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Research Article

ESTIMATION AND MINIMIZATION OF SCRAP IN A PRODUCTION LINE WITH IMMEDIATE FEEDBACK AND SINGLE SERVER WORKSTATIONS

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ABSTRACT

In this paper, we have made efforts to frame a policy for minimization of loss in a production line caused by unsuccessful processing of items or jobs at various workstations constituting the production line. We have considered a production line consisting of an arbitrary number of workstations, all arranged in a series in a specific order. At each of the workstations, there is facility of reprocessing of an item at the same workstations, if its processing is not done correctly at that workstations as well as there is chance of its processing once more. This type of activity in queuing is called immediate feedback. We have assumed single server facility at each of the workstations. At the first workstation, arrival of jobs is according to Poisson rule and at each of the workstations, service times are assumed to be exponentially distributed. We have applied queuing theoretical approach to model the above said production line.

Keywords: *Queuing Network, Scrap, Single Server workstation, Immediate Feedback, Production Line*

INTRODUCTION

A production line is a serial arrangement of an arbitrary number of workstations arranged in a specific order. The number and the order of workstations in a production line depend upon the type of the product to be manufactured. The processing of raw material/ a job/ a work part starts at the first workstation and lasts at the last workstation. Completion of processing of an item/ a job/ a work part at a workstation results in the following two ways:

1. That the processing of a job/ material/ a work part at a workstation is done successfully and it is transferred to the next workstation for processing there.

2. The other kind is that the processing at a workstation is not done correctly. It again give rise to two possibilities:

(a) That it still possesses the quality of being processed once more at the same workstation.

(b) The other is that it does not possess the quality of being processed once more at the same workstation.

If the material/ a job/ a work part with unsuccessful processing at a workstation, has no quality to be processed once more at the same workstation, then the material/a job/ a work part is useless and it is termed as scrap.

Scrap results in a loss to the manufacture unit in two ways as follows:

i) Material/a job/a work part after unsuccessful processing at a workstation and having no quality of reprocessing once more at the same workstation does not remain even raw material. To bring it into raw material form, some cost is required and this can be done in some cases only, otherwise it is lost.

ii) Cost of processing at the current workstation and at all the previous workstations is lost.

Seeing this, efforts to minimize the amount of scrap are required. Our study is a sincere effort in this direction.

After the successful completion of processing at the first workstation, the material /a job / a work part is transferred to the second workstation for other type of processing. As the processing at the second workstation is successfully over, the material/a job/a work part is transferred to the third workstation for another type of processing. After the successful processing at all intermediate workstations, the material/a job/ a work part reaches the last workstation and after getting successful processing there, it comes out as a product ready for use.

In ours model there is facility of check-up of each processed item /job/work part, for its successful or unsuccessful processing at a workstation as its processing is over, and this facility is provided at each of the workstations.

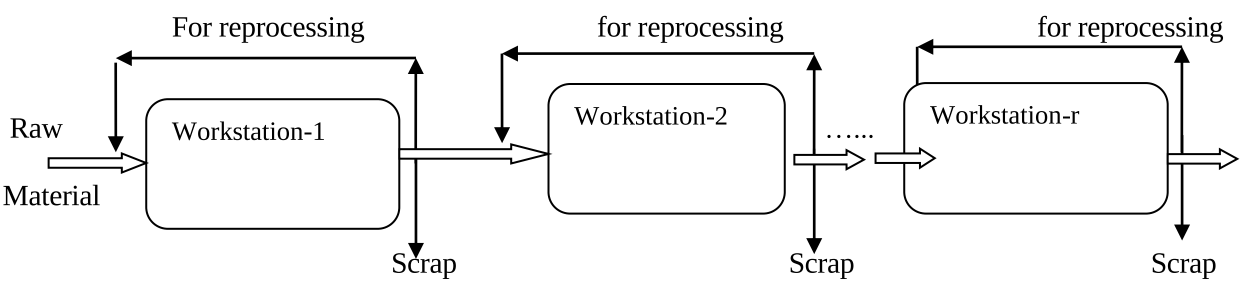

Figure 1: A simple Production Line

In a manufacturing system with several workstations, material/jobs completing service at a workstation wait for access to the next. It requires waiting space/buffer space before each of the workstations. In some cases, at a workstation, a machine may remain idle for some time interval waiting for jobs to process. This whole requires a tool to decide the buffer space before each workstation as well as the number of machines at a workstation for smooth running of the production system with least buffer space before each workstation and minimum number of machines at a workstation.

