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Review Article

ON CR-STRUCTURE AND F-STRUCTURE SATISFYING $F^{4k+3}+F=0$

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ABSTRACT
In this paper, we have studied a relationship between CR-structure and F-structure satisfying $F^{4k+3}+F=0$, where K is a positive integer greater than or equal to 1. Nijenhuis tensor and integrability conditions have also been discussed.

Keywords: *Projection Operators, Distributions, Nijenhuis Tensor, Integrability Conditions and CR-structure*

INTRODUCTION

Let *M* be an n-dimensional differentiable manifold of class C^∞. Let *F* be a non-zero tensor of type (1, 1) and class C^∞ defined on *M*, such that

$$F^{4K+3}+F=0 \tag{1.1}$$

where *K*is a positive integer greater than or equal to 1

Let rank $((F))=r$, which is constant everywhere. We define the operators on *M* as

$$l=-F^{4K+2},\ m=F^{4K+2}+I \tag{1.2}$$

where I is the identity operator on *M*.

Theorem (1.1) Let *M* be an *F*- structure satisfying (1.1) then

(a) $l+m=I$

(b) $l^2=l$

(c) $m^2=m$

(d) $lm=ml=0$ (1.3)

Proof: From (1.1) and (1.2), we get the results.

Let D_l and D_m be the complementary distributions corresponding to the operators *l* and *m* respectively. then

$$\dim((D_l))=r,\quad \dim((D_m))=n-r$$

Theorem (1.2) Let *M* be an *F*-structure satisfying (1.1). Then

(a) $lF=Fl=F,\quad mF=Fm=0$

(b) $F^{4K+2}\,l=-l,\quad F^{4K+2}m=0$ (1.4)

Proof: From (1.1), (1.2), (1.3)(a), (b), we get the results.

From (1.4) (b), it is clear that F^{2K+1} acts on D_l as an almost complex structure and on D_m as a null operator.

Nijenhuis Tensor

Definifion (2.1) Let *X* and *Y* be any two vector fields on *M*, then their Lie bracket [*X*, *Y*] is defined by

$[X, Y] = XY- YX$ (2.1)

and Nijenhuis tensor

N(*X*,*Y*) of *F* is defined as

$$N(X,Y)=[FX,FY]-F[FX,Y]-F[X,FY]+F^2[X,Y]. \tag{2.2}$$

Theorem (2.1) *A* necessary and sufficient condition for the *F*-structure to be integrable is $N(X,Y)=0$, for any two vector fields X & Y on *M*.

Theorem (2.2) Let the *F*-structure satisfying (1.1) be integrable, then

$$\left(-F^{4K+1}\right)\left([FX,FY]+F^2[X,Y]\right)=l\left([FX,Y]+[X,FY]\right). \tag{2.3}$$

Proof: using theorem (2.1) in (2.2), we get

$$[FX,FY]+F^2[X,Y]=F\left([FX,Y]+[X,FY]\right) \tag{2.4}$$

Operating by $\left(-F^{4K+1}\right)$ on both the sides of (2.4) and using (1.2), we get the result.

Theorem (2.3) On the *F*-structure satisfying (1.1)

(a) $m\,N(X,Y)=m[FX,FY]$ (2.5)

(b) $m\,N\left(F^{4K+1}X,Y\right)=m\left[F^{4K+2}X,FY\right]$

Proof: Operating *m* on both the sides of (2.2) and using (1.4) (a) we get (2.5) (a). Replacing X by $F^{4K+1}X$ in (2.5) (a), we get (2.5) (b).

Theorem (2.4): On the *F*-structure satisfying (1.1), the following conditions are all equivalent

(a) $m\,N(X,Y)=0$ (2.6)

(b) $m[FX,FY]=0$

(c) $m\,N\left(F^{4K+1}X,Y\right)=0$

(d) $m\left[F^{4K+2}X,FY\right]=0$

(e) $m\left[F^{4K+2}lX,FY\right]=0$

Proof: Using (1.4) (a), (b) in (2.5) (a), (b), we get the results.

