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Research Article

ANISOTROPIC BIANCHI TYPE-III COSMOLOGICAL MODELS IN GENERAL RELATIVITY

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ABSTRACT
In this paper we have obtained exact solutions of the field equations for Bianchi type-III space times with variable gravitational constant G(t) and cosmological constant $\Lambda(t)$ in the presence of perfect fluid. We have discussed physical behavior of the model in detail. Also the model satisfies a Machian cosmological solution, i.e. $G_{\epsilon} \sim H^2$ which follows from the model of Kalligas *et al.*, (1992).

Keywords: *Bianchi Type-III, Gravitational Constant, Cosmological Constant, Hubble parameter.*

INTRODUCTION

The Einstein field equations have two parameters, the gravitational constant G and the cosmological constant Λ. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein field equation. In an evolving universe, it is natural to look at this constant as a function of time. Dirac (1937, 1937, 1938, 1975) suggested a possible time varying gravitational constant. The large number hypothesis proposed by Dirac leads to a cosmology where G varies with time. Many other extensions of Einstein theory with time-dependent G have also been proposed by Hoyle and Narlikar (1964), Canuto *et al.*, (1977a, 1977b) Dersarkissian (1985). The Λ term arises naturally in general relativistic quantum field theory where it is interpreted as the energy density of the vacuum (Zeldovich, 1967, 1968; Ginzburg *et al.*, 1971; Fulling *et al.*, 1974). The Λ term has also been interpreted in terms of the Higgs scalar field by Bergmann. 1968 Dreitlan1974 suggested that the mass of the Higgs boson is connected with Λ, and Linde (1974) proposed that Λ is a function of temperature and is related to the process of broken symmetries.

Recently, several models with the Friedman-Robertson-Walker (FRW) metric where G and Λ are the functions of the time have been studied. For these models, the energy-momentum tensor is described as a perfect fluid (Abdel-Rahman, 1992; Berman, 1991; Abdussattar and Vishwakarma, 1977, 1995, 1996). Also Arbab (1997) discussed the bulk viscous models. Beesham (1986a, 1986b) and Kilinc (2006) discussed the Bianchi type-I model with variables G and Λ. Wang (2003, 2004, 2005, 2006) studied the Bianchi type-III model with bulk viscosity.

In this paper, we consider space-time of the Bianchi type III model in a general form with variable G and Λ. We apply the equation of state $p = \omega\varepsilon$ and scalar of expansion proportional to the shear scalar $\theta \propto \sigma$.

Model and Field Equations

We consider the Bianchi type-III metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\propto x} dy^2 + C^2 dz^2, \tag{1}$$

Where A, B and C are the function of cosmic time t alone, and $\propto$ is a constant.

Einstein's field equations with variables G and Λ in suitable units are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} + \Lambda g_{ij} \tag{2}$$

The energy momentum tensor for a perfect fluid is

$$T_{ij} = (\varepsilon + p)v_i v_j + pg_{ij}, \tag{3}$$

Where ε is the energy density of cosmic matter and p is its pressure.
Einstein's field equation (2) for the metric (1) leads to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G(t)p + \Lambda(t), \tag{4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi Gp + \Lambda, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\propto^2}{A^2} = -8\pi Gp + \Lambda, \tag{6}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\propto^2}{A^2} = 8\pi G \in + \Lambda, \tag{7}$$

$$\propto \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \tag{8}$$

where dots on A, B and C denote the ordinary differentiation with respect to t.
In view of the vanishing divergence of the Einstein tensor,
Eq. (2) gives

$$\varepsilon + (\varepsilon + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \varepsilon\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0 \tag{9}$$

We now assume that the law of conservation of energy $\left(T^{ij}_{;j} = 0\right)$ gives

$$\varepsilon + (\varepsilon + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{10}$$

