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Research Article

ANISOTROPIC LRS MODEL IN GENERAL RELATIVITY

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ABSTRACT
We have investigated the LRS Bianchi type-I cosmological model containing stiff matter with cosmological term proportional to Hubble parameter suggested by Schűtzhold (2002a) on the basis of quantum field estimation of the vacuum density in an expanding background. We have discussed physical behavior of the model in detail. The model asymptotically tends to a deSitter universe. Gravitational constant G is constant initially and increases exponentially with time increases. Which is similar result of Abdussattar and Vishwakarma (1997).

Keywords: *LRS Bianchi Type-I, Gravitational Constant, Cosmological Constant, Hubble Parameter*

INTRODUCTION
For simplification and description of large scale behavior of the actual universe, locally rotationally symmetric (LRS) Bianchi-I space time has been widely studied. In order to study problems like the formation of galaxies and the process of homogenization and isotropization of the universe, it is necessary to study problems relating to in homogenous and anisotropic space-time. Mazumdar (1994) has obtained cosmological solutions for LRS Bianchi-I space time filled with a perfect fluid with arbitrary cosmic scale functions and studied kinematical properties of particular form of the solution. Hajj – Boutros (1985) presented a generating technique which converts known LRS Bianchi-I perfect fluid solutions into new solution of same type.
Hajj and Sfeila (1987) and Ram (1989) also obtained some solutions for the same field equations by using solution generation technique. LRS Bianchi type-I space-time has been widely studied by many researches like Pradhan *et al.*, (2001); Pradhan and Vishwakarma (2004); Charkraborty and Pradhan, (2001); Singh (2009); Akarsu and Kilinc (2010).
Some authors have argued for dependence $\Lambda \sim t^{-2}$, See, e.g. Endo and Fukui (1977); Canuto *et al.*, (1977); Lau (1985); Berman (1991a, 1991b). Keeping in mind the dimensional consideration in the spirit of quantum cosmology, Chen and Wu (1990) considered Λ varying as R^{-2}, Carvalho and Lima (1992) generalized it by taking $\Lambda = \alpha R^{-2} + \beta H^2$, where R is the scale factor of Robertson-Walker metric, H is Hubble parameter and α and β are adjustable dimension less parameters. On the basis of quantum field estimation in the curved expanding background. Schűtzhold (2002a, 2002b) recently proposed a vacuum density proportional to Hubble parameter this leads to a vacuum energy density decaying as $\Lambda \approx m^3 H$.
The idea of gravitational constant G in the framework of general relativity was first proposed by Dirac (1937). A lot of work have been done by Saha (2005, 2006a, 2006b) in studying FRW models and Bianchi type-I cosmological model in general relativity with varying G and Λ.
Singh and Tiwari (2008) and Tiwari (2008, 2009, 2010, 2011) have studied perfect fluid Bianchi type-I model with variable G and Λ by taking different conditions for Λ. Recently Tiwari (2011) considered as cosmological term is proportional to Hubble parameter in Bianchi type-I model with varying G and Lambda.
In this paper we study LRS Bianchi type-I model with variable G and Λ. We obtain solution of Einstein equations assuming cosmological term proportional to Hubble parameter for stiff matter.

Model and Field Equations

The spatially homogeneous and anisotropic LRS Bianchi type-I space time is described by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)[dy^2 + dz^2] \tag{1}$$

The spatial volume of model is given by

$$V = AB^2 \tag{2}$$

We define $R = \left(AB^2\right)^{1/3}$ as the average scale factor so that Hubble parameters is defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \tag{3}$$

Here and elsewhere a dot stands for ordinary time derivative of the concerned quantity.

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{4}$$

where $H_1 = \frac{\dot{A}}{A},\ H_2 = \frac{\dot{B}}{B}\ and\ H_3 = \frac{\dot{C}}{C}$ are directional Hubble factor in the direction of X, Y and Z respectively. We assume the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)\upsilon_i\upsilon_j + pg_{ij} \tag{5}$$

where ρ, p are energy density, thermo dynamical pressure and υ_i is the four velocity vector of the fluid satisfying $\upsilon_i \upsilon^i = -1$.