Literature Survey

Queuing theory is a branch of applied mathematics that deals with phenomenon of waiting lines and arises from the use of powerful mathematical analysis to describe production processes. A lot of work has been done on the applications of Queuing Theory to study and analyze the production systems. After 1920, a lot of work has been done for developing the Queuing Theory as well as on its applications to model various phenomenon. Queueing theory's history goes back nearly 100 years. Johannsen's "Waiting Times and Number of Calls" (an article published in 1907 and reprinted in Post Office Electrical Engineers Journal, London, October, 1910) seems to be the first paper on the subject. But the method used in this paper was not mathematically exact and therefore, from the point of view of exact treatment, the paper that has historic importance is AK Erlang's, "The Theory of Probabilities and Telephone Conversations". Several researches have made efforts to apply queuing theory to model manufacturing systems. We are making effort to apply this theory to determine a policy to minimize the amount of scrap, which is produced due to ill-processing of material or jobs at various workstations of the production line.

Saaty (1961) in his book "Elements of Queuing Theory" has discussed various queuing models and their measures of performances. Jackson (1954**)** has found the steady state solution of a network of two queues in series. Arya (1972) had found that the steady state distribution of queue length taking two queues in the system, where each of the two non-serial servers is separately in service with the other two non-serial servers. Sharma (1973) studied the stationary behavior of a finite space queuing model consisting of queues in series. He had found the steady state solution of serial network of queues with multi server facility and infinite waiting space. In our model, we have introduced immediate-feedback at each of the processing units. Jackson (1957) has discussed the network of queues. Arrivals to the first processing unit are according to Poisson rule and service times at each of the processing units are exponentially distributed. Trivedi (2002) in his book "Probability & Statistics with Reliability, Queuing and Computer Science Applications" has discussed the network of queues and has found the steady state solution of serial network of queues. Bhat (2008) has discussed the steady state behavior of queues and found the steady state solution of a serial network of an arbitrary number of queues.

Leonard (1976) in his book "Queueing Systems, Computer Applications" Volume - 1, has found the steady state probability distribution of number of units in the system. Gross (1985) in their book "Fundamentals of Queuing Theory" have discussed major queuing models. Warland (1998**)** in his book "An Introduction to Queuing Networks" has discussed a queuing system with immediate feedback and has suggested how the parameters change when immediate feedback is removed. Groover (2004) in his book "Automation, Production Systems, and Computer-Integrated Manufacturing" has discussed various kinds of assembly line/ production line. Curry *et al.,* (2011) in their book "Manufacturing System Modeling and Analysis, have discussed the M/M/1 queuing model with applications. Singh (2012**)** has discussed measures of performance of a production line with immediate feedback. Singh *et al.,* (2014) have discussed the policy for minimization of scrap in a production line with immediate feedback and multi server facility at all the Workstations. Singh *et al.,* (2013) have discussed the policy for minimization of loss in a production line with immediate feedback and multi server facility at all the workstations. Singh *et al.,* (2012**)** have studied the average losses caused by ill-processing in a production line with immediate feedback and multi server facility at each of the workstations.

Assumptions

a) Arrival is only at the first workstation, with mean arrival rate λ, following Poisson rule.

b) Service times at all the workstations are according to negative exponential distribution.

c) Transfer times between the workstations are negligible.

d) Quality check times at the workstations are negligible.

Nomenclature

λ = Mean arrival rate to the first workstation from an infinite source.

μ_i = Mean service rate at the i^{th} workstation.

n_i= Number of unprocessed items/ jobs/ work parts before the i^{th} workstation, including one in service, if any.

$p_{i,i+1}$ = Probability that the processing of an item/job/work part at the i^{th} workstation is done correctly and it is transferred to the next workstation.

$p_{i,i}$ = Probability that the processing of an item/job/work part at the i^{th} workstation is not done correctly and it is transferred to the same workstation for reprocessing.

