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Research Article

SELF-SIMILAR FLOW OF A MIXTURE OF A NON-IDEAL GAS AND SMALL SOLID PARTICLES WITH INCREASING ENERGY BEHIND A SHOCK WAVE UNDER A GRAVITATIONAL FIELD

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ABSTRACT
Similarity solutions are obtained for one-dimensional unsteady adiabatic flow of a dusty gas behind a spherical shock wave with time dependent energy-input under the influence of a gravitational field. The dusty gas is assumed to be a mixture of small solid particles and a non-ideal gas. It is assumed that the viscous stress and heat-conduction of the mixture are negligible. An equation of state of the dusty gas is derived. The shock-Mach number is not infinite, but has a finite value. Effects of a change in the value of the parameter of non-idealness of the gas in the mixture, the mass concentration of the solid particles in the mixture, the ratio of the density of the solid particles to the initial density of the gas and variation of parameter of gravitation are obtained.

Keywords: *Self-similar Solutions, Adiabatic Flow, Dusty Gas*

INTRODUCTION
The study of shock wave in a mixture of a gas and small solid particles is of great importance due to its application to nozzle flow, lunar ash flow, coal-mine explosions, bomb blasts and many other engineering problems (see Pai *et al.*, 1980; Miura and Glass, 1983). Miura and Glass (1985) obtained an anlytic solution for a planer dusty gas flow with constant velocities of the shock and the piston moving behind it. As they neglected the volume occupied by the solid particles mixed into the perfect gas, the dust virtually has a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of the dust upon the shock propagation. Pai *et al.*, (1980) generalized the well known solution of a strong explosion due to an instantaneous release of energy in gas (Sedov, 1959), Korobeinikov (1976)) to the case of two phase flow of a mixture of perfect gas and small solid particles, and brought out the essential effects due to the presence of dusty particles on such a strong shock wave. As they considered the non-zero volume fraction of solid particles in the mixture, their results reflects the influence of both the decrease of mixture's compressibility and the increase of mixture's inertia on the shock propagation (Steiner and Hirschler, 2002; Vishwakarma and Pandey, 2003; Vishwakarma and Nath, 2006).
Carrus *et al.*, (1951) have studied the propagation of shock waves in a gas under the gravitational attraction of a central body of fixed mass (Roche Model) and obtained similarity solutions by numerical methods. Rogers (1957) has discussed a method for obtaining analytical solution of the same problem. Ojha *et al.*, (1998) have discussed the dynamical behaviour of an unstable magnetic star by employing the concept of the Roche Model in an electrically conducting atmosphere. Singh (1982) has studied the self-similar flow of a non-conducting perfect gas, moving under the gravitational attraction of a central body of fixed mass, behind a spherical shock wave driven out by a propelling contact surface into quite solar wind region. Total energy content between the inner expanding surface and the shock front is assumed to be increasing with time.
The assumption that the gas is ideal is no more valid when the flow takes place at extreme conditions. Anisimov and Spiner (1972) have taken an equation of state for low density non-ideal gases in a simplified form, and investigated the effect of parameter for non-idealness on the problem of a strong point explosion.

In the present paper, we therefore investigated the self-similar flow behind a spherical shock wave propagation in a dusty gas, which is a mixture of small solid particles and non-ideal gas. The medium is assumed to be under a gravitational field due to heavy nucleus at the origin (Roche Model). The unsteady model of Roche consists of gas distributed with spherical symmetry around a nucleus having a large mass *m*. It is assumed that the gravitational effect of the mixture itself can be neglected compared with the attraction of the heavy nucleus. The total energy of the flow-field behind the shock is supposed to be increasing with time (Freeman, 1968; Director and Dabora, 1977). This increase can be obtained by the pressure exerted on the mixture by inner expanding surface (Rogers, 1958). In order to obtain the similarity solutions of the problem the density of the undisturbed medium is assumed to be constant. Effects of a change in the value of the parameter of non-idealness of the gas in the mixture $\bar{\alpha}$, the mass concentration of the solid particles in the mixture k_p, the ratio of the density of the solid particles to the initial density of the gas G_1 and variation of parameter of gravitation *h* are obtained.

