [8]: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.2 $\tau_R(X)$ satisfies the following axioms:

i) U and $\phi \in \tau_R(X)$.
ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
iii) The intersection of all elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X, $(U, \tau_R(X))$ is called the Nano topological space.

Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$.

Remark 2.4 [8]: If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5 [8]: If $(U, \tau_R(X))$ is a Nano topological space with respect X where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by $NInt(A)$. N$Int(A)$ is the largest Nano open subset of A.
- The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by $NCl(A)$. N$Cl(A)$ is the smallest Nano closed set containing A.

Definition 2.6 [8]: Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

- Nano semi open if $A \subseteq NCl(NInt(A))$
- Nano pre open if $A \subseteq NInt(NCl(A))$
- Nano α open if $A \subseteq NInt[NCl(NInt(A))]$
- Nano regular open if $A = NInt(NCl(A))$

$NSO(U,X), NPO(U,X), \tau_R^\alpha(X)$ and $NRO(U,X)$ respectively denote the families of all Nano semi open, Nano pre open, Nano α open, and Nano regular open subsets of U.

Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$, A is said to be nano semi-closed, nano pre-closed, nano α-closed and nano regular closed if its complement is respectively Nano semi open, Nano pre open, Nano α open, and Nano regular open.

Definition 2.7 [3], [4]: A subset A of $(U, \tau_R(X))$ is called

(i) Nano b-closed set (briefly Nb-closed) if $NCl(NInt(A)) \cap NInt(NCl(A)) \subseteq A$.

(ii) Nano generalized closed set (briefly Ng-closed) if $NCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

(iii) Nano generalized b-closed set (briefly Ngb-closed) if $NbCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

(iv) Nano regular generalized b-closed set (briefly Nrgb-closed) if $NbCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano regular open in $(U, \tau_R(X))$.

Nano Regular Generalized Star b-closed Set

Definition 3.1: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be regular generalized star closed set (briefly Nrg*-closed) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ U is nano regular open in $(U, \tau_R(X))$.

Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$, A is said to be nano regular generalized star open set (briefly Nrg*-open) if its complement is Nrg*-closed.

Definition 3.2: Let $(U,\tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be regular generalized star b-closed set (briefly Nrg*b-closed) if $Nbcl(A) \subseteq U$ whenever $A \subseteq U$ U is Nrg*-open in $(U,\tau_R(X))$.

Nrg*O(U,X) denotes the family of all nano rg*-open subsets of U.

Example 3.3: Let U= {a,b,c,d} with U / R ={{a},{b,c},{d}} and X = {a,b}. Then the nano topology $\tau_R(X)=$ {{a}, {a,b,c}, {b,c}, U, ϕ}. The nano closed sets are U, ϕ, {b,c,d}, {d} and {a,d}.Then Nrg*O(U,X) = {{a}, {b}, {c}, {d}, {a,b}, {b,c}, {a,c}, {b,d}, {c,d},{a,b,c}, {a,b,d}, U, ϕ }. Then rg*b-closed sets are {a}, {b}, {c}, {d}, {b,c}, {a,d}, {b,d}, {c,d}, {b,c,d}, {a,b,d}, {a,c,d}, U and ϕ.

Theorem 3.4: Every nano closed set is nano rg*b-closed.

Proof: Let A be a nano closed set in $(U,\tau_R(X))$ and G be nano rg*-open such that $A \subseteq G$. Since every nano closed set is nano b-closed, $Nbcl(A) \subseteq Ncl(A) = A \subseteq G.$ Therefore A is nano rg*b-closed.

Remark 3.5: Converse of the above theorem need not be true as seen from the following example.

Example 3.6: Let U = {a,b,c,d} with U / R = {{a}, {b,c}, {d}} and X = {a,b}. Then the nano topology $\tau_R(X)=$ {{a}, {a,b,c}, {b,c}, U, ϕ }.

Then the sets {b,c} and {b,d} are rg*b-closed, but not closed.

The following theorems can also be proved in a similar way.

Theorem 3.7: Let $(U,\tau_R(X))$ be a nano topological space and $A \subseteq U.$ Then

(i) Every nano semi-closed set is nano rg*b-closed.
(ii) Every nano pre-closed set is nano rg*b-closed.
(iii) Every nano $\alpha-$closed set is nano rg*b-closed.
(iv) Every nano regular closed set is nano rg*b-closed.

Remark 3.8: Reverse implications of the theorem need not be true as seen from the following example.

Example 3.9: Let U= {x,y,z,w} with U / R ={{x,y}, {z}, {w}} and X = {x,z}. Then the nano topology $\tau_R(X)=$ {{z}, {x,y}, {x,y,z}, U, ϕ}. Here the sets {x} and {y} are nano rg*b-closed, but not nano semi-closed. The set {z} is nano rg*b-closed, but not nano pre-closed.