CR-Structure

Definition (3.1) Let $T_c(M)$ denotes the complexified tangent bundle of the differentiable manifold *M*. A CR-structure on *M* is a complex sub-bundle *H* of $T_c(M)$ such that

(a) $H_p\cap\tilde{H}_p=\{0\}$ (3.1)

(b) *H* is involutive that is $X,Y\in H\Rightarrow[X,Y]\in H$ for complex vector fields *X* and *Y*.

For the integrable F-structure satisfying (1.1) rank $((F))=r=2m$ on *M*.

we define

$$H_p=\left\{X-\sqrt{-1}FX : X\in X\left(D_l\right)\right\} \tag{3.2}$$

where $X(D_l)$ is the $F(D_m)$ module of all differentiable sections of D_l.

Theorem (3.1) If *P* and *Q* are two elements of *H*, then

$$[P,Q]=[X,Y]-[FX,FY]-\sqrt{-1}(-1)\left([FX,Y]+[X,FY]\right) \tag{3.3}$$

Proof: Defining $P=X-\sqrt{-1}(-1)FX,\ Q=Y-\sqrt{-1}(-1)FY$ and simplifying, we get (3.3)

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Theorem (3.2) for $X, Y, \in X\left(D_l\right)$

$$l\left(\left[FX, Y\right]+\left[X, FY\right]\right)=\left[FX, Y\right]+\left[X, FY\right] \tag{3.4}$$

Proof: Using (1.4) (a) and (2.1), we get the result as

$$\begin{aligned} l\left(\left[FX, Y\right]+\left[X, FY\right]\right) &= l\left(FXY-YFX+XFY-FYX\right) \\ &= FXY-YFX+XFY-FYX \\ &= \left[FX, Y\right]+\left[X, FY\right] \end{aligned} \tag{3.5}$$

Theorem (3.3) The integrable F-structure satisfying (1.1) on *M* defines a CR-structure *H* on it such that

$$R_e\left(H\right)=D_l \tag{3.6}$$

Proof since $\left[X, FY\right], \left[FX, Y\right] \in X\left(D_l\right)$ then from (3.3), (3.4), we get

$$l\left[P, Q\right]=\left[P, Q\right] \tag{3.7}$$

$$\Rightarrow \left[P, Q\right] \in X\left(D_l\right)$$

Thus *F* structure satisfying (1.1), defines a CR-structure on *M*.

Definition (3.2) Let $\tilde{K}$ be the complementary distribution of $R_e\left(H\right)$ to *TM*. We define a morphism $F: TM \longrightarrow TM$, given by

$F\left(X\right)=0, \forall X \in X\left(\tilde{K}\right)$ such that

$$F\left(X\right)=\frac{1}{2}\sqrt{-1}\left(-1\right)\left(P-\tilde{P}\right) \tag{3.8}$$

where $P=X+\sqrt{-1}\left(-1\right)Y \in X\left(H_p\right)$ and $\tilde{P}$ is complex conjugate of *P*.

Corollary (3.1): From (3.8) we get

$$F^2 X=-X \tag{3.9}$$

Theorem (3.4): If *M* has CR-structure then $F^{4K+3}+F=0$ and consequently F-structure satisfying (1.1) is defined on *M* s.t. D_l and D_m coincide with $R_e\left(H\right)$ and $\tilde{K}$ respectively.

Proof. from (3.9)

$$F^2 X=-X$$

$$F^3 X=-F\left(X\right)$$

$$F^4 X=-F^2\left(X\right)=X \text{ etc.}$$

Then

$$\begin{aligned} F^{4K+3}X &= F^3 X \\ &= F\left(F^2\left(X\right)\right) \\ &= F\left(-X\right) \end{aligned}$$

Thus

$$F^{4K+3}+F=0$$

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