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Research Article

BIANCHI TYPE III DARK ENERGY MODEL IN $f(R,T)$ GRAVITY

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ABSTRACT
In this paper we investigate Bianchi type III dark energy model with equation of state (EoS) parameter filled with perfect fluid in a scalar tensor theory of gravitation proposed by Harko *et al.,* To obtain a determinate solution, a special law of variation proposed by Berman is used. We have also assumed that the scalar expansion is proportional to shear and the EoS parameter is proportional to skewness parameter. It is been observed that EoS parameter and skewness parameter are function of time. Some physical and kinematical behaviours of the cosmological model are also discussed.

Keywords: *Bianchi Type III,* $f(R,T)$ *Gravity, Dark Energy, Constant Deceleration Parameter*

INTRODUCTION
Observations on large scale structures (Scranton *et al.,* 2004) and cosmic microwave background radiation (Bennett *et al.,* 2003; Caldwell and Kamionkowski, 2009; Eardley *et al.,* 1992; Spergel *et al.,* 2003; Spergel *et al.,* 2007) indicates that universe is spatially flat and there exit an exotic cosmic fluid called Dark energy with negative pressure, which accounts about 70% of the total energetic content of the universe. It is puzzling phenomenon that the most abundant form of matter-energy in the universe is mysterious (Peebles and Rathra, 2003). Several models have been proposed to explain dark energy (Cardone *et al.,* 2005; Peebles and Rathra, 1988; Padmanabhan, 2003; Rathra and Peebles, 1988; Tortora and Demianski, 2005). The dark energy is a prime candidate for explaining the recent cosmic observations. In view of the late time acceleration of the universe and the existence of dark energy and dark matter, several modified theories of gravity have been developed and studied. Noteworthy amongst them are $f(R)$ theory of gravity formulated by Nojiri and Odintsov (2003) and $f(R,T)$ theory of gravity proposed by Harko *et al.*, (2011).

Harko *et al.*, (2011) developed $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor. The dependence of T may be introduced by exotic imperfect fluids or quantum effects. They have obtained the field equations from a Hilbert-Einstein type variational principal and also obtained the covariant divergence of the stress-energy tensor. They have derived some particular models corresponding to specific choices of the function $f(R,T)$. They have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of the function $f(T)$.

Paul *et al.,* (2009) obtained accelerating universe in modified theory of gravity. Shamir (2010) obtained exact vacuum solutions of bianchi type-I and type-V cosmological models in $f(R,T)$ gravity.

Rao *et al.,* (2011) have discussed anisotropic Bianchi type I universe with cosmic strings and bulk viscosity in a scalar-tensor theory of gravitation. Reddy *et al.,* (2012), presented Bianchi type III and Kaluza-Klein cosmological model in $f(R,T)$ gravity.

Several authors (Adhav, 2012; Chaubey and Shukla, 2013; Naidu *et al.,* 2012; Naidu *et al.,* 2013) studied cosmological model with $f(R,T)$.

Motivated by above discussion, in this paper we investigate Bianchi type III dark energy cosmological model, with EoS parameter, in $f(R,T)$ gravity choosing an appropriate form of $f(T)$ proposed by Harko *et al.,* (2011).

Metric and Field Equations

We consider the Bianchi type-III space time in the form

$$ds^2 = dt^2 - A^2 dx^2 - e^{-2mx} B^2 dy^2 - C^2 dz^2 \tag{1}$$

where m is a constant and A, B and C are function of cosmic time t.

The energy momentum tensor for anisotropic dark energy is given by

$$T_i^{\,j} = diag\left[\rho, -p_x, -p_y, -p_z\right]$$
$$= diag\left[1, -\omega_x, -\omega_y, -\omega_z\right]\rho$$
$$= diag\left[1, -\omega, -(\omega+\delta), -(\omega+\gamma)\right]\rho \tag{2}$$

where ρ is the energy density of fluid, p_x, p_y and p_z are the pressure and ω_x, ω_y and ω_z are the directional EoS parameters along the x, y and z axis respectively. ω is the deviation-free EoS parameter of the dark energy and one can parameterized the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameters δ and γ that are deviation from ω along y and z axis respectively.

The field equation in $f(R,T)$ theory of gravity for the function $f(R,T) = R + f(T)$ when the matter source is a perfect fluid is given by (Harko *et al.*, 2011)

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2 f'(T) T_{ij} + [2 p f'(T) + f(T)] g_{ij} \tag{3}$$

where $T_{ij} = (\rho + p) u_i u_j - p g_{ij}$ (4)

We choose the function $f(T)$ of the trace of the energy tensor of the matter so that

$$f(T) = \lambda T, \tag{5}$$

where λ is a constant.

Now with the help of (2), (4) and (5), the field equations (3) for the metric (1) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \rho\left[(8\pi + 2\lambda)\omega - \lambda(1 - 3\omega - \delta - \gamma)\right] - 2p\lambda \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \rho\left[(8\pi + 2\lambda)(\omega + \delta) - \lambda(1 - 3\omega - \delta - \gamma)\right] - 2p\lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \rho\left[(8\pi + 2\lambda)(\omega + \gamma) - \lambda(1 - 3\omega - \delta - \gamma)\right] - 2p\lambda \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -\rho\left[(8\pi + 2\lambda) + \lambda(1 - 3\omega - \delta - \gamma)\right] - 2p\lambda \tag{9}$$

$$m\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{10}$$

where an overhead dot $(\cdot)$ denotes differentiation with respect to time.

