International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2014 Vol. 4 (4) October-December, pp. 34-42/Khanna and Banyal
Research Article

LIMITATIONS TO THE GROWTH RATE OF PERTURBATIONS IN MAGNETO-ROTATORY THERMAL INSTABILITY IN COUPLE-STRESS FLUID

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ABSTRACT
The thermal instability of a couple-stress fluid acted upon by uniform vertical magnetic field and rotation heated from below is investigated. Following the linearized stability theory and normal mode analysis, the paper through mathematical analysis of the governing equations of couple-stress fluid convection with a uniform vertical magnetic field and rotation, for the case of rigid boundaries shows that the complex growth rate σ of oscillatory perturbations, neutral or unstable for all wave numbers, must lie inside a semi-circle

$$\sigma_r^{\,2} + \sigma_i^{\,2} \langle Q^2 \left[1 - \frac{T_A}{\pi^4 (1 + \pi^2 F)} \right]^{-2} ,$$

in the right half of a complex σ-plane, where Q is the Chandrasekhar number, T_A is the Taylor number, and F is the couple-stress parameter, which prescribes the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in a rotatory couple-stress fluid heated from below.
Key Words: Thermal convection; Couple-Stress Fluid; Rotation; Magnetic Field; PES; Chandrasekhar number; Taylor number.

INTRODUCTION

The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics etc. A detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar (1981). The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Sharma et al (1976) has considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics. The fluid has been considered to be Newtonian in all above studies. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stokes (1966) proposed and postulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. According to the theory of Stokes (1966), couple-stresses are found to appear in noticeable magnitude in fluids having very large molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluid, Walicki and Walicka(1999) modeled synovial fluid as couple-stress fluid in human joints. An electrically conducting couple-stress fluid heated from below in porous medium in the presence of uniform horizontal magnetic field has been studied by Sharma and Sharma (2001). Sharma and Thakur (2000) have studied the thermal convection in couple-stress fluid in porous medium in hydromagnetics. Sharma and Sharma (2004) and Kumar and Kumar (2011) have studied the effect of dust particles, magnetic field and rotation on couple-stress fluid heated from below and for the case of stationary convection, found that dust particles have destabilizing effect on the system, where as the rotation is found to have stabilizing effect on the system, however couple-stress and magnetic field are found to have both stabilizing and destabilizing effects under certain conditions.

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However, in all above studies the case of two free boundaries which is a little bit artificial except the stellar atmospheric case is considered. Banerjee et al (1981) gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee (1984) established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al.(1986). However no such result existed for non-Newtonian fluid configurations, in general and for couple-stress fluid configurations, in particular. Banyal (2011) and Banyal and Singh (2012, 2013)) have characterized the non-oscillatory motions in couple-stress fluid.

Keeping in mind the importance of non-Newtonian fluids, the present paper is an attempt to prescribe the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude, in a layer of incompressible couple-stress fluid heated from below in the presence of uniform vertical magnetic field and rotation opposite to force field of gravity, when the bounding surfaces are of infinite horizontal extension, at the top and bottom of the fluid are rigid.

Formulation of the Problem and Perturbation Equations

Considered an infinite, horizontal, incompressible couple-stress fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0, ρ_0 respectively and at the upper surface z = d are T_d, ρ_d and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained.

The fluid is acted upon by a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ and uniform vertical magnetic field $\vec{H}(0,0,H)$.. Let ρ, p, T and $\vec{q}(u,v,w)$ denote respectively the density, pressure, temperature and velocity of the fluid. Then the momentum balance, mass balance equations of the couple-stress fluid in the presence of uniform vertical magnetic field and rotation (Stokes (1966) and Chandrasekhar (1981)) are

$$\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=-\nabla\left(\frac{p}{\rho_0}-\frac{1}{2}\left|\vec{\Omega}\times r\right|^2\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}+2\left(\vec{q}\times\vec{\Omega}\right)+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}, \qquad (1)$$

$$\nabla.\vec{q}=0, \qquad (2)$$

$$\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\kappa\nabla^2 T, \qquad (3)$$

$$\nabla.\vec{H}=0, \qquad (4)$$

$$\frac{\partial \vec{H}}{\partial t}=\left(\nabla.\vec{H}\right)\vec{q}+\eta\nabla^2 T, \qquad (5)$$

