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Research Article

BOUNDARY LAYER FLOW OF A NANOFLUID PAST A LINEAR AND EXPONENTIAL STRETCHING SHEET-A COMPARATIVE STUDY

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ABSTRACT
The present work considers the problem of steady laminar 2D boundary layer flow, and heat transfer of nanofluids with comparative study concerned to linear stretching sheet (LSS) as well as exponential stretching sheet (ESS) respectively. The governing boundary value problems with their usual boundary conditions are solved numerically. The BVP which is in the form of P.D.E's has been converted into nonlinear ordinary differential equations with the usage of a similarity transformation. The transformed form of coupled higher order non-linear ordinary differential equations with the boundary conditions are numerically solved by using fourth order Runge-Kutta method, along with shooting technique. The velocity, temperature and nanoparticle concentration profiles are analysed for both LSS and ESS, considering their effects on the involved parameters namely, Prandtl number Pr, Lewis number Le, Brownian motion parameter Nb and thermophoresis parameter Nt. The variation of the reduced Nusselt number and reduced Sherwood numbers with Nb and Nt for various values of Pr and Le is provided in tabular and graphical forms. In ESS the value of reduced Nusselt number and reduced Sherwood number is found to be quantitatively higher (greater) compared to LSS. In both the cases of LSS and ESS it is found that the reduced Nusselt number is a decreasing function, while the reduced Sherwood number is an increasing function for each of the dimensionless parameters Pr, Le, Nb and Nt considered. A comparative study connecting the previously published results and the present results, are found to be in excellent agreement

Keywords: *Nanofluid; Stretching Sheet; Brownian Motion; Thermophoresis; Heat Transfer; Similarity Transformation; Boundary Layer Flow; Linear Stretching Sheet (LSS); Exponential Stretching Sheet (ESS)*

INTRODUCTION
During the last many years, the study of boundary layer flow and heat transfer over a stretching sheet has achieved a lot of success because of its large number of applications in industry and technology. Few of these applications are materials manufactured by polymer extrusion, drawing of copper wires, continuous stretching of plastic films, artificial fibres, hot rolling, wire drawing, glass fibre, metal extrusion and metal spinning etc. The boundary layer flow and heat transfer due to nanofluids over a stretching sheet are thrust areas of current research. Such investigations find applications over a broad spectrum of science and engineering disciplines. An important aspect of boundary layer flow of a nanofluid over a stretching sheet is the heat transfer characteristics. It is crucial to understand the heat transfer characteristics of the stretching sheet so that the finished product meets the desired quality. This is due to the fact that the quality of a final product mainly depends on the rate of heat transfer and the stretching rate.
Subsequent to the pioneering work by Sakiadis (1961), a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surfaces. Thereafter several researchers have extensively considered to study the various aspects of boundary layer flow and heat transfer problems over linear as well as nonlinear stretching sheet, (see Liu, 2004; Khan *et al.*, 2003; Cortell, 2006; Dandapat *et al.*, 2007; Nadeem *et al.*, 2010; Bachok and Ishak, 2010; Bachok *et al.*, 2010; Bachok *et al.*, 2011; Bachok *et al.*, 2011). Accordingly, Kuznestov and Nield (2010) have studied the natural convective boundary-layer flow of a nanofluid past a vertical plate analytically. They used a model in which Brownian motion and thermophoresis effects were taken into account. Ibrahim and Shanker (2012) have studied the boundary-layer flow and heat transfer of nanofluid

over a vertical plate taking into account the convective surface boundary condition. Further, Makinde and Aziz (2011) conducted a numerical study of boundary layer flow of a nanofluid past a stretching sheet with convective boundary condition. Mustafa *et al.*, (2011) investigated stagnation point flow of a nanofluid towards a stretching sheet. Khan and Pop (2010) studied the boundary layer flow of a nanofluid past a stretching sheet with a constant surface temperature.