$p_{i,o}$ = Probability that the processing of an item/job/work part at the i^{th} workstation is neither done correctly nor it is found suitable for reprocessing at the same workstation, consequently it is rejected and put into the scrap.

$p_{0,1}$ = 1 = Probability that the material/a job/a work part, requiring service joins the first workstation only.

$p_{0,0}$ = 0 = Probability that the material/a job/a work part, requiring service goes back without taking service.

$P(n_1, n_2, ...n_r, t) =$ Probability that there are n_1 jobs for processing before the first workstation, n_2 jobs before the second workstation , and so on, n_r jobs before the r^{th} workstation at time t, with $n_i \geq 0 (1 \leq i \leq r)$, and $P(n_1, n_2, ...n_r, t)$ =0, if any n_i <0.

S = the amount of scrap obtained from all the workstations of the production line due to unsuccessful processing of material / jobs/ work parts.

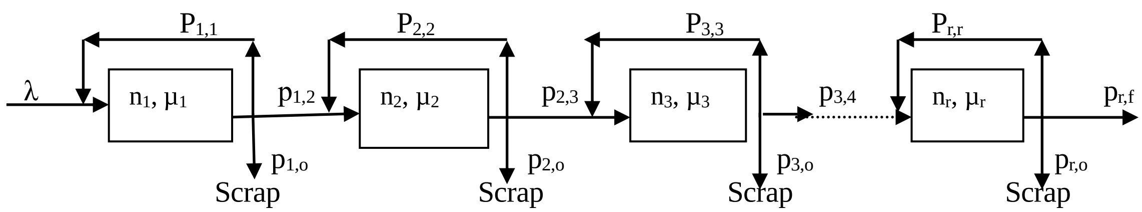

Figure 2: Modeled Production Line

Governing Equations of the Above Production Line

The above production line can be supposed as a serial network of queues where each workstation is supposed as a node of the queuing network. At the first node of the queuing network (i.e. the first workstation) there are n_1 units waiting for service (including one in service, if any), a single server with service rate μ_1 and the respective routing probabilities. Similarly, the second and other all workstations are replaced by the corresponding nodes of the queuing network with single servers and respective number of units waiting for service with corresponding routing probabilities. The supposed serial network of queues has immediate feedback at each of the queues. To analyze this serial network of queues, firstly we remove the immediate feedback from each of the queues. After removing the immediate feedback, the equivalent serial network of queues representing the production line becomes as follows:

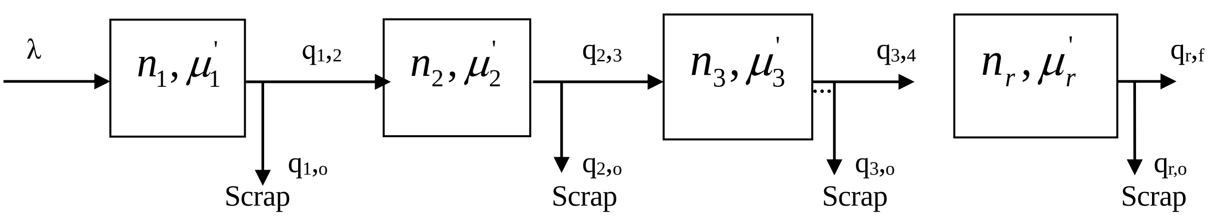

Figure 3: Equivalent Serial Queuing Network representing the Production Line

Here μ_i' is the effective service rate at the i^{th} node (workstation) after the removal of the immediate feedback, and is given by $\mu_i' = \mu_i(1 - p_{i,i})$, Warland (1998) and the routing probabilities become as follows:

$$q_{i,o} = \frac{p_{i,o}}{(1 - p_{i,i})} \quad , \; q_{i,i+1} = \frac{p_{i,i+1}}{(1 - p_{i,i})} \tag{1}$$

At any instant of time 't', the above serial network of queues representing the production line is governed by the following differential –difference equation