The equation of state

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right], \qquad (6)$$

Here $\vec{g}(0,0,-g)$ is acceleration due to gravity. The kinematic viscosity ν, the magnetic permeability μ_e, the electrical resistivity η, couple-stress viscosity μ', thermal diffusivity $\kappa=\frac{q}{\rho_0 c_v}$, and coefficient of

thermal expansion α are all assumed to be constants and c_v denote the heat capacity of the fluid at constant volume.
The basic motionless solution is

$$\vec{q} = (0,0,0)\ ,\ T = T_0 - \beta z\ ,\ \vec{\Omega} = (0,0,\Omega)\ \text{and}\ \rho = \rho_0(1+\alpha\beta z). \tag{7}$$

Assume small perturbations around the basic solution state and let $\delta\rho$, δp, θ, $\vec{h}(h_x, h_y, h_z)$ and $\vec{q}(u,v,w)$ denote respectively the perturbations in density ρ, pressure p, temperature T, magnetic field $\vec{H}(0,0,H)$ and perturbation fluid velocity (0,0,0) .
The change in density $\delta\rho$ caused mainly by the perturbation θ in temperature is given by

$$\delta\rho = -\alpha\rho_0\theta\ , \tag{8}$$

Then the linearized perturbation equations of the couple-stress fluid becomes

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}\,\alpha\theta + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} + 2\left(\vec{q}\times\vec{\Omega}\right) + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}\ , \tag{9}$$

$$\nabla.\vec{q} = 0\ , \tag{10}$$

$$\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta\ , \tag{11}$$

$$\nabla.\vec{h} = 0\ , \tag{12}$$

$$\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}.\nabla\right)\vec{q} + \eta\nabla^2\vec{h}\ , \tag{13}$$

Within the framework of Boussinesq approximation, equations (9) - (13), gives

$$\frac{\partial}{\partial t}\nabla^2 w = \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^4 w + \frac{\mu_e H}{4\pi\rho_0}\nabla^2\left(\frac{\partial h_z}{\partial z}\right) + g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - 2\Omega\frac{\partial\varsigma}{\partial z}\ , \tag{14}$$

$$\frac{\partial\varsigma}{\partial t} = \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\varsigma + 2\Omega\frac{\partial w}{\partial z} - \frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}\ , \tag{15}$$

$$\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta\ , \tag{16}$$

$$\frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \eta\nabla^2 h_z\ , \tag{17}$$

$$\frac{\partial\xi}{\partial t} = H\frac{\partial\varsigma}{\partial z} + \eta\nabla^2\xi\ , \tag{18}$$

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Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and, $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ and $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ denote the z-component of current density and vorticity, respectively.

Normal Mode Analysis

Analyzing the disturbances into normal modes, we assume that the Perturbation quantities are of the form

$$[w, \theta, \varsigma, h_z, \xi] = [W(z), \Theta(z), Z(z), K(z), X(z)] \text{ Exp}\left(ik_x x + ik_y y + nt\right), \tag{19}$$

Where k_x, k_y are the wave numbers along the x and y-directions respectively $k = \left(k_x^{\ 2} + k_y^{\ 2}\right)^{\frac{1}{2}}$, is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using (19), equations (14) – (18), in non-dimensional form, become

$$\left(D^2 - a^2\right)\left[\sigma + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]W = -Ra^2\Theta - T_A DZ + QD(D^2 - a^2)K, \tag{20}$$

$$\left[\left\{1 - F\left(D^2 - a^2\right)\right\}\left(D^2 - a^2\right) - \sigma\right]Z = -DW - QDX, \tag{21}$$

$$\left(D^2 - a^2 - p_1\sigma\right)\Theta = -W, \tag{22}$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW, \tag{23}$$

$$\left(D^2 - a^2 - p_2\sigma\right)X = -DZ, \tag{24}$$

Where Where we have introduced new coordinates (x', y', z') = (x/d, y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number; $p_2 = \frac{\nu}{\eta}$, is the magnetic Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\mu'/(\rho_0 d^2)}{\nu}$, is the dimensionless couple-stress viscosity parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta\varepsilon}$, is the Chandrasekhar number and $T_A = \frac{4\Omega^2 d^4}{\nu^2\varepsilon^2}$, is the Taylor number. Also we have Substituted $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$, $Z = \frac{2\Omega d}{\nu\varepsilon}Z_\oplus$, $K = \frac{Hd}{\varepsilon\eta}K_\oplus$, $X = \left(\frac{Hd}{\varepsilon\eta}\right)\left(\frac{2\Omega d}{\varepsilon\nu}\right)X_\oplus$ and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the case where both the boundaries are rigid and perfectly conducting and are maintained at constant temperature, and then the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (20)--(24), must possess a solution are

W = DW = 0, $\Theta = 0$, Z=0, K=0 and DX = 0 at z = 0 and z = 1. (25)

An equation (20)--(24), along with boundary conditions (25), poses an eigenvalue problem for σ and we wish to characterize σ_i when $\sigma_r \geq 0$.