Recently, Aminreza *et al.*, (2012) investigated the effect of partial slip condition on the flow and heat transfer of nanofluids past stretching sheet, with prescribed constant wall temperature. This problem is solved by using Runge-Kutta-Fehlberg scheme with shooting method. They indicated that the reduced Nusselt number and Sherwood number are strongly influenced by the velocity slip parameter. Wubshet and Bandari (2013) investigated the boundary layer flow and heat transfer over a permeable stretching sheet considering a nano fluid with the effect of magnetic field, slip boundary condition and thermal radiation. It seems that Magyari and Keller (1999) were the first to consider the boundary layer flow and heat transfer over exponentially stretching sheet. Bidin and Nazar (2009), Ishak (2011) and Nadeem *et al.*, (2010, 2011) numerically examined the flow and heat transfer over an exponentially stretching surface with thermal radiation. Elbashbeshy (2001) numerically examined the flow and heat transfer over an exponentially stretching surface considering wall mass suction. Sanjayanand and Khan (2005) studied the visco-elastic boundary layer flow and heat transfer due to and exponentially stretching sheet. Ishak (2011) studied the MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. Singh and Agarwal (2012) explained the effects of heat transfer for two types of viscoelastic fluids over and exponentially stretching sheet with thermal conductivity and radiation in porous medium

Thus motivated by the above mentioned investigations and applications of linear and exponential stretching sheets, we contemplate to fulfil this gap (i.e comparative study) by considering both LSS and ESS at a time. The present work involves the problem of steady laminar two dimensional boundary layer flow and heat transfer of nanofluids past a LSS and ESS, a comparative study.

Mathematical Formulation

We consider a steady, incompressible, laminar, two dimensional boundary layer flow of a viscous nanofluid past a flat stretching sheet (LSS and ESS) coinciding with the plane y=0 and the flow being confined to y>0.The flow is generated due to stretching of the sheet caused by the simultaneous application of two equal and opposite force along the x-axis. Keeping the origin fixed, the sheet is then stretched with a velocity $u_w(x) = ax$ (in case of LSS), where "a" is constant and x is the coordinate measured along the linear stretching surface and $u = u_w(x) = U_0 \exp(x/L)$ (in case of ESS), where U_0 is the reference velocity, L is the reference length and x is the coordinate measured along the stretching surface varying exponentially with the distance from the slit as shown in Figure 1.

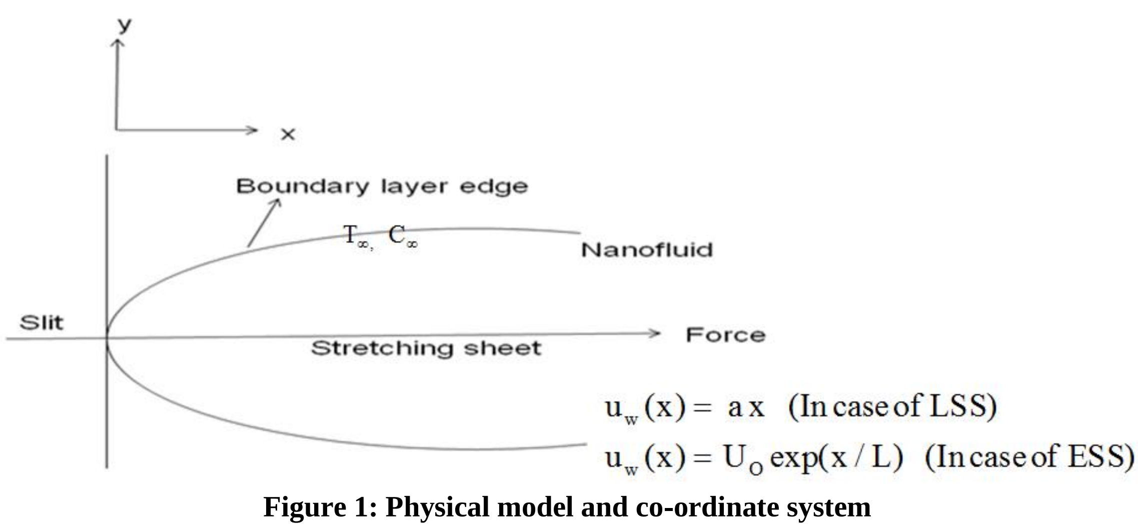

Figure 1: Physical model and co-ordinate system

It is assumed that at the stretching surface, the temperature T and the nanoparticles fraction C take constant values T_w and C_w respectively.