Then, $\frac{\partial}{\partial t} p\left(n_1, n_2, \ldots\ldots n_r, t\right) = -\left[\lambda + \sum_{i=1}^{r}\left(1-\delta_{n_i,0}\right)\mu_i\right].\mathrm{P}\left(n_1, n_2 \ldots\ldots n_r, t\right)$
$+\lambda.\mathrm{P}\left(n_1-1, n_2, n_3, \ldots. n_r, t\right) + \mu_1^{'}.q_{1,2}.\mathrm{P}\left(n_1+1, n_2-1, n_3, \ldots. n_r, t\right)$
$+ \mu_2^{'}.q_{2,3}.\mathrm{P}\left(n_1, n_2+1, n_3-1, n_4 \ldots\ldots\ldots. n_r, t\right) + \ldots + \mu_{r-1}^{'}.q_{r-1,r}.\mathrm{P}\left(n_1, n_2, \ldots\ldots n_{r-1}+1, n_r-1, t\right)$
$+\mu_r^{'} q_{r,f}.\mathrm{P}\left(n_1, n_2, \ldots\ldots. n_r+1, t\right) + \mu_1^{'}.q_{1,o}.\mathrm{P}\left(n_1+1, n_2 \ldots\ldots n_r, t\right)$
$+ \mu_2^{'}.q_{2,o}.\mathrm{P}\left(n_1, n_2+1, n_3, \ldots.. n_r, t\right) + \mu_{r-1}^{'}.q_{r-1,o}\mathrm{P}\left(n_1, n_2, n_3, \ldots. n_{r-1}+1, n_r, t\right)$, such that $n_i \geq 0$ (1≤ i≤ r),
and $\mathrm{P}\left(n_1, n_2, n_3, \ldots.. n_r, t\right)$=0, if any n_i <0. (2)

Initially the work in the production line starts at the first workstation, its output is transferred to the second workstation and so on. After an interval of time, all workstations in the production line are operational. System starts working smoothly and all the probabilities become independent of time. This state of the system is referred to as steady state condition. Under steady state condition, all the queues behave independently.

Thus, under the steady state condition $\mathrm{P}\left(n_1, n_2, \ldots\ldots n_r, t\right) = \mathrm{P}\left(n_1, n_2, \ldots\ldots n_r\right)$

$$\therefore \quad \frac{\partial}{\partial t}\mathrm{P}\left(n_1, n_2, \ldots.. n_r, t\right) = 0$$

Consequently, under steady state condition, the above set of governing equations reduces to

$$\left[\lambda + \mu_1 + \mu_2 + \ldots.. + \mu_r\right].\mathrm{P}\left(n_1, n_2, \ldots.. n_r\right) = \lambda\,\mathrm{P}\left(n_1-1, n_2 n_3, \ldots\ldots n_r\right) + \sum_{i=1}^{r}\mu_i^{'}.q_{i,i+1}.P\left(n_1, n_2, \ldots. n_i+1, n_{i+1}-1, \ldots\ldots.. n_r\right) + \sum_{i=1}^{r}\mu_i^{'}.q_{i,o}.P\left(n_1, n_2, \ldots.. n_i+1.n_{i+1}, \ldots. n_r\right)$$
(3)

Solution for Infinite Queuing System

Under steady state condition, all the queues behave independently i.e. as if all the workstations were working independently.
As under the steady state condition, all the queues behave independently, consequently, the solution of the above open queuing network, in product form, is given by

$\mathrm{P}\left(n_1, n_2, \ldots\ldots. n_r\right) = \prod_{i=1}^{r}\left(1-\rho_i\right)\rho_i^{n_i}$, Curry (2011) and Bhat (2008) and Trivedi (2002).
Where, $n_i \geq 0\left(1 \leq i \leq r\right)$ and $\rho_i < 1\ \left(1 \leq i \leq r\right)$ (4)

If any ρ_i $\left(1 \leq i \leq r\right)$>1, then the stability is disturbed and the behavior of the system will not remain stationary i.e. the system will not remain in steady state condition, consequently, the solution mentioned above will no longer be valid.

λ_i , the mean arrival rate at the i[th] workstation, is given by

$\lambda_i = \lambda \prod_{k=1}^{i} \frac{p_{k-1,k}}{(1-p_{k-1,k-1})}$, with $p_{0,0} = 0$, and $p_{0,1} = 1$, Singh (2012)

And therefore

$$\rho_i = \frac{\lambda}{\mu_i} \prod_{k=1}^{i} \frac{p_{k-1,k}}{(1-p_{k,k})}, \text{ with } p_{0,1} = 1 \quad (5)$$

It can be observed that $\sum_{i=1}^{r} \lambda_i . q_{i,o} + \lambda_r . q_{r,f} = \lambda$ (6)

Equation (6) confirms the principle of conservation.