We first note that since W and Z satisfy $W(0) = 0 = W(1)$, K(0)=0=K(1) and $Z(0) = 0 = Z(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality (1973)

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz \,;\, \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz \text{ and } \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz, \tag{26}$$

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Further, for $W(0)=0=W(1)$, K(0)=0=K(1) and $Z(0)=0=Z(1)$, Banerjee et al (1992), have show that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz \quad \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2 dz \text{ and } \int_0^1 \left|D^2Z\right|^2 dz \geq \pi^2 \int_0^1 |DZ|^2 dz, \qquad (27)$$

Mathematical Analysis

We prove the following lemmas:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz \leq \frac{1}{p_2|\sigma|}\int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (23) and its complex conjugate, and integrating by parts each term on both sides of the resulting equation for an appropriate number of times and making use of boundary conditions on K namely $K(0)=0=K(1)$ along with appropriate boundary condition (25), we get

$$\int_0^1 \left|\left(D^2-a^2\right)K\right|^2 dz + 2p_2\sigma_r \int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz + p_2^{\,2}|\sigma|^2 \int_0^1 |K|^2 dz = \int_0^1 |DW|^2 dz, \qquad (28)$$

Since $p_2 \rangle 0$ and $\sigma_r \geq 0$, therefore the equation (28) give,

$$\int_0^1 \left|\left(D^2-a^2\right)K\right|^2 dz \langle \int_0^1 |DW|^2 dz, \qquad (29)$$

And

$$\int_0^1 |K|^2 dz \langle \frac{1}{p_2^{\,2}|\sigma|^2}\int_0^1 |DW|^2 dz, \qquad (30)$$

It is easily seen upon using the boundary conditions (25) that

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz = \text{Real part of } \left\{-\int_0^1 K^*\left(D^2-a^2\right)K dz\right\} \leq \left|\int_0^1 K^*\left(D^2-a^2\right)K dz\right|,$$

$$\leq \int_0^1 \left|K^*\left(D^2-a^2\right)K\right| dz \leq \int_0^1 |K^*|\left|\left(D^2-a^2\right)K\right| dz,$$

$$= \int_0^1 |K|\left|\left(D^2-a^2\right)K\right| dz \leq \left\{\int_0^1 |K|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 \left|\left(D^2-a^2\right)K\right|^2 dz\right\}^{\frac{1}{2}},$$

(Utilizing Cauchy-Schwartz-inequality)

Upon utilizing the inequality (29) and (30), we get

$$\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz \leq \frac{1}{p_2|\sigma|}\int_0^1 |DW|^2 dz, \qquad (31)$$

This completes the proof of lemma 1**.**

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz \leq \frac{1}{\pi^4(1+\pi^2 F)}\int_0^1 |DW|^2 dz.$$

Proof: Multiplying equation (21) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (24) and appropriate boundary conditions (25), it follows that

$$\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\}dz + F\int_0^1 \left(|D^2Z|^2 + 2a^2|DZ|^2 + a^4|Z|^2\right)dz$$

$$+\sigma_r \int_0^1 |Z|^2\,dz + Q\int_0^1 \left\{|DX|^2 + (a^2 + p_2\sigma_r)|X|^2\right\}dz = \text{Real part of } \left\{\int_0^1 DWZ^*dz\right\},$$

$$\leq \left|\int_0^1 DWZ^*dz\right| \leq \int_0^1 |DW^*Z|dz \leq \int_0^1 |DW^*||Z|dz,$$

$$= \int_0^1 |DW||Z|dz \leq \left\{\int_0^1 |Z|^2dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2dz\right\}^{\frac{1}{2}}, \quad (32)$$

(Utilizing Cauchy-Schwartz-inequality),

This gives that

$$\int_0^1 |DZ|^2dz \leq \left\{\int_0^1 |Z|^2dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2dz\right\}^{\frac{1}{2}}, \quad (33)$$

Inequality (32) on utilizing inequalities (26) and (33), gives

$$\left\{\int_0^1 |Z|^2dz\right\}^{\frac{1}{2}} \leq \frac{1}{\pi^2}\left\{\int_0^1 |DW|^2dz\right\}^{\frac{1}{2}}, \quad (34)$$

Since $\sigma_r \geq 0$ and $p_2 \rangle 0$, hence inequality (32) on utilizing (34) and (27), give

$$\int_0^1 |Z|^2dz \leq \frac{1}{\pi^4(1+\pi^2F)}\int_0^1 |DW|^2dz, \quad (35)$$

This completes the proof of lemma 2**.**

We prove the following theorem:

Theorem 1: If R$\rangle$ 0 , F$\rangle$ 0, $Q\rangle 0$, $T_A\rangle$ 0, $p_1\rangle 0$, $p_2\rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, K, Z, X) of equations (20) – (24), together with boundary conditions (25) is that

$$\left(\frac{Qp_2\pi^2}{2\pi^2 - 1}\right) + \frac{T_A}{\left\{2\pi^2(1+\pi^2F) - 1\right\}} \rangle 1 .$$

Proof: Multiplying equation (20) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\sigma\int_0 W^*\left(D^2 - a^2\right)Wdz + F\int_0^1 W^*\left(D^2 - a^2\right)^3 Wdz - \int_0^1 W^*\left(D^2 - a^2\right)^2 Wdz$$

$$= -Ra^2\int_0^1 W^*\Theta dz - T_A\int_0^1 W^*DZdz + Q\int_0^1 W^*D(D^2 - a^2)Kdz, \quad (36)$$

Taking complex conjugate on both sides of equation (22), we get

$$\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* = -W^*, \quad (37)$$

Therefore, using (37), we get

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$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* dz\,, \tag{38}$$

Also taking complex conjugate on both sides of equation (21), we get

$$\left[\left\{1 - F\left(D^2 - a^2\right)\right\}\left(D^2 - a^2\right) - \sigma^*\right]Z^* = -DW^* - QDX^*, \tag{39}$$

Therefore, using (39) and (24), we get

$$\int_0^1 W^* DZdz = -\int_0^1 DW^* Zdz = \int_0^1 Z\left\{\left(D^2 - a^2\right) - F\left(D^2 - a^2\right)^2 - \sigma^*\right\}Z^* dz + Q\int_0^1 (D^2 - a^2 - p_2\sigma)X^* dz\,, \tag{40}$$

Also taking complex conjugate on both sides of equation (23), we get

$$\left[D^2 - a^2 - p_2\sigma^*\right]K^* = -DW^*, \tag{41}$$

Therefore, equation (41), using appropriate boundary condition (25), we get

$$\int_0^1 W^* D\left(D^2 - a^2\right)Kdz = -\int_0^1 DW^*\left(D^2 - a^2\right)Kdz = \int_0^1 K\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma^*\right)K^* dz\,, \tag{42}$$

Substituting (38), (40) and (42) in the right hand side of equation (36), we get

$$\sigma\int_0^1 W^*\left(D^2 - a^2\right)Wdz + F\int_0^1 W^*\left(D^2 - a^2\right)^3 Wdz - \int_0^1 W^*\left(D^2 - a^2\right)^2 Wdz$$

$$= Ra^2\int_0^1 \Theta\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* dz - T_A\int_0^1 Z\left\{\left(D^2 - a^2\right) - F\left(D^2 - a^2\right)^2 - \sigma^*\right\}Z^* dz\,, \tag{43}$$

$$+ Q\int_0^1 K(D^2 - a^2)\left(D^2 - a^2 - p_2\sigma^*\right)K^* dz - T_A Q\int_0^1 X(D^2 - a^2 - p_2\sigma)X^* dz$$

Integrating the terms on both sides of equation (43) for an appropriate number of times by making use of the appropriate boundary conditions (25), we get

$$\sigma\int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz + F\int_0^1 \left\{|D^3W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2\right\}dz$$

$$+ \int_0^1 \left\{|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz = Ra^2\int_0^1 \left\{|D\Theta|^2 + a^2|\Theta|^2 + p_1\sigma^*|\Theta|^2\right\}dz - T_A\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\}dz$$

$$- T_A F\int_0^1 \left\{|D^2Z|^2 + 2a^2|DZ|^2 + a^4|Z|^2\right\}dz - T_A\sigma^*\int_0^1 |Z|^2 dz - T_A Q\int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz\,, \tag{44}$$

$$- T_A Q p_2\sigma\int_0^1 |X|^2 dz - Q\int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz - Qp_2\sigma^*\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz$$

And equating imaginary parts on both sides of equation (44), and cancelling $\sigma_i (\neq 0)$ throughout from imaginary part, we get

$$\int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz = -Ra^2 p_1\int_0^1 |\Theta|^2 dz + T_A\int_0^1 |Z|^2 dz + Qp_2\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz - T_A Q p_2\int_0^1 |X|^2 dz\,, \tag{45}$$

Now R $\rangle$ 0, $Q\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (31) and (35), the equation (45) gives,

$$\int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz = -Ra^2 p_1\int_0^1 |\Theta|^2 dz + T_A\int_0^1 |Z|^2 dz + Qp_2\int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz - T_A Q p_2\int_0^1 |X|^2 dz\,, \tag{45}$$