When y tends to infinity, the ambient values of temperature T and nanoparticles fraction C are denoted by T_∞ and C_∞ respectively. It is assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium.

The basic steady conservation of mass, momentum, thermal energy and nanoparticles equations for nanofluids can be written in Cartesian co-ordinates x and y as, see Kuznetsov and Nield ()[30-31]. In both the cases of LSS and ESS the flow and heat transfer characteristics under the boundary layer approximations are governed by the following equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial y}\right)^2\right]\right\} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The boundary conditions in both the cases of LSS and ESS are

$$v=0,\quad u=u_w(x),\quad T=T_w,\quad C=C_w,\quad \text{at}\quad y=0 \tag{5}$$

$$u=v=0,\ T=T_\infty,\quad C=C_\infty,\ \text{as}\ y\to\infty \tag{6}$$

Where u and v are the velocity components along x and y axis respectively, $\upsilon=\mu/\rho_f$ is the kinematic viscosity, $\alpha=\kappa/(c\rho)_f$ is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient and $\tau=(\rho a)_p/(\rho a)_f$ is the ratio between the effective heat capacity of the nanoparticles material and heat capacity of the nano fluid.

T is the temperature inside the boundary layer, T_∞ is the temperature far away from the sheet. In case of LSS: $u_w(x)=ax$ is the stretching velocity of the sheet, $T_w=T_\infty+b(x/l)^2$ is the temperature of stretching surface, $C_w=C_\infty+c(x/l)^2$ is nanoparticles volume fraction at the stretching surface. In case of ESS: $u_w(x)=U_o\exp(x/L)$ is the stretching velocity of the sheet, $T_w=T_\infty+b\exp(x/2L)$ is the temperature of stretching surface, $C_w=C_\infty+c\exp(x/2L)$ is nanoparticles volume fraction at the stretching surface.

We are interested in similarity solution of the above boundary value problem therefore we introduce the following similarity transformations (dimensionless quantities).

In Case of LSS

$$\eta=\left(\frac{a}{\upsilon}\right)^{1/2}y,\quad \psi=(a\upsilon)^{1/2}x\,f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

$$u=axf'(\eta),\quad v=-\left(\frac{a\mu}{\rho}\right)^{\frac{1}{2}}f(\eta) \tag{7}$$

In Case of ESS

$$\eta = y\sqrt{\frac{U_0}{2\nu L}}\exp(x/2L)\,,\ \psi = \sqrt{2\nu L U_0}\exp(x/2L)f(\eta),\quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

$$u = U_0\exp(x/L)f^{'}(\eta),\quad v = -\sqrt{\frac{\nu U_0}{2L}}\exp(x/2L)\{f(\eta) + \eta f^{'}(\eta)\ \} \tag{8}$$

In eqns (7) and (8), f denotes the non-dimensional stream function, the prime denotes differentiation with respect to η and the stream function ψ is defined in the usual way as $u = \partial\psi/\partial y,\ v = -\partial\psi/\partial x$. Making use of transformations (7) and (8) in (1), we can realize incompressibility condition (i.e. continuity equation) is identically satisfied and the governing eqns (2) - (4) takes the form of non-linear ordinary differential equations:

In Case of LSS

$$f''' + ff'' - f'^2 = 0 \tag{9}$$

$$\theta'' + \Pr f' + \Pr Nb\phi'\theta' + \Pr Nt\theta'^2 = 0 \tag{10}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta' = 0 \tag{11}$$

In Case of ESS

$$f''' + ff'' - f'^2 = 0 \tag{12}$$

$$\theta'' + \Pr f' + \Pr Nb\phi'\theta' + \Pr Nt\theta'^2 = 0 \tag{13}$$

$$\phi'' + Le(f\phi' - f'\phi) + \frac{Nt}{Nb}\theta' = 0 \tag{14}$$

The Boundary Conditions in Both the Cases of LSS and ESS are

$$f(0) = 0,\ f'(0) = 1, \theta(0) = 1, \phi(0) = 1\ at\ \eta = 0$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0,\ as\ \eta \to \infty \tag{15}$$