Fundamental Equations and Boundary Conditions

We consider the medium to be a mixture of small solid particles and a non-ideal gas. The solid particles are continuously distributed in the non-ideal gas and considered as pseudo-fluid. It is assumed that the velocity and temperature equilibrium conditions are maintained in the flow-field. The equation of state of the non-ideal gas in the mixture is taken to be (Anisimov and Spiner, 1972); Ranga and Purohit, 1976; Vishwakarma and Pandey, 2006).

$$p_g = R^* \bar{\rho}_g (1 + \bar{b}\bar{\rho}_g) T, \tag{1}$$

where p_g and $\bar{\rho}_g$ are the partial pressure and partial density of the gas in the mixture, *T* is the temperature of the gas (and of the solid particles as the equilibrium flow condition is maintained), R^* is the gas constant and $\bar{b}$ is the internal volume of the molecules of the gas. In this equation the deviations of an actual gas from the ideal state are taken into account, which result from interaction between its component molecules. It is assumed that the gas is still so rarefied that triple, quadruple, etc., collisions between molecules are negligible, and their interaction is assumed to occur only through binary collisions. The quantity $\bar{b}$ is, in general, a function of temperature *T*, but at high temperature range it tends to a constant value equal to the internal volume of the gas molecules (Anisimov and Spiner, 1972; Landau and Lifshitz, 1958). The effects of dissociation and ionization of gas molecules are assumed to be negligible.

The equation of state of the solid particles in the mixture is, simply,

$$\rho_{sp} = \text{constant}, \tag{2}$$

where ρ_{sp} is the species density of the solid particles. Proceeding on the same lines as in Pai (1977), we obtain the equation of state of the mixture as

$$p = \frac{(1-k_p)}{(1-z)}[1 + \bar{b}\rho(1-k_p)]\rho R^* T, \tag{3}$$

Where p and ρ are the pressure and density of the mixture, $Z = \frac{V_{sp}}{V_m}$ is the volume fraction and $k_p = \frac{M_{sp}}{M_m}$ is the mass fraction (concentration) of the solid particles in the mixture, where M_{sp} and V_{sp} are the total mass and the volumetric extension of the solid particles and V_m and M_m are the total volume and total mass of the mixture.

The relation between k_p and *Z* is given by (Pai, 1977)

$$k_p = \frac{Z\rho_{sp}}{\rho}. \tag{4}$$

In the equilibrium flow, k_p is constant in whole flow-field. Therefore, from (4)

$$\frac{Z}{\rho} = \text{constant} \tag{5}$$

in the whole flow-field. Also, we have the relation (Pai, 1977)

$$Z = \frac{k_p}{G(1-k_p)+k_p}, \tag{6}$$

where $G = \frac{\rho_{sp}}{\rho_g}$ is the ratio of the density of solid particles to the species density of the gas.

The internal energy per unit mass of the mixture may be written as

$$U_m = [k_p C_{sp} + (1-k_p)C_v]T = C_{vm}T, \tag{7}$$

where C_{sp} is the specific heat of solid particles, C_v specific heat of the gas at constant volume and C_{vm} the specific heat of the mixture at constant volume process.

The specific heat of mixture at constant pressure process is

$$C_{pm} = k_p C_{sp} + (1-k_p)C_p, \tag{8}$$

where C_p is the specific heat of the gas at constant pressure process.

The ratio of the specific heats of the mixture is given by (Pai *et al.*, 1980; Pai, 1977; Marble, 1970)

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \gamma \frac{(1+\delta\beta'/\gamma)}{1+\delta\beta'}, \tag{9}$$

where

$$\gamma = \frac{C_p}{C_v}, \quad \delta = \frac{k_p}{(1-k_p)} \quad \text{and} \quad \beta' = \frac{C_{sp}}{C_v}. \tag{10 a-c}$$

Now,

$$C_{pm} - C_{vm} = (1-k_p)(C_p - C_v) = (1-k_p)R^*, \tag{11}$$

neglecting the term containing $\bar{b}^2\rho^2$ (Anisimov and Spiner, 1972). The internal energy per unit mass of the mixture is, therefore, given by

$$U_m = \frac{p(1-Z)}{\rho(\Gamma-1)[1+\bar{b}\rho(1-k_p)]} = \frac{(1-k_p)R^*T}{(\Gamma-1)}. \tag{12}$$