Estimation of Scrap

As we start the production line, its first workstation starts working and the output of this workstation is moved to the second workstation and so on, finally the work part/a job/material is processed at the last workstation. This process produces some scrap also. Under steady state condition, A, the amount of scrap produced per unit of time is given by

S = The amount of scrap obtained from all the workstations of the production line due to unsuccessful processing of material/a job/a work part

= Sum of amounts of scrap obtained at all of the workstations of the production line

= amount of scrap obtained at the first workstation + amount of scrap obtained at the second workstation + ... + amount of scrap obtained at the last workstation

$$= \lambda\, q_{1,o} + \lambda\, q_{1,2}\, q_{2,o} + \ldots + \lambda\, q_{1,2}\, q_{2,3} \ldots q_{r-1,r}\, q_{r,o}$$

$$= \sum_{i=1}^{r} \lambda\, q_{1,2}.q_{2,3}...q_{i-1,i} q_{i,o}, \text{ With } q_{0,1} = 1$$

$$= \lambda \sum_{i=1}^{r} \frac{p_{1,2}}{(1-p_{1,1})} \frac{p_{2,3}}{(1-p_{2,2})} \ldots \frac{p_{i-1,i}}{(1-p_{i-1,i-1})} \frac{p_{i,o}}{(1-p_{i,i})}, \text{ with } p_{0,0} = 0, and\ p_{0,1} = 1$$

$$\therefore S = \lambda \sum_{i=1}^{r} \prod_{k=1}^{i} \left(\frac{p_{k-1,k}}{1-p_{k-1,k-1}} \right) \frac{p_{i,o}}{(1-p_{i,i})} \quad \text{with } p_{0,0} = 0, and \quad p_{0,1} = 1 \quad (7)$$

Minimization of Scrap

The value of an expression can be minimized by minimizing the values of its individual terms. The value of a term can be minimized by minimizing its numerator or by maximizing its denominator or both.

Thus the amount of scrap which is given by the equation (7), can be minimized in two ways as:

(i) By maximizing the denominator in each term i.e. by maximizing the value of $(1-p_{i,i})$ $(1\le i\le r)$, or by minimizing the term $p_{i,i}(1\le i\le r)$. This shows that the scrap can be minimized by installing the machines rated with minimum reprocessing rate. Figure 4, 5, 6 show the variations of Scrap with the variation of reprocessing (feedback) probabilities at the first, second, and third workstations respectively. From the figures it is clear that the scrap amount increases as the there is an increase in feedback probability.

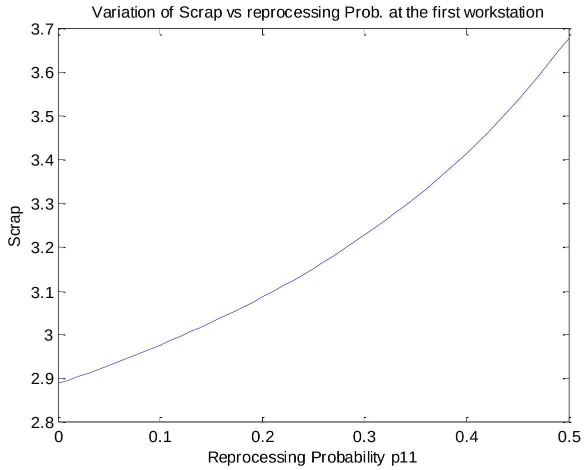

Figure 4: Variation of Scrap with the Variation of Feedback Probability at the First Workstation