Where f, θ, and ϕ are dimensionless velocity, temperature and nanoparticles concentration, respectively. η is the similarity variable, the prime denote differentiation with respect to η and the governing parameters appearing in eqs (9) to(14) are defined by

$$\left.\begin{aligned}
&\Pr = \frac{\upsilon}{\alpha} && \to \text{Prandtl number}\\
&Le = \frac{\upsilon}{D_B} && \to \text{Lewis number}\\
&Nb = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f \upsilon} && \to \text{Brownian motion parameter}\\
&Nt = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f T_\infty \upsilon} && \to \text{Thermophoresis parameter}
\end{aligned}\right\} \tag{16}$$

It is important to note that this boundary value problem reduces to the classical problem of flow and heat and mass transfer due to a stretching surface in a viscous fluid when Nb and Nt are zero in eqs. (10)-(11)in case of LSS and in eqs. (13)-(14) in case of ESS. (The boundary value problem for ϕ then becomes ill-posed and is of no physical significance).

The important physical quantities of interest in this problem are local Skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x are defined as**:**

$$C_f = -\frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \tag{17}$$

Where wall shear stress τ_w, wall heat flux q_w, mass flux q_m are given by**:**

$$\tau_w = \rho\nu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \quad q_m = -D_B\left(\frac{\partial \phi}{\partial y}\right)_{y=0} \tag{18}$$

In Case of LSS

By solving eqs. (17) using eqs. (7),(18).we get

$$C_f\sqrt{Re_x} = -f''(0), \qquad \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0) = Shr, \qquad \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0) = Nur \tag{19}$$

Where C_f, $Nu_x(Nur)$, $Sh_x(Shr)$, Re_x are the skin friction, local Nusselt number, local Sherwood number and local Reynolds number respectively.

In Case of ESS

By solving eqs. (17) using eqs. (8),(18).We get

$$C_f\sqrt{2Re_x} = -f''(0), \qquad \sqrt{\frac{2}{X}}\left(\frac{Nu_x}{\sqrt{Re_x}}\right) = -\theta'(0) = Nur, \qquad \sqrt{\frac{2}{X}}\left(\frac{Sh_x}{\sqrt{Re_x}}\right) = -\phi'(0) = Shr \tag{20}$$

Where X=x/L is dimensionless coordinate along the sheet, L is the length of the sheet, C_f, $Nu_x(Nur)$, $Sh_x(Shr)$, Re_x are the skin friction, local Nusselt number, local Sherwood number and Reynolds number respectively.

Numerical Solution

An efficient fourth order Runge-Kutta method along with Shooting technique has been employed to study the flow model of the above coupled non-linear ordinary differential equations (9)-(11) and (12)-(14) for different values of governing parameters viz. Prandtl number Pr, Lewis parameter Le, Brownian motion parameter Nb and thermophoresis parameter Nt. The non-linear differential equations are first decomposed into a system of first order differential equations. The coupled ordinary differential eqs.(9)-(11) and (12)-(14) are third order in f and second order in θ and ϕ which have been reduced to a system of seven simultaneous equations for seven unknowns. In order to numerically solve this system of equations using Runge-Kutta method, the solutions requires seven initial conditions but two initial conditions in f one initial condition in each of θ and ϕ are known. However, the values of f, θ and ϕ are known at $\eta \to \infty$.These end conditions are utilized to produce unknown initial conditions at $\eta = 0$ by Shooting technique. The most important step of this scheme is to choose the appropriate finite value of η_∞.Thus to estimate the value of η_∞, we start with some initial guess value and solve the boundary value problem consisting of Eqs. (9)-(11) and (12)-(14) to obtain $f''(0), \theta'(0)$ and $\phi'(0)$.The solution process is repeated with another larger value of η_∞ until two successive values of $f''(0), \theta'(0)$ and $\phi'(0)$ differ only after desired significant digit. The last value η_∞ is taken as the finite value of the limit η_∞ for the particular set of physical parameters for determining velocity, temperature, and concentration, respectively are $f(0), \theta(0)$ and $\phi((0)$ in the boundary layer. After getting all the initial conditions we solve this system of simultaneous equations using fourth order Runge-Kutta integration scheme. The value of η_∞ is selected to vary from 5 to 20 depending on the physical parameters governing the flow so that no numerical oscillation would occur.