Now R $\rangle$ 0, $Q\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (31) and (35), the equation (45) gives,

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$$\left[1-\frac{T_A}{\pi^4(1+\pi^2F)}-\left(\frac{Q}{|\sigma|}\right)\right]\int_0^1|DW|^2dz+a^2\int_0^1|W|^2dz+Ra^2p_1\int_0^1|\Theta|^2dz+T_AQp_2\int_0^1|X|^2dz\,\langle\,0, \qquad (46)$$

and therefore , we must have

$$|\sigma|\langle Q\left[1-\frac{T_A}{\pi^4(1+\pi^2F)}\right]^{-1}. \qquad (47)$$

Hence, if

$$\sigma_r\geq 0 \text{ and } \sigma_i\neq 0\text{, then } |\sigma|\langle Q\left[1-\frac{T_A}{\pi^4(1+\pi^2F)}\right]^{-1}. \qquad (48)$$

And this completes the proof of the theorem.

Conclusion

The inequality (48) for $\sigma_r \geq 0$ and $\sigma_i \neq 0$, can be written as

$$\sigma_r^{\ 2}+\sigma_i^{\ 2}\langle Q^2\left[1-\frac{T_A}{\pi^4(1+\pi^2F)}\right]^{-2},$$

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of couple-stress fluid of infinite horizontal extension heated form below, having top and bottom bounding surfaces rigid, in the presence of uniform vertical magnetic field and rotation parallel to the force field of gravity, the complex growth rate of an arbitrary oscillatory motions of growing amplitude, must lie inside a semi-circle in the right half of the $\sigma_r\,\sigma_i$ - plane whose centre is at the origin and radius is $Q\left[1-\frac{T_A}{\pi^4(1+\pi^2F)}\right]^{-1}$, where Q is the Chandrasekhar number, T_A is the Taylor number and F is the couple-stress parameter.

REFERENCES

Banerjee MB, Katoch DC, Dube GS and Banerjee K (1981). Bounds for growth rate of perturbation in thermohaline convection. *Proceedings of the Royal Society A* **378** 301-304

Banerjee MB and Banerjee B (1984). A characterization of non-oscillatory motions in magnetohydronamics. *Indian Journal of Pure and Applied Mathematics* **15**(4) 1984, 377-382

Banerjee MB, Gupta JR and Prakash J (1992). On thermohaline convection of Veronis type, *Journal of Mathematical Analysis and Applications* **179** 327-334.

Banyal AS (2011). The necessary condition for the onset of stationary convection in couple-stress fluid, *International Journal of Fluid Mechanics Research* **38**(5) 450-457.

Banyal AS and Singh K (2012). A Characterization of Magneto-convection in Couple-Stress Fluid, *International Journal of Mathematical Sciences and Engineering Applications* **6**(1) 281-289.

Banyal AS and Singh K (2013). Onset of Instability in Magneto-Rotatory Thermal Convection in Couple-Stress Fluid, *International Journal of Physics and Mathematical Sciences* **3**(2) 54-63.

Chandrasekhar S (1981). *Hydrodynamic and Hydromagnetic Stability* (Dover Publication) New York.

Gupta JR, Sood SK and Bhardwaj UD (1986). On the characterization of non-oscillatory motions in rotatory hydromagnetic thermohaline convection, *Indian Journal of Pure and Applied Mathematics* **17**(1) 100-107.

Kumar V and Kumar S (2011). On a couple-stress fluid heated from below in hydromagnetics, *Applications and Applied Mathematics* **05**(10) 1529-1542

Schultz MH (1973). *Spline Analysis* (Prentice Hall) Englewood Cliffs, New Jersey.

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Sharma RC, Prakash K and Dube SN (No Date). Effect of suspended particles on the onset of Bénard convection in hydromagnetics, *Journal of Mathematical Analysis Applications*, USA **60** 227-35.
Sharma RC and Sharma S (2001). On couple-stress fluid heated from below in porous medium, *Indian Journal of Physics* **75B** 59-61.
Sharma RC and Thakur KD (2000). Couple stress-fluids heated from below in hydromagnetics, Czech. *Journal of Physics* **50** 753-58.
Sharma RC and Sharma M (2004). Effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field, *Indian Journal of Pure and Applied Mathematics*, **35**(8) 973-989.
Stokes V K (1981). Couple-stress in fluids, *Physics of Fluids* **9** 1976, 1709-15, 1981.
Walicki E and Walicka A (1999). Inertial effect in the squeeze film of couple-stress fluids in biological bearings, *International Journal of Applied Mechanics and Engineering* **4** 363-73.