From the first law of thermodynamics and the equation of state (3), we may calculate the speed of sound in the mixture of non-ideal gas and small solid particles, as

$$a = \left(\frac{dp}{d\rho}\right)_S^{1/2} = \left[\frac{\{\Gamma + (2\Gamma - Z)\bar{b}\rho(1-k_p)\}p}{(1-Z)\{1+\bar{b}\rho(1-k_p)\}\rho}\right]^{1/2}, \tag{13}$$

where $\left(\frac{dp}{d\rho}\right)_S$ denotes the derivative of p with respect to ρ at constant entropy *S*.

The compressibility (adiabatic) of the mixture may be calculated as (Moelwyn-Hughes (1961))

$$l = -\rho\left(\frac{d(1/\rho)}{dp}\right)_S = \frac{1}{\rho a^2} = \frac{(1-Z)\{1+\bar{b}\rho(1-k_p)\}}{\{\Gamma + (2\Gamma - Z)\bar{b}\rho(1-k_p)\}p}. \tag{14}$$

The equations of motion for one-dimensional adiabatic unsteady spherically symmetric flow of a mixture of non-ideal gas and small solid particles under the influence of a gravitational field are (Rogers, 1957; Vishwakarma, 2000)

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{2\rho u}{r}=0, \tag{15}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{G^{*}m}{r^{2}}=0, \tag{16}$$

$$\frac{\partial U_m}{\partial t}+u\frac{\partial U_m}{\partial r}-\frac{p}{\rho^2}\left(\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}\right)=0, \tag{17}$$

where u is the flow velocity, r the radial distance, t the time, \$m\$$m$ the mass of heavy nucleus at the centre and G^{*} the gravitational constant. Here, it is assumed that the gravitating effect of the medium itself is negligible in comparison with the attraction of the heavy nucleus.

We consider that a spherical shock wave is propagating into a medium (mixture of small solid particles and non-ideal gas) of constant density ρ_1 which is at rest.

The pressure immediately ahead of the shock front is given by, from equation (16),

$$p_1=\frac{\rho_1 m G^{*}}{R}, \tag{18}$$

where R is the shock radius.

The jump conditions across the moving shock are as follows :

$$u_2=(1-\beta)\dot{R},\ \ \rho_2=\frac{\rho_1}{\beta},\ \ p_2=p_1+(1-\beta)\rho_1\dot{R}^2,\ \ Z_2=\frac{Z_1}{\beta}, \tag{19 a-d}$$

where $\dot{R}\left(=\frac{dR}{dt}\right)$ denotes the shock velocity, and the subscripts "1" and "2" refer to the values just ahead and just behind the shock front. The quantity β is given by the equation

$$\begin{aligned}&\gamma(\gamma-1)^2(\Gamma+1)\{1+\bar{\alpha}(1-k_p)\}\beta^2-[(\gamma-1)^2\{2\Gamma M^{-2}+\gamma(2Z_1+\Gamma-1)\}\\&+2\bar{\alpha}(\gamma-1)^2(1-k_p)(\Gamma M^{-2}+Z_1\gamma)-\gamma\bar{\alpha}^2(\gamma-1)^2(1-k_p)^2(\Gamma-1)]\beta\\&-[\bar{\alpha}(\gamma-1)^2(1-k_p)\{2(\Gamma-Z_1)M^{-2}+\gamma(\Gamma-1)\}\\&+(\Gamma-1)\bar{\alpha}^2(\gamma-1)^2(1-k_p)^2(\gamma+2M^{-2})]=0,\end{aligned} \tag{20}$$

where

$$Z_1=\frac{k_p}{G_1(1-k_p)+k_p}, \tag{21}$$

$\bar{\alpha}=\bar{b}\rho_1$ is the parameter of non-idealness of the gas in the mixture, M is the shock-Mach number referred to the speed of sound in the dust-free ideal gas $\left(\frac{\gamma p_1}{\rho_1}\right)^{1/2}$, and G_1 the ratio of the density of the solid particles to the initial density of the gas.