In this study, the boundary value problem is first converted into an initial value problem (IVP).Then the IVP is solved by appropriately guessing the missing initial value using the shooting method for several sets of parameters. The step size h=0.1 is used for the computational purpose**.**The error tolerance of 10^{-6} is also being used. The results obtained are presented through tables and graphs, and the main features of the problems are discussed and analyzed.

RESULTS AND DISCUSSION

The numerical solutions are obtained for temperature and concentration profiles for different values of governing parameters. The obtained results are displayed through graphs Figures 2-9, for temperature and concentration profiles respectively.

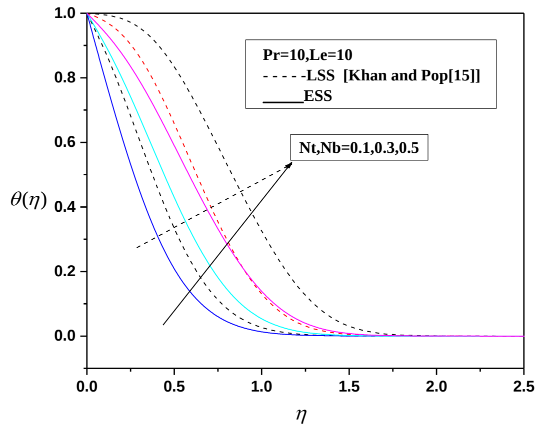

Figure 2: Effects of Nt and Nb on temperature profiles

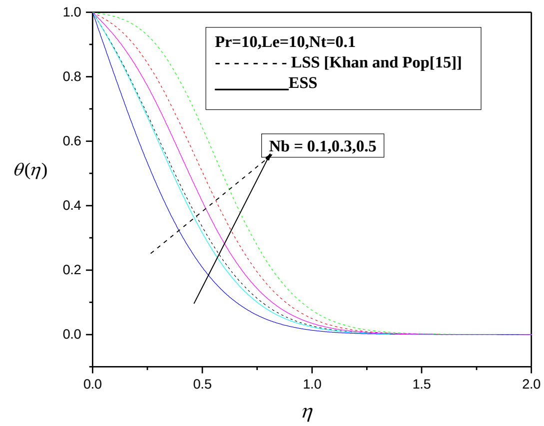

Figure 3: Effects of Nb on temperature profiles

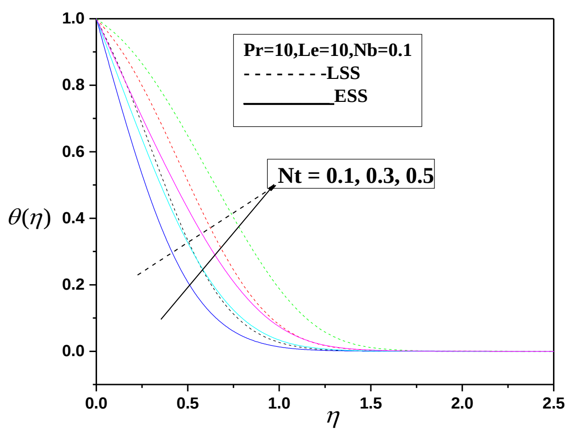

Figure 4: Effects of Nt on temperature profiles

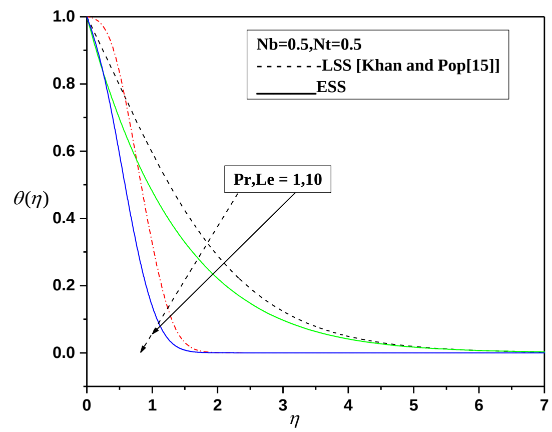

Figure 5: Effects of Pr and Le on temperature profiles

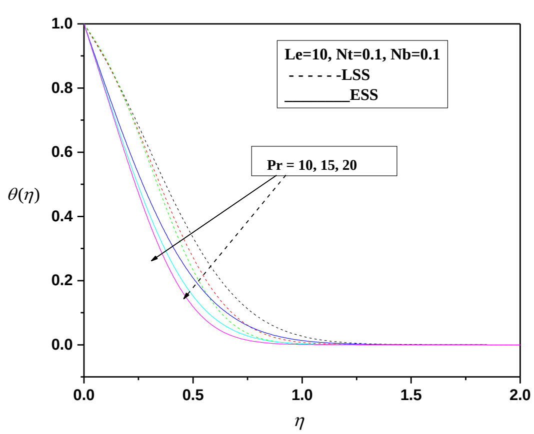