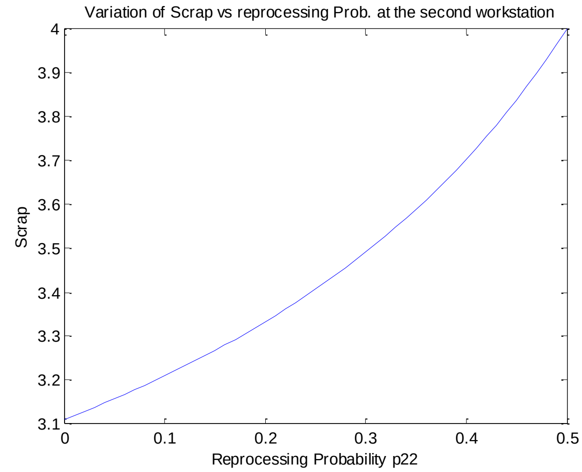

Figure 5: Variation of Scrap with the Variation of Feedback Probability at the Second Workstation

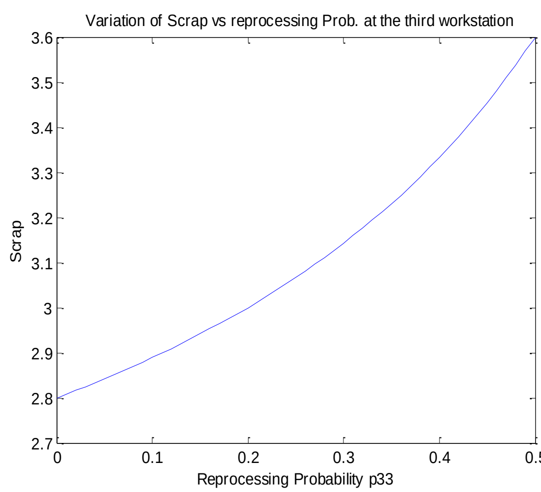

Figure 6: Variation of Scrap with the Variation of Feedback Probability at the Third Workstation

Figure 7: Variation of Scrap with the Variation of Rejection Probability at the First Workstation

(ii) Scrap can be minimized by minimizing the numerator in each term i.e. minimizing the values of $p_{i,o}$ $(1\le i\le r)$. This shows that for minimizing the scrap, machines rated with minimum rejection rate must be installed at the workstations in the production line. Figures 7, 8, and 9 shows the variation of scrap with the variation of rejection probabilities at the first, second, and the third workstations respectively. These figures show that the scrap amount varies linearly with the rejection probability.

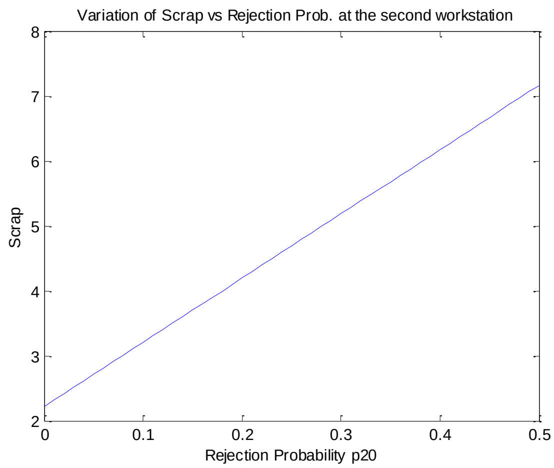

Figure 8: Variation of Scrap with the Variation of Rejection Probability at the Second Workstation

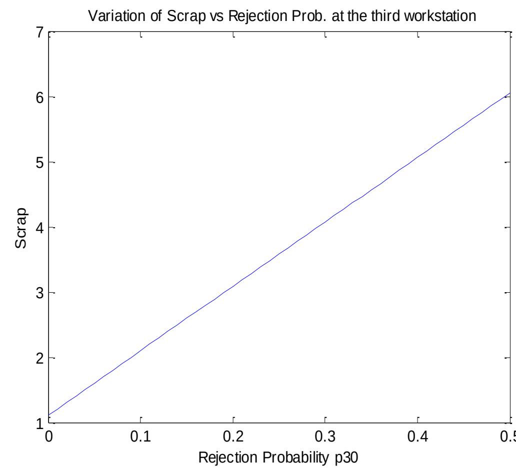

Figure 9: Variation of Scrap with the Variation of Rejection Probability at the Third Workstation

Conclusion

We have framed a policy for minimizing the amount of scrap obtained from a production line due to ill processing of material/jobs/work parts at the workstations. Our policy can be applied for minimizing the scrap in a production line. This can be used to design a production system. One can predict the amount of scrap caused by ill processing. Work can further be extended by incorporating the quality check times at the workstation.

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