Using Eq. (9) yields

$$\dot{G} = -\frac{\dot{\Lambda}}{8\pi\varepsilon}, \tag{11}$$

indicating that G increases or decreases as Λ decrease or increases. We also consider the perfect fluid equation of state,

$$p = \omega\varepsilon \tag{12}$$

where ω suggested by Wang (2003) may be defined by

$$\omega = \frac{1}{3}\frac{\varepsilon_r}{\varepsilon_m + \varepsilon_r} \tag{13}$$

with $\varepsilon = \varepsilon_m + \varepsilon_r, \varepsilon_m \; and \; \varepsilon_r$ being the matter crest mass and radiation energy densities. As the variation of ω (t) is slow as compared with the expansion of the universe, except near the time when matter and radiation energy densities are equal, we can approximate $\omega(t)$ as a step function:

$$\omega \cong \begin{cases} 1/3, & in \, the \, radiation \, do\min ated \, (RD) \, universe, \\ 0, & in \, the \, matter \, do\min ated \, (MD) \, universe \end{cases} \tag{14}$$

From Eq. (9), we have

$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}(\text{sin}ce\ \alpha\neq 0), \tag{15}$$

which leads to

$$A=k_1 B, \tag{16}$$

where k_1 is a constant of integration.
From Eqs. (4) and (6), using Eq. (16), we have

$$\frac{\dot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=\frac{\alpha^2}{k_1^2 B^2} \tag{17}$$

Solution of the Field Equations

There are only six independent equations in the seven unknowns A, B, C, $p, \varepsilon,$ G and Λ, an extra equation is needed to solve the system completely. We assume that scalar of expansion is proportional to the shear scalar $\theta \propto \sigma,$ which leads to a relation between metric potential

$$B=C^2 \tag{18}$$

.Using Eqs. (18) and (17), we have

$$\frac{\ddot{C}}{C}+4\frac{\dot{C}^2}{C^2}=\frac{\lambda^2}{k_1^2 C^4},\ k_1\neq 0, \tag{19}$$

Integrating Eq. (19), we obtain

$$\dot{C}=\sqrt{\frac{\propto}{3k_1^2 c^3+k_3}} \tag{20}$$

where k_3 is constant of integration. With the help of Eqs. (16), (18) and (20), the line element (1) reduces to

$$ds^2=-\left[\frac{k_1^2 3C^3+k_3}{\propto^2}\right]dC^2+k_1^2 C^4 dx^2$$
$$+C^4 e^{-2\alpha x}dy^2+C^2 dz^2 \tag{21}$$

By suitable transformation of coordinates, the line element (21) reduces to

$$ds^2=-\left[\frac{3T^3}{\alpha^2}\right]dT^2+k_1^2 T^4 dx^2=0$$
$$+T^4 e^{-2\alpha x}dy^2+T^2 dz^2 \tag{22}$$

For the model (22) the physical and geometrical parameters can be easily obtained. The expressions for the energy density $\in,$ gravitational constant G(t), and cosmological constant Λ(t) are given by

$$\in=\frac{k_4}{T^{5(w+1)}},\ k_4=\text{const}, \tag{23}$$

$$G(t)=\frac{T^{3(\omega+1)}}{8\pi k_4\alpha^2(\omega+1)}. \qquad [k_1^2-18T^{2n-1}] \tag{24}$$

$$\Lambda(t)=\frac{1}{(\omega+1)T^2\alpha^2}\left[k_1^2 T^3(6)(5+2\omega)+4(\omega+1)\right]$$

$$+\frac{4}{T^2\alpha^2}\left[k_1^2(3)T^3+k_3\right]-\frac{\alpha^2}{k_1^2T^4} \tag{25}$$

The expansion scalar θ and shear σ for the model (22) are

$$\theta=\frac{(5)\left[k_1^2(3)T^3+k_3\right]^{1/2}}{\alpha T} \tag{26}$$

$$\sigma=\frac{(2)\left[k_1^2(3)T^3+k_3\right]^{1/2}}{\sqrt{3}\,T\alpha} \tag{27}$$

For $\in>0,$ we require $k_4>0.$ The model has singularity at T = 0. The model starts with $\rho,\theta,\sigma,\Lambda$ all being infinite and continues to expand till $T=\infty$. For this model, the scale factors are zero at T = 0, which shows that the space time exhibits point type singularity. Gravitational constant G (t) is zero initially and gradually increases and tends to infinity at late times. Since $\frac{\sigma}{\theta}$ = constant, the model does not approach isotropy for large values of T. Therefore, the model describes a continuously expanding, shearing, non-rotating universe with the big-bang start. In this model we observe that the cosmological term Λ is infinite initially, gradually decreases, and becomes zero at late times.