Figure 5: Effects of Pr number on temperature profiles

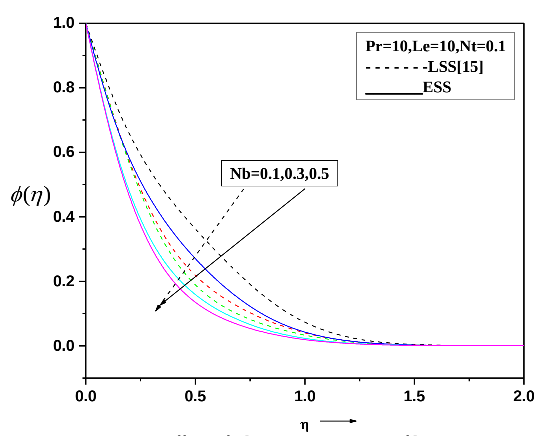

Figure 5: Effects of Nb on concentration profiles

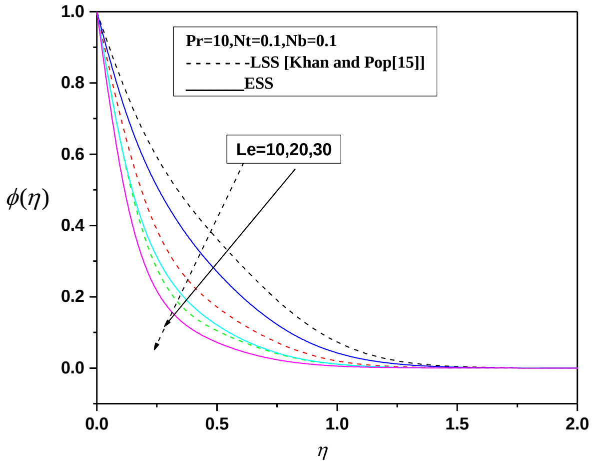

Figure 8: Effects of Le on concentration profiles

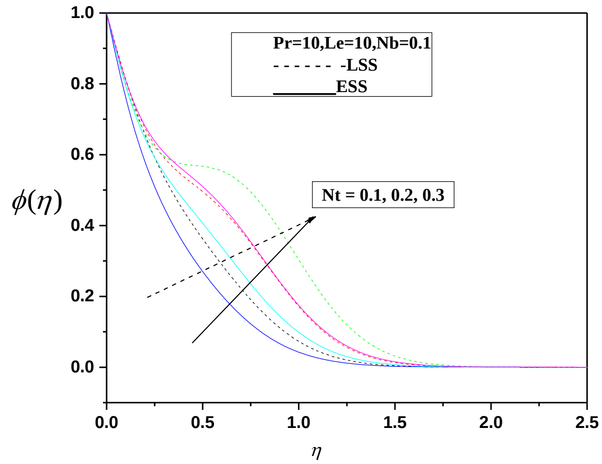

Figure 9: Effects of Nt on concentration profiles

Figures 2, 3 and 4 shows the effects of Nt and Nb parameters for the selected values of Pr and Le numbers and for the LSS and ESS cases respectively. As expected, the boundary layer profiles for the temperature function $\theta(\eta)$ are essentially the same form as in the case of a regular fluid. In both the cases of LSS and ESS, it is observed that the temperature increases as the parameters (figure 2, Nt, Nb=0.1, 0.3, 0.5),(figure 3, Nb=0.1,0.2,0.3) and (figure 4, Nt=0.1,0.3,0.5) increases, which results in thickening of thermal boundary layer thickness of the fluid. It is noticed that the temperature (heat fraction) and thermal boundary layer thickness of the fluid increases more in case of LSS compared to ESS. **In all the graphs where ever we are getting the LSS, we have compared present work with those reported by Khan and Pop (2010)**