We assume that matter content obey an equation of state

$$p = \omega\rho, \quad o \le \omega \le 1 \tag{6}$$

The Einstein's field equations with time dependent G and Λ is given by

$$R_{ij} - \frac{1}{2} R_l^l g_{ij} = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij} \tag{7}$$

For the metric (1) and energy momentum tensor (5) in co-moving system of coordinates, the equation (7) yields

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -8\pi Gp + \Lambda \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi Gp + \Lambda \tag{9}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi G\rho + \Lambda \tag{10}$$

In view of vanishing of divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0 \tag{11}$$

The usual energy conservation equation $T_{i,j}^{j} = 0$, yields

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right)=0 \tag{12}$$

Equation (11) together with (12) gives

$$8\pi\rho\dot{G}+\dot{\Lambda}=0 \tag{13}$$

Implying that Λ is constant whenever G is constant.

$$\rho=\frac{k_1}{(R^3)^{1+\omega}} \tag{14}$$

where k_1 = constant >0

The non-vanishing components of shear tensor σ_{ij} defined by $\sigma_{ij}=u_{i;j}+u_{j;i}-\frac{2}{3}g_{ij}u;_k^k$.
Thus the shear scalar σ is obtained as

$$\sigma=\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right) \tag{15}$$

From (3) and (15) we obtain

$$\frac{\dot{\sigma}}{\sigma}=-\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right)=-3H \tag{16}$$

Einstein's field equations (8-10) can be also written in terms of Hubble parameter H, shear scalar σ and declaration parameter q as:

$$H^2(2q-1)-\sigma^2=8\pi Gp-\Lambda \tag{17}$$

$$3H^2-\sigma^2=8\pi G\rho+\Lambda \tag{18}$$

where

$$q=-1-\frac{\dot{H}}{H^2}=-\frac{R\ddot{R}}{\dot{R}^2} \tag{19}$$

Thus we have,

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_2}{AB^2} \tag{20}$$

where k_2 is constant of integration, from (20) we obtain

$$\frac{3\sigma^2}{\theta^2}=1-\frac{24\pi G\rho}{\theta^2}-\frac{3\Lambda}{\theta^2} \tag{21}$$

Implying that $\Lambda\geq 0$

$$\theta<\frac{\sigma^2}{\theta^2}<\frac{1}{3},\ 0<\frac{8\pi G\rho}{\theta^2}<\frac{1}{3}$$

Thus the presence of positive Λ lowers the upper limit of anisotropy whereas a negative Λ gives more room for anisotropy.
Equation (21) can also be written as

$$\frac{\sigma^2}{3H^2}=1-\frac{8\pi G\rho}{3H^2}-\frac{\Lambda}{3H^2}=1-\frac{\rho}{\rho_c}-\frac{\rho_\upsilon}{\rho_c} \tag{22}$$

where $\rho_c = \frac{3H^2}{8\pi G}$ is critical density and $\rho_\upsilon = \Lambda / 8\pi G$ is the vacuum density.
Form (21) and (22) we get

$$\frac{d\theta}{dt} = -12\pi G\rho - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G\left(\rho + p\right) - 3\sigma^2 \tag{23}$$

Showing that the rate of volume expansion decreases during time evolution and presence of positive Λ slows down the rate of this decrease whereas a negative Λ would promote it. Thus we get

$$\Lambda = (2-q)H^2 - 4\pi(1-\omega)G\rho \tag{24}$$

Implying $\Lambda \le 0 \;\; and \;\; q \ge 2$

Solution of the Field Equations

The system of equations (8)-(10) supply only five equations in six unknowns $(A, B, \rho, p, G \, and \, \Lambda)$ one extra equation is needed to solve the system completely, for this purpose we take a cosmological term is proportional to Hubble parameter. This variation law for vacuum density has initially proposed by Schűtzhold (2002b). Recently Borges and Carnerio (2005) have considered a cosmological term proportional to H. Thus we take the decaying vacuum energy density

$$\Lambda = aH \tag{25}$$

Where a is positive constant of order m^3, let

$$\Omega = \Lambda / \rho$$

be the ratio between the vacuum and matter densities.

$$a = \frac{3H\Omega}{(8\pi G + \Omega)}\left(1 - 3\sigma^2/\theta^2\right) \tag{26}$$

