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Research Article

EXISTENCE OF STIFF FLUID MODEL AS A COUPLING EFFECT OF PERFECT FLUID AND MAGNETIC FIELD IN BIMETRIC RELATIVITY

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ABSTRACT
Einstein's field equation in Bimetric relativity in presence of perfect fluid and magnetic field for a four dimensional spherically symmetric homogeneous metric has been studied. The solution is obtained by taking, $p = \rho$ (Stiff fluid model) to satisfy which we have taken $\rho = 3\eta$ to avoid the complicacy in solving the equations. Various physical properties have been studied which shows the sharp decrease in the value of $p, \rho \, \& \eta$ with increase in 'r'. The volume expansion is independent of 't' and the model is neither accelerating decelerating.

***Keywords**: Bimetric Relativity, Perfect Fluid, Magnetic Field*

INTRODUCTION
Rosen (1973, 1975) proposed a bimetric theory of gravitation incorporating the covariance and equivalence principle. In this theory, at each point of space-time there exists two metric tensors g_{ij} and the background flat space time metric tensor γ_{ij}. The tensor g_{ij} describes the geometry of curved space-time and gravitational field while γ_{ij} refers to inertial force. The field equations of Rosen Bimetric theory of gravitation derived from variational principles are

$$N_j^i - \frac{1}{2} N\delta_j^i = -8\pi k T_j^i ,$$

where $N_j^i = \frac{1}{2}\gamma^{ab}\left(g^{hi}\ g_{hj;\,a}\right)_{;b}$ (';' indicates for covariant differentiation)

$$N = \sum N_j^i = N_1^1 + N_2^2 + N_3^2 + N_4^4 ,$$

$$g = \det\left(g_{ij}\right), \gamma = \det\left(\gamma_{ij}\right), k = \left(\frac{g}{\gamma}\right)^{\frac{1}{2}} \text{ and}$$

$T_j^i \rightarrow$ energy momentum tensor of the matter.

In particular, Mohanty and Sahoo (2002) and Mohanty *et al.*, (2002) have established the non-existence of anisotropic spatially homogeneous Bianchi type cosmological models in bimetric theory when the source of gravitation is governed by perfect fluid. Reddy *et al.*, (2006, 2008) have discussed the non-existence of Kantowski-Sachs Cosmological model in case of string as well as perfect fluid distribution. Also, Jena *et al.*, (2015) have shown the non-existence of string cosmological models in presence of magnetic field in Bimetric theory.

Magnetic field plays an important role in the description of the energy distribution in the universe as it contains highly ionized matter. It is believed that the presence of electromagnetic field cloud alter the rate of creation of particles in the anisotropic model. Also, it directly affects the rate of expansion of the universe. Inspite of these lot of works we need the following further investigation.

In the previous paper we have obtained the four dimensional spherically symmetric space-time coupled with Cosmic String and Magnetic field and found that magnetic field has no contribution. So this study has been extended to a coupling effect of perfect fluid and magnetic field on the space-time in Bimetric relativity.

Field Equation

Here we have considered the four dimensional spherically symmetric space-time

$$ds^2 = -e^{\lambda}\, dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{\mu}\, dt^2, \tag{1}$$

where λ and μ are functions of 't' only.

The background of flat space-time corresponding to equation (1) is

$$d\delta^2 = -dr^2 - d\theta^2 - d\phi^2 + dt^2. \tag{2}$$

The existing components of energy momentum tensor for electromagnetic field along X-axis in Bimetric theory are

$$E^1_{1(mag)} = \eta, \qquad E^3_{3(mag)} = 3\eta,$$
$$E^2_{2(mag)} = 3\eta \text{ and } \qquad E^4_{4(mag)} = -\eta, \tag{3}$$

where $\eta \; \dfrac{-A\operatorname{cosec}^2\theta}{2r^4}$,

$A = F_{23}$ = constant = Magnetic field intensity along X-axis

and $F^{23} = g^{22}g^{33}\, F_{23} = \dfrac{A \operatorname{cosec}^2\theta}{r^4}$.