Figures 5 and 6 shows the effects of Pr and Le numbers on the temperature profiles for the selected values of Nb and Nt parameters and for both the LSS and ESS cases respectively. In both the cases of LSS and ESS, it is observed that the temperature decrease as the parameters in (figure 5, Pr, Le=1, 10) and (figure 6, Pr=10, 15, 20) increases, which results in thinning of thermal boundary layer thickness of the fluid. It is noticed that the temperature (heat fraction) and thermal boundary layer thickness of the fluid decreases more in case of ESS compared to LSS.

Figures 7, 8 and 9 shows the effects of Nb, Le and Nt parameters on the concentration profiles for the selected values of other parameters and for the LSS and ESS cases respectively. In both the cases of LSS and ESS, it is observed that the concentration decrease as the parameters (figure 7, Nb=0.1, 0.3, 0.5), (figure 8, Le=10, 20, 30) increases, while concentration increases as the parameter (figure 9, Nt=0.1, 0.2, 0.3) increases. In figures 7 and 8 which results in thinning of concentration boundary layer thickness of the fluid. It is noticed that the concentration (mass fraction) and concentration boundary layer thickness of the fluid decreases more in case of ESS compared to LSS. Whereas figure 9 which results in thickening of concentration boundary layer thickness of the nano fluid. It is seen that the concentration profile and concentration boundary layer thickness of the fluid increases more in case LSS compared to ESS.

Finally, a comparison with published papers available in the literature has been done in order to check the accuracy of the present results. Table 1 compares results for the local Nusselt number $-\theta'(0)$ and local Sherwood number $-\phi'(0)$ with Nt and Nb for Le=Pr=10 obtained in the present work, are in good agreement with the results reported by Khan and Pop (2010) and Aminreza *et al.*, (2012).

From table 2, it shows a test of accuracy of the solution, the values of local Nusselt number $-\theta'(0)$ for different values of Prandtl number are compared with solutions reported by Magyari and Keller (1999), Bidin and Nazar (2009), Ishak (2011). The table shows the numerical solution obtained by the present fourth order Runge-Kutta method along with Shooting technique are in very good agreement. Therefore, we are confident that our results are highly accurate to analyze the flow problem.

Table 1: Comparison of results for the local Nusselt number $-\theta'(0)$ and local Sherwood number $-\phi'(0)$ in case of LSS, when Le=Pr=10

Nt	Nb	Khan and Pop (2010) $-\theta'(0)$	Aminreza *et al.*, (2012) $-\theta'(0)$	Present result $-\theta'(0)$	Khan and Pop (2010) $-\phi'(0)$	Aminreza Noghrehabad (2012) $-\phi'(0)$	Present result $-\phi'(0)$
0.1	0.1	0.9524	0.9523768	0.952398	2.1294	2.1293938	2.129346
0.2		0.6932	0.6931743	0.693215	2.2740	2.2740215	2.273857
0.3		0.5201	0.5200790	0.520130	2.5286	2.5286382	2.528362
0.4		0.4026	0.4025808	0.402636	2.7952	2.7951701	2.794799
0.5		0.3211	0.3210543	0.321110	3.0351	3.0351425	3.034698
0.1	0.2	0.5056	0.5055814	0.505589	2.3819	2.3818706	2.381470
	0.3	0.2522	0.2521560	0.252444	2.4100	2.4100188	2.409953
	0.4	0.1194	0.1194059	0.119402	2.3997	2.3996502	2.399450
	0.5	0.0543	0.0542534	0.054253	2.3836	2.3835712	2.383571
0.2	0.3	--	--	0.181881	--	--	2.514821
0.3		--	--	0.135775	--	--	2.608559

Table 2: Comparison of results for the local Nusselt number $-\theta'(0)$ in case of ESS for Nt =Nb=Le=0