In the special case of $k_3 = 0$, from Eq. (20) the line element (1) reduces to

$$ds^2=-dT^2+\frac{4}{3}dx^2$$

$$+\frac{4\alpha^2T^2}{3\alpha_1^2}e^{-2\alpha x}dy^2+\left(\frac{2\alpha T}{k_1\sqrt{3}}\right)^{2/n}dz^2 \tag{28}$$

After suitable transformation Eq. (28) reduces to

$$ds^2=-dT^2+T^2dx^2+T^2e^{-2\alpha x}dy^2+T\ dz^2 \tag{29}$$

The physical and gravitational parameters of the model (29) are

$$\in=\frac{k_5}{T^{\frac{5}{2}}},\ k_5=\text{const}, \tag{30}$$

$$G(t)=\frac{1}{4\pi(\omega+1)k_2}T^{\frac{(5)\omega+1}{2}} \tag{31}$$

$$\Lambda(t)=\left\{\frac{(5)\omega+1}{4(\omega+1)}\right\}\frac{1}{T^2} \tag{32}$$

The shear σ and expansion scalar θ are given by

$$\sigma=\frac{1}{2\sqrt{3}T^2} \tag{33}$$

$$\theta=\frac{5}{T^2} \tag{34}$$

Since $\frac{\sigma}{\theta}=const,$ the model does not approach isotropy. We can obtain the deceleration parameter $q^2=\frac{1}{5},$ which shows that q is constant. The model of constant deceleration parameter has been considered by Berman and Som (1990). The Hubble parameter H(T) reads

$$\mathrm{H(T)}=\frac{5}{4\mathrm{T}}, \tag{35}$$

Which can be rewritten as

$$\mathrm{H}=\frac{1}{6\mathrm{qT}} \tag{36}$$

For the present phase p,

$$\mathrm{T_p}=\frac{1}{6\mathrm{q_p H_p}} \tag{37}$$

It is evident that negative q_p would increases the present age of the universe. From Eq. (31), we obtain

$$\frac{\dot{\mathrm{G}}}{\mathrm{G}}=\frac{5}{2}\frac{1}{\mathrm{T}} \tag{38}$$

and the present value is

$$\left(\frac{\dot{\mathrm{G}}}{\mathrm{G}}\right)_{\mathrm{p}}=\frac{6\mathrm{n}[(5)\omega+1]}{\mathrm{n}(2)}\mathrm{q_p}\ \ \mathrm{H_p} \tag{39}$$

We can find that the quantity $G_\in$ satisfies the condition for a Machian cosmological solution i.e. $G_\in \sim H^2,$ which follows from the model of Kalligas *et al.*, (1992).

For the energy density to be positive definite, we must have $k_5>0.$ The energy density decreases as time increases and tends to zero as T tends to infinity.

We also observe that the spatial volume is zero at T = 0. Thus, the singularity exists at T = 0 in the model. The gravitational constant G(t) is zero initially and gradually increases and tends to infinity at late times provided $n>0,$ where as cosmological term $\Lambda(t)$ varies as square of the age of universe and tends to zero as $T\rightarrow\infty.$ Deceleration parameter is constant for all time. For $n=1, T_0H_0=1.$ This is within the current limits for the universe age $0.8< T_0H_0<1.3$ and in good agreement with the best estimation $T_0H_0\approx 1.$ (Abdussattar and Vishwakarma, 1997).

CONCLUSION

In summary, we have obtained exact solutions of the field equations for Bianchi type-III space times with variable gravitational constant G (t) and cosmological constant $\Lambda(t)$ in the presence of perfect fluid. In general, the space time exhibits point type singularity at initial stage and gravitational constant is zero but cosmological term varies as square of the age of universe. Cosmological term Λ is infinite at the beginning of the model and it decreases to become zero at late times. Deceleration parameter is constant for all time. Also the model satisfies a Machian cosmological solution, i.e. $G_\in \sim H^2$ which follows from the model of Kalligas *et al.*, (1992).

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