The energy momentum tensor for perfect fluid is given by

$$T^i_{j(p)} = (p+\rho)\, V_i V_j - P g_{ij} \tag{4}$$

together with $g_{ij}\, V^i\, V^j = 1$,

where $\quad V^i \to$ four velocity vector of the fluid

$\quad P \to$ proper pressure

$\quad \rho \to$ energy density.

The existing components of energy momentum tensor for perfect fluid are

$$T^1_{1(P)} = -P \qquad T^3_{3(P)} = -P$$
$$T^2_{2(P)} = -P \qquad T^4_{4((P)} = \rho. \tag{5}$$

The energy momentum tensor for perfect fluid coupled with magnetic field is

$$T^i_j = T^i_{j(p)} + E^i_{j(mag)} \tag{6}$$

and the existing components are (from equation (3))

$$T^1_1 = -p+\eta,\ T^2_2 = -p+3\eta,\ T^3_3 = -p+3\eta,\ T^4_4 = \rho-\eta. \tag{7}$$

With the help of equation(6), equation (1) yields

$$\lambda_{44} - \mu_{44} = 32\pi k(P-\eta) \tag{8}$$
$$\lambda_{44} + \mu_{44} = 32\pi k(3\eta - p) \tag{9}$$
$$\mu_4 - \lambda_{44} = 32\pi k(\eta-\rho) \tag{10}$$

Stiff Fluid Model $(P=\rho)$**:**

Using this condition in equation (8), (9) and adding, we get

$$\lambda_{44} = 32\pi k\eta. \tag{11}$$

Again adding equations (9) and (10, we get

$$\mu_{44} = 32\pi k(2\eta-\rho). \tag{12}$$

To avoid the complicacy in finding the solution, let us take

$$\rho = 3\eta \tag{13}$$

to satisfy $\lambda = -\mu$ (14)

and the solutions are obtained from equation (11) and (12) as

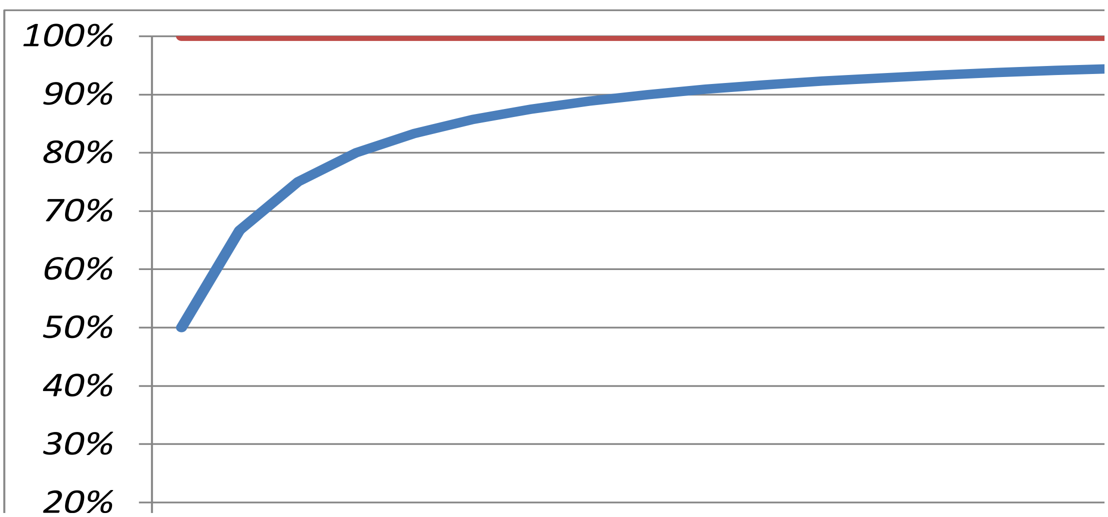
$$\mu(=-\lambda) = \frac{8\pi A^2 t^2}{r^2 \sin\theta} - C_1 t - C_2 \tag{15}$$

Putting equation (14) in equation (1), the line element takes the form

$$ds^2 = -e^{-\left(\frac{8\pi A^2 t^2}{r^2 \sin^2\theta} - C_1 t - C_2\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + e^{\left(\frac{8\pi A^2 t^2}{r^2 \sin\theta} - C_1 t - C_2\right)} dt^2 \tag{16}$$

***Some Physical and Geometrical Properties*:**

(1) volume $(V) = (-g)^{\frac{1}{2}} = r^2 \sin\theta$ i.e., $V \alpha\, r^2$ i.e. volume is independent of 't' and function of 'r' only. When 'r' increases 'V' increases which has shown in the graph.