Pr	Magyari and Keller (1999)	Bidin and Nazar (2009)	Anur Ishak (2011)	Present Results
1.0	0.954782	0.9548	0.9548	0.951556
2.0	--------	1.4714	1.4715	1.465304
3.0	1.869075	1.8691	1.8691	1.859997
5.0	2.500135	-------	2.5001	2.485222
10.0	3.660379	-------	3.6604	3.630831

Nomenclature

LSS linear stretching sheet
ESS exponential stretching sheet
a,b,c constant
c_f skin friction coefficient
u_w is the velocity of the stretching sheet
C_w is nanoparticles volume fraction at the stretching surface
C_∞ ambient nanoparticles volume fraction
D_B brownian diffusion coefficient
D_T thermophoresis diffusion coefficient
$f(\eta)$ dimensionless stream function
κ thermal conductivity
Pr prandtl number
Le lewis number
Nb brownian motion parameter
Nt thermophoresis parameter

Nu_x (Nur)	local Nusselt number
Sh_x (Shr)	local Sherwood number
Re_x	local Reynolds number
T_W	uniform temperature over the surface of the sheet
T_∞	ambient temperature or is the temperature far away from the sheet
T	temperature of the fluid inside the boundary layer
u,v	velocity component along x and y-direction
p	is the fluid pressure

Greek symbols

η	dimensionless similarity variable
μ	dynamic viscosity of the fluid
υ	kinematic viscosity of the fluid
ϕ	dimensionless concentration function
ρ_f	density of the fluid
$(a\rho)_f$	heat capacity of the fluid
$(a\rho)_p$	effective heat capacity of a nano fluid
ψ	stream function
α	thermal diffusivity
θ	dimensionless temperature
τ	parameter defined by $(a\rho)_p/(a\rho)_f$

Subscripts

∞	condition at the free stream
w	condition of the surface

Concluding Remarks

In this paper, effects of Prandtl number (Pr), Lewis number (Le), Brownian motion parameter (Nb), thermophoresis parameter (Nt) on temperature profiles, concentration profiles, local Nusselt number and local Sherwood number, on the boundary layer flow and heat transfer of nanofluids past a linear and exponential stretching sheet is investigated.

In both the cases of LSS and ESS, the variation of the reduced Nusselt number and reduced Sherwood numbers with Nb and Nt for various values of Pr and Le is presented in tabular and graphical forms. The numerical results obtained are in excellent agreement with the previously published data in limiting condition and for some particular cases of the present study. Some of the important findings of our investigations are as mentioned below

Similarity between LSS and ESS

The increase in Brownian motion parameter Nb and thermophorosis parameter Nt is to enhance temperature in the thermal boundary layer which results in reducing temperature at the surface, where as the reverse effect is noticed in case of Pr, Le.

It is established that the reduced Nusselt number is a decreasing function, while the reduced Sherwood number is an increasing function for each of the dimensionless parameters Pr, Le, Nb and Nt considered.

The reduced Sherwood number is an increasing function of higher values of Pr, and a decreasing function of lower values of Pr, while reduced Nusselt number is a decreasing function for lower values of Pr and increasing function for higher values of Pr for each of Le, Nb and Nt.

The reduced Nusselt number is a decreasing function of higher values of Le and an increasing function of lower values of Le, while reduced Sherwood number is an increasing function of higher values of Le and decreasing function of lower values of Le, for each of M Pr, Nb, and Nt.

Differences between LSS and ESS:

In LSS the value of reduced Nusselt number and reduced Sherwood number is found to be quantitatively lower (smaller) compared to ESS.

In case of ESS, it is noticed that the reduced Nusselt number is a decreasing function, while reduced Sherwood number is an increasing function for (Nb=0.1 to Nb=0.5 keeping Nt =0.1, 0.2, 0.3, 0.4, 0.5 fixed) and initially decreasing function for (Nt=0.1 to Nt=0.5 keeping Nb=0.1,0.2 fixed) afterwards increasing function for (Nt=0.1 to 0.5 for Nb=0.3, 0.4, 0.5 fixed) and for each of the dimensionless parameters Pr, Le, Nb and Nt considered.

In figure 12(b), Shr vs Nt: In case of LSS as we increase Nb=0.3, 0.4, 0.5 the reduced Sherwood number decreases where as reverse effects is noticed in case of ESS for the selected values of Pr=10, Le=10.

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