Thus the value of a in anisotropic background is smaller in comparison to its value in isotropic space Borges and Carnerio (2005). From current value of H, $\Omega \;\; and \;\; \sigma/\theta,$ precise value of a can be obtained.
For stiff fluid $(\omega = 1)$ leads to differential equation

$$\dot{H} + 3H^2 - aH = 0 \tag{27}$$

Determining the time evolution of Hubble parameter integrating (27), we get

$$H = \frac{a}{3\left(1 - e^{-at}\right)} \tag{28}$$

From (28) we obtain scale factor

$$R^3 = m\left(e^{at} - 1\right) \tag{29}$$

Where m is constant of integration, the metric (1) become in from

$$ds^2 = -dt^2 + m^{2/3}\left(e^{at} - 1\right)^{2/3}\left[\left(1 - e^{-at}\right)^{4k_2/3am} dx^2 + \left(1 - e^{-at}\right)^{-2k_2/3am}\left(dy^2 + dz^2\right)\right] \tag{30}$$

For the model (30) matter density $\rho,$ pressure p, gravitational constant G, cosmological term Λ are given by

$$\rho = p = \frac{k_1}{m^2\left(e^{at} - 1\right)^2} \tag{31}$$

$$G=\frac{1}{24\pi k_1}\left[-a^2m^2e^{2at}-k_2^2+2a^2m^2e^{at}+\frac{a^2m^2\left(e^{at}-1\right)^2}{3\left(1-e^{at}\right)}\right] \tag{32}$$

$$\Lambda=\frac{a^2}{3\left(1-e^{-at}\right)} \tag{33}$$

Expansion scalar θ *and* σ are given by

$$\theta=\frac{a}{\left(1-e^{-at}\right)} \tag{34}$$

$$\sigma=\frac{1}{\sqrt{3}}\left[\frac{k_2}{3m\left(e^{at}-1\right)}\right] \tag{35}$$

The ratio between the vacuum matter and densities are given by

$$\Omega=\frac{\Lambda}{\rho}=\frac{a^2m^2\left(e^{at}-1\right)e^{at}}{3k} \tag{36}$$

$$a=\frac{3H\Omega}{8\pi G+\Omega}\left(\frac{1-\sigma^2}{3H^2}\right) \tag{37}$$

Thus the presence of anisotropy lowers the upper limit a from the observed current values of H, σ, G *and* Ω, a precise value of a can be obtained. The declaration parameter q for the model is

$$q=-1+3e^{-at}=1-9H/a \tag{38}$$

The vacuum energy density ρ_υ and critical density ρ_c are given by

$$\rho_c=\frac{a^2k_1}{\left(1-e^{-at}\right)^2}\frac{}{\left[-a^2m^2e^{2at}+k_2^2+2a^2m^2e^{at}+\frac{a^2m^2\left(e^{at}-1\right)^2}{3\left(1-e^{-at}\right)}\right]} \tag{39}$$

$$\rho_\upsilon=\frac{3a^2k_1}{\left(1-e^{-at}\right)}\frac{}{\left[-a^2m^2e^{2at}+k_2^2+2a^2m^2e^{at}+\frac{a^2m^2\left(e^{at}-1\right)^2}{3\left(1-e^{-at}\right)}\right]} \tag{40}$$

$$\rho=\frac{k}{mR^6} \tag{41}$$

$$\Lambda=\frac{a^2}{3\left[1-\left(1-R^3/m\right)^{-1}\right]} \tag{42}$$

For model (1) we observe that the spatial volume V is zero at t = 0 and expansion scalar θ is infinite at t=0. Pressure, energy density, Hubble factor, shears scalar and cosmological term diverses at initial singularity.

The universe model matter density varies at R^{-6} in accordance with standard model. Whereas vacuum density decays as R^{-3}.we obtain from $\rho\approx 0$ *and* $\Lambda=a^{2/3}$

CONCLUSION

We have investigated the LRS Bianchi type-I cosmological model containing stiff matter with cosmological term proportional to Hubble parameter suggested by Schűtzhold (2002a); on the basis of quantum field estimation of the vacuum density in an expanding background. We have found that cosmological term Λ being very large at initial times relaxes to a genuine cosmological constant at the late time, which is accordance with the observations. The model asymptotically tends to a deSitter universe. Gravitational constant G is constant initially and increases exponentially with time increases. Which is similar result of abdussattar and Vishwakarma (1997).

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