(2) $\theta = V_i^i = 0$

$\therefore$ the model has no scalar expansion.

(3) Since $\dot{V}_i = 0$ i.e., there is no acceleration and the model is geodesic in nature.

(4) $$\sigma^2 = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{23}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{44,4}}{g_{44}}\right)^2 + \left(\frac{g_{44,4}}{g_{44}} - \frac{g_{11,4}}{g_{11}}\right)^2\right] \neq 0,$$

So that the model is anisotropy in nature.

(5) Since $R_{ij} \neq \frac{1}{4} g_{ij}$, so it shows that

the model is not an Einstein's space.

(6) $w_{ij} = 0$, hence the model is non-rotating.

(7) Deceleration parameter

$$q = -3\theta^2 \left[\theta_{;\alpha} V^\alpha + \frac{1}{3}\theta^2\right] = 0 \qquad (\because \theta = 0)$$

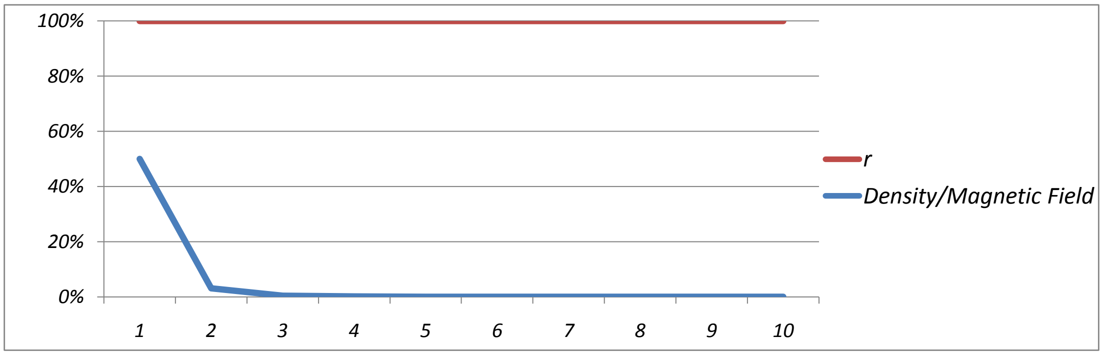
i.e. the model is neither accelerating or decelerating. But is independent of time ‘t’.

(8) By equation (13) $\rho = 3\eta$

$\therefore P = \rho = 3\eta$ [by stiff fluid model]

$= \dfrac{3A^2 \operatorname{cosec}^2 \theta}{2r^4}$ i.e., $p \text{ or } \rho \text{ or } \eta \ \alpha \dfrac{1}{r^4}$

i.e. the proper pressure, energy density and electromagnetic field varies inversely as fourth power of parameter ‘r’. When ‘r’ increases, p, ρ & η decreases rapidly and then becomes zero which has shown in the graph taking r along X-axis and $p \text{ or } \rho \text{ or } \eta$ along Y-axis.



CONCLUSION

In this paper we have investigated spatially homogeneous anisotropic cosmological model of the universe in bimetric relativity in presence of magnetic field and perfect fluid. The solution shows that $p = \rho$. Hence the model does not exist in form of false vacuum model, Radiating Model or dust Distribution Model, but only stiff fluid model exist. p, ρ & η decreases rapidly with increase in value of ‘r’. Volume expansion is a function of ‘r’ only but independent of ‘t’ i.e., model is neither accelerating or decelerating. Moreover the model is non rotating since the verticity tensor $w_{ij} = 0$ and also shows anisotropy throughout the evolution.

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