Example 3.10: Let U = {a,b,c,d} with U / R = {{a}, {b,c}, {d}} and X = {a,b}. Then the nano topology $\tau_R(X)=$ {{a}, {a,b,c}, {b,c}, U, ϕ }. The sets {b} and {b,c} are rg*b-closed, but not nano $\alpha-$closed. Also the sets {b,c} and {a,d} are rg*b-closed, but not nano regular closed.

Theorem 3.11: Let $(U,\tau_R(X))$ be a nano topological space and $A \subseteq U.$ Then

(i) Every nano rg*b-closed set is nano rgb-closed.
(ii) Every nano rg*b-closed set is nano gb-closed.

Proof: Let A be a nano rg*b-closed set in $(U,\tau_R(X))$ and G be nano regular open such that $A \subseteq G$. Since every nano regular open set is nano rg*-open, G is nano rg*-open. Since A is nano rg*b-closed, $Nbcl(A) \subseteq G.$ Therefore A is nano rgb-closed.

(ii) The proof is similar.

Remark 3.12: The converse of the theorem need not be true as seen from the following example.

Example 3.13: Let U= {a,b,c,d,e} with U / R ={{a,e}, {b,d}, {c}} and X = {c,d}. Then the nano topology $\tau_R(X)=$ {{c}, {b,d}, {b,c,d}, U, ϕ }. Here the set {b,c}is nano rgb-closed, but not nano rg*b-closed and the set {a,b,c}is nano gb-closed, but not nano rg*b-closed

Remark 3.14: The nano rg*b-closed sets are independent of the nano closed sets like nano g-closed sets and nano rg*-closed sets as shown by the following example.

Example 3.15: Let U= {a,b,c,d,e} with U / R ={{a,e}, {b,d}, {c}} and X = {c,d}. Then the nano topology $\tau_R(X)=$ {{c}, {b,d}, {b,c,d}, U, ϕ}. Here the set {b,c}is nano rg*-closed, but not nano rg*b-closed. The set {b}is nano rg*b -closed, but not nano rg*-closed.
The set {a,b,c}is nano g-closed, but not nano rg*b-closed. The set {b,d}is nano rg*b -closed, but not nano g-closed.
Theorem 3.16: The union of two nano rg*b-closed sets need not be nano rg*b-closed as seen from the following example.
Example 3.17: Let U= {a,b,c,d,e} with U / R ={{a,e}, {b,d}, {c}} and X = {c,d}. Then the nano topology $\tau_R(X)=$ {{c}, {b,d}, {b,c,d}, U, ϕ}. Here the sets {b} and {c} are nano rg*b-closed, but their union $\{b\}\bigcup\{c\}=\{b,c\}$ not nano rg*b-closed.

Theorem 3.18: Let A be a nano rg*b-closed set in $(U,\tau_R(X))$. Then Nbcl(A) - A has no non-empty nano rg*-closed set.
Proof: Let A be a nano rg*b-closed. Let F be a nano rg*-closed set in Nbcl(A) - A. That is, $F\subseteq Nbcl(A)-A \Rightarrow F\subseteq Nbcl(A)\bigcap A^c$. $F\subseteq A^c \Rightarrow A\subseteq F^c$, where F^c is a nano rg*-open set. Since A is nano rg*b-closed, $Nbcl(A)\subseteq F^c$. That is, $F\subseteq (Nbcl(A))^c$. Thus $F\subseteq Nbcl(A)\bigcap (Nbcl(A))^c=\phi$. Therefore $F=\phi$.

Theorem 3.19: Let A be nano rg*b-closed set in $(U,\tau_R(X))$. Then A is nano b-closed iff Nbcl(A) - A is nano rg*-closed.
Proof: Let A be a nano rg*b-closed set. If A is nano b-closed, we have Nbcl(A) - A = ϕ, which is nano rg*-closed.
Conversely let Nbcl(A) - A is nano rg*-closed. Then by theorem 3.18, Nbcl(A) - A = ϕ, which implies Nbcl(A) = A. Therefore A is nano b-closed.
Theorem 3.20: If Ais nano rg*b-closed set and B is any set such that $A\subseteq B\subseteq Nbcl(A)$, then B is nano rg*b-closed.
Proof: Let $B\subseteq G$, where G is a nano rg*-open set. Since A is nano rg*b-closed and $A\subseteq G$, $Nbcl(A)\subseteq G$. Also $Nbcl(B)\subseteq Nbcl(Nbcl(A))=Nbcl(A)\subseteq G$. Thus $Nbcl(B)\subseteq G$ and hence B is nano rg*b-closed.
Theorem 3.21: If A is nano rg*-open and nano rg*b-closed, then A is nano b-closed.
Proof: Let A be nano rg*-open and nano rg*b-closed. Then $Nbcl(A)\subseteq A$, but always $A\subseteq Nbcl(A)$. Therefore A = Nbcl(A). Hence A is nano b-closed.

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