Also, the relation between M and the effective shock-Mach number M_e is

$$M^2=\frac{M_e^2}{\left[\frac{\gamma(1-Z_1)\{1+\bar{\alpha}(1-k_p)\}}{\{\Gamma+(2\Gamma-Z_1)\bar{\alpha}(1-k_p)\}}\right]}, \tag{22}$$

where M_e is defined by

$$M_e=\frac{\dot{R}}{a_1}=\frac{\dot{R}}{\left[\frac{p_1\{\Gamma+(2\Gamma-Z_1)\bar{\alpha}(1-k_p)\}}{\rho_1(1-Z_1)\{1+\bar{\alpha}(1-k_p)\}}\right]^{1/2}}. \tag{23}$$

The total energy *E* of the flow behind the shock is assumed to be varying as (Rogers, 1958; Freeman 1968; Director and Dabora, 1977)

$$E = E_c t^s, \tag{24}$$

where *s* is a non-negative number and E_c a constant. The positive values of *s* correspond to the class in which the total energy increases with time. This increase can be achieved by the pressure exerted on the fluid by an expanding surface (a contact surface or a piston). This surface may be, physically, the surface of the stellar corona or the condensed explosives or the diaphragm containing a very high-pressure driver gas. By sudden expansion of the stellar corona or the detonation products or the driver gas into the ambient gas, a shock wave is produced in the ambient gas. The shocked gas is separated from this expanding surface which is a contact discontinuity. This contact surface acts as a 'piston' for the shock wave.

Similarity Solutions

Following the general similarity analysis of Sedov (1959), we define two characteristic parameter 'a^*' and 'b' with independent dimensions as

$$[a^*] = [mG^* \rho_1], \tag{25a}$$

$$[b] = [mG^*] = \left[\frac{E_c}{\rho_1}\right]^{3/5}. \tag{25b}$$

The single dimensionless independent variable in this case will be

$$\eta = (\alpha m G^*)^{-\delta^*/2} r t^{-\delta^*}, \tag{26a}$$

where

$$\delta^* = \frac{2}{3} = \frac{2+s}{5}, \tag{26b}$$

and α is constant to be determined by the condition that η assumes the value '1' at the shock front.

Second of the equations (26b) show that the similarity solution of the present problem exists only when the total energy of the flow-field behind the shock increases as $t^{4/3}$, that is only when *s* = 4/3.

From (26a), we find that

$$\dot{R}^2 = \frac{4\alpha m G^*}{9R}, \tag{27a}$$

$$\frac{d\dot{R}}{dt} = -\frac{\dot{R}^2}{2R}. \tag{27b}$$

From equations (18) and (27a), we obtain the following expression for α in terms of the shock-Mach number *M* :

$$\frac{mG^*}{\dot{R}^2 R} = \frac{9}{4\alpha} = \frac{1}{\gamma M^2}. \tag{28}$$

The quantity $\frac{9}{4\alpha}(= h,\ \text{say})$ may be taken as a parameter of gravitation.

To obtain similarity solutions, we write the unknown variables in the following form (Vishwakarma and Yadav, 2003b)

$$u = \dot{R} v(\eta), \tag{29a}$$

$$\rho = \rho_1 g(\eta), \tag{29b}$$

$$p = \rho_1 \dot{R}^2 P(\eta), \tag{29c}$$

$$Z = Z_1 g(\eta), \tag{29d}$$

where *v*, *g* and *P* are functions of the non-dimensional variable (similarity variable) η only.

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The condition to be satisfied at the inner expanding surface is that the velocity of the fluid is equal to the velocity of the surface itself. This kinematic condition, from equations (26a) and (29), can be written as

$$v(\bar{\eta}) = \bar{\eta}, \tag{30}$$

where $\bar{\eta}$ is the value of η at the inner expanding surface.
Using the similarity transformations (29), the equations of motion are transformed into

$$-(\eta - v)g' + g\left(v' + \frac{2v}{\eta}\right) = 0, \tag{31}$$

$$-(\eta - v)v' - \frac{v}{2} + \frac{P'}{g} + \frac{9}{4\alpha\eta^2} = 0, \tag{32}$$

$$-(\eta - v)(1 - Z_1 g)P' + \frac{(\eta - v)P}{g}\left[\frac{\Gamma + (2\Gamma - Z_1 g)\bar{\alpha}g(1 - k_p)}{1 + \bar{\alpha}g(1 - k_p)}\right]g' - P(1 - Z_1 g) = 0, \tag{33}$$