Integrating equation (10), we obtain

$$A = k_1 B \tag{11}$$

where k_1 is positive constant of integration and without loss of generality we take $k_1 = 1$.

Using equation (11) in equation (7) and subtract the result from equation (6), we obtain that the skewness parameter on y-axis is null i.e.

$$\delta = 0 \tag{12}$$

Thus system of equations from (6) - (9) reduces to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=\rho\left[(8\pi+2\lambda)\omega-\lambda(1-3\omega-\gamma)\right]-2p\lambda \quad (13)$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{m^2}{B^2}=\rho\left[(8\pi+2\lambda)(\omega+\gamma)-\lambda(1-3\omega-\gamma)\right]-2p\lambda \quad (14)$$

$$2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}-\frac{m^2}{B^2}=-\rho\left[(8\pi+2\lambda)+\lambda(1-3\omega-\gamma)\right]-2p\lambda \quad (15)$$

Solution of the Field Equations

The average scale factor (R), spatial volume (V), Hubble parameter (H), scalar expansion (θ) shear scalar (σ) and mean anisotropic parameter (A_m) are given by

$$R^3=ABC \quad (16)$$

$$V=R^3=ABC \quad (17)$$

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \quad (18)$$

$$\theta=3H \quad (19)$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-\frac{\theta^2}{3}\right) \quad (20)$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \quad (21)$$

We have three linearly independent equations (13) - (15) with six unknown parameters. Hence to find deterministic solution we use the following conditions,

(i) The shear (σ) is proportional to scalar expansion (θ) which gives

$$B=C^n \quad (22)$$

where $n>1$ is a constant.

(ii) Variation of Hubble parameter proposed by Berman (1983) that yields constant deceleration parameter models of the universe defined by

$$q=-\frac{R\ddot{R}}{\dot{R}^2} \quad (23)$$

which admits the solution

$$R=(at+b)^{1/1+q} \quad (24)$$

where $a\neq 0$ and b are the constants of integration and $1+q>0$ for accelerating expansion of the universe.

(ii) The EoS parameter ω is proportional to skewness parameter γ such that

$$\omega+\gamma=0 \quad (25)$$

Now from equations (11), (16), (22) and (24) we obtain

$$A=B=(at+b)^{3n/(1+q)(2n+1)} \quad (26)$$

$$C=(at+b)^{3/(1+q)(2n+1)} \quad (27)$$

Hence the metric (1) with the suitable choice of coordinates and constants can be written as

$$ds^2 = dT^2 - T^{6n/(1+q)(2n+1)}\left(dX^2 + e^{-2mX}dY^2\right) - T^{6/(1+q)(2n+1)}dZ^2 \tag{28}$$

Some Physical Properties of the Model

Equation (28) represents the Bianchi type III dark energy model in $f(R,T)$ gravity with the following physical and kinematical parameters which plays a vital role in the discussion of cosmology:

Spatial volume

$$V = R^3 = T^{3/1+q} \tag{29}$$

Scalar of expansion

$$\theta = 3H = \frac{3a}{(1+q)T} \tag{30}$$

The generalized Hubble parameter

$$H = \frac{a}{(1+q)T} \tag{31}$$

The shear scalar

$$\sigma^2 = \frac{3a^2(n-1)^2}{(1+q)^2(2n+1)^2T^2} \tag{32}$$

The mean anisotropic parameter

$$A_m = \frac{2(n-1)^2}{(2n+1)^2} \tag{33}$$

It is observed that at initial epoch $(T=0)$, the spatial volume will be zero. For large value of T, the spatial volume tends to infinity. The volume increases as time increases, i.e., the model is expanding. It is also observed that expansion scalar (θ), shear scalar (σ) and Hubble parameter (H) becomes zero for large value of T and they tends to infinity when $T \to 0$.

Using equations (14), (15) and (25), the energy density is

$$\rho = -p = \frac{6na^2\left[(1-2q)n-(q+4)\right]}{(8\pi+2\lambda)(1+q)^2(2n+1)^2}\frac{1}{T^2} \tag{34}$$

With the help of equations (13), (15) and (34), the EoS and Skewness parameter are

$$\omega = -\gamma = -1 + \frac{1}{\rho(8\pi+2\lambda)}\left\{\frac{3a^2(1+n)\left[3n-(1+q)(2n+1)\right]+9a^2\left[1-2n-n^2\right]}{(1+q)^2(2n+1)^2}\frac{1}{T^2} + \frac{m^2}{T^{6n/(1+q)(2n+1)}}\right\} \tag{35}$$

CONCLUSION

In this paper we have studied anisotropic Bianchi type III dark energy models with variable EoS parameter ω in $f(R,T)$ gravity in the presence of perfect fluid. To obtain a determinate solution of the highly non-linear field equations of this theory, we have taken the help of special law of variation for Hubble parameter proposed by Berman (1983). It has been observed that EoS parameter and skewness parameter are function of time t. The EoS parameter ω tends to infinity as $T \to 0$. Since $\frac{\sigma}{\theta} \neq 0$, the models do not approach isotropy at any time. However the model become isotropic for $n = 1$.

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