where a prime denotes differentiation with respect to η.
From the equations (31) to (33), we have

$$v' = \left(-\frac{vg(\eta - v)(1 - Z_1 g)}{2} + \frac{(\eta - v)(1 - Z_1 g)M^{-2}g}{\gamma\eta^2}\right.$$
$$+\frac{2Pv}{\eta}\left\{\frac{\Gamma + (2\Gamma - Z_1 g)\bar{\alpha}g(1 - k_p)}{1 + \bar{\alpha}g(1 - k_p)}\right\} - (1 - Z_1 g)P\Big) \Big/ \Big[(\eta - v)^2(1 - Z_1 g)g$$
$$-P\left\{\frac{\Gamma + (2\Gamma - Z_1 g)g\bar{\alpha}(1 - k_p)}{1 + \bar{\alpha}g(1 - k_p)}\right\}\Big], \tag{34}$$

$$g' = \frac{g}{(\eta - v)}\left(v' + \frac{2v}{\eta}\right), \tag{35}$$

$$P' = (\eta - v)v'g + \frac{vg}{2} - \frac{M^{-2}g}{\gamma\eta^2}. \tag{36}$$

The transformed shock conditions are

$$v(1) = (1 - \beta), \tag{37a}$$

$$g(1) = \frac{1}{\beta}, \tag{37b}$$

$$P(1) = \frac{1}{\gamma M^2} + (1 - \beta), \tag{37c}$$

where β is given by the equation (20).
For exhibiting the numerical solutions, it is convenient to write the field variables in the non-dimensional form as

$$\frac{u}{u_2} = \frac{v(\eta)}{v(1)}, \qquad \frac{\rho}{\rho_2} = \frac{Z}{Z_2} = \frac{g(\eta)}{g(1)}, \qquad \frac{p}{p_2} = \frac{P(\eta)}{P(1)}. \tag{38a -c}$$

The ordinary differential equations (34) to (36) with boundary conditions (37) can now be numerically integrated to obtain the solution for the flow behind the shock surface.

RESULTS AND DISCUSSION

The flow variables in the flow-field behind the shock are obtained by numerical integration of the equations (34) to (36) with boundary conditions (37) by Runge-Kutta method of order four. The values of the constant parameters for the purpose of numerical calculation are taken as γ = 1.4; $\bar{\alpha} = 0$, 0.025, 0.05; M^{-2} = 0.014, 0.14; k_p = 0, 0.2, 0.4; G_1 = 1, 10, 100; and β' = 1. The case $\bar{\alpha}$ = 0, k_p = 0 corresponds to a perfect gas; the case $\bar{\alpha} = 0$, $k_p \neq 0$ to a mixture of a perfect gas and small solid particles; and the case $\bar{\alpha} \neq 0$, $k_p \neq 0$ to a mixture of a non-ideal gas and small solid particles. The solutions are shown in figures 1-6.

Figures 1 and 4 show that the reduced fluid velocity $\frac{u}{u_2}$ is higher at the inner expanding surface than that at the shock front. Figures 2 and 5, and 3 and 6 show that the reduced pressure $\frac{p}{p_2}$ and reduced density $\frac{\rho}{\rho_2}$ decrease as we move inward from the shock front.

As can be seen from equations (35) for *g* (non-dimensional density), there is a singularity at the inner expanding surface where $v = \eta$, because this equation becomes singular there. The inner expanding surface is decelerating as its velocity $\frac{d\bar{r}}{dt}$ varies as, $t^{-1/3}$ (from (26a)), and the derivative of density tends to negative infinity there (as shown in figures 3 and 6). This singularity can be physically interpreted as follows (Steiner and Hirschler, 2002) : the path of the decelerated inner surface diverges from the path of the particle immediately ahead rarifying the gas. This can also the interpreted from the adiabatic integral as follows :

By taking certain linear combinations of equations (31) and (33), we can obtain the adiabatic integral, for $k_p = 0$,

$$Cg^{(\gamma-\frac{1}{3})}(1+\bar{\alpha}g)^{\gamma} = P(\eta - v)^{1/3}\eta^{2/3}, \tag{39}$$

where *C* is a constant to be determined by (37). This relation shows that as the inner surface (at which $v = \eta$) is approached, the non-dimensional density *g* tends abruptly to zero.

The quantity *h* which is a parameter of gravitation depends upon γ and *M* and is tabulated in table 1 at γ = 1.4 and M^{-2} = 0.014, 0.14. The density ratio β across the shock front is tabulated in table 2 at γ = 1.4, β' = 1 and at various values of *h*, $\bar{\alpha}$, k_p and G_1. Positions of inner expanding surface $\bar{\eta}$ are shown in the table 3 at different values of $\bar{\alpha}$, k_p, *h* and G_1. The initial dimensionless compressibility of the mixture

$$l_1 = \frac{p_1}{\rho_1 a_1^2} = \frac{(1-Z_1)\{1+\bar{\alpha}(1-k_p)\}}{\{\Gamma + (2\Gamma - Z_1)(1-k_p)\bar{\alpha}\}}, \tag{40}$$

is calculated for various values of $\bar{\alpha}$, k_p and G_1, and tabulated in table 4.

It was found that the effects of an increase in the value of the parameter of the non-idealness of the gas $\bar{\alpha}$ are

(i) to increase the value of β, i.e. to decrease the shock strength (see table 2);

(ii)to increase the reduced velocity $\frac{u}{u_2}$, reduced pressure $\frac{p}{p_2}$ and reduced density $\frac{\rho}{\rho_2}$ at any point in the flow field-field behind the shock (see figures 1 to 3); and

(iii) to increase the distance of inner expanding surface from the shock front (see table 3).

This concludes the same result as in (i), i.e. an increase in $\bar{\alpha}$ decreases the shock strength. Actually, an increase in $\bar{\alpha}$, decreases the compressibility of the initial medium (see table 4) and this decrease of compressibility results in the decrease of shock strength.

It was found that the effects of an increase in k_p, the mass concentration of the solid particles in the mixture are

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(i) to increase the value of β when $G_1 = 1$ and to decrease it, when G_1 is higher ($\geq$10) (see table 2);
(ii)to increase $\bar{\eta}$, i.e. to decrease the distance of inner expanding surface from the shock front, when G_1 ($\geq$ 10). At $G_1 = 1$, the effects is of opposite nature;
(iii) to decrease the initial dimensionless compressibility of the mixture when $G_1 = 1$, and to increase it, when G_1 is higher ($\geq$ 10) (see table 4); and
(iv) to decrease reduced pressure $\frac{p}{p_2}$ and reduced density $\frac{\rho}{\rho_2}$ when $G_1 = 10$ (see figures 1 to 3).

The effects of an increase in G_1, the ratio of the density of the solid particles to the initial density of the gas, are
(i) to decrease the distance between the shock front and the inner expanding surface (see table 3);
(ii)to decrease β, i.e. to increase the shock strength (see table 2);
(iii) to increase the dimensionless compressibility of the initial medium l_1. Physically, this decrease in l_1 causes more compression of the medium behind the shock, which results in the increase of the shock strength (see table 4); and
(iv) to decrease the reduced velocity $\frac{u}{u_2}$, reduced pressure $\frac{p}{p_2}$ and reduced density $\frac{\rho}{\rho_2}$ at any point in the flow-field behind the shock (see figures 4 to 6).

Table 1: Values of *h* (a parameter of gravitation) for different values of M^{-2} and γ = 1.4

M^{-2}	0.014	0.028	0.07	0.14
H	0.01	0.02	0.05	0.1

Table 2: Density ratio β across the shock front for *h* = 0.01,0.1; $\bar{\alpha}$ = 0, 0.025, 0.05; k_p = 0, 0.2, 0.4; G_1 = 1, 10, 100; γ =1.4; and β' = 1

$\bar{\alpha}$	k_p	G_1	***h***	**β**
0	0	-	0.01	0.178333
			0.1	0.283333
0	0.2	10	0.01	0.170345
			0.1	0.272758
0.025	0	-	0.01	0.196795
			0.1	0.302420
0.025	0.2	1	0.01	0.327958
			0.1	0.432168
0.025	0.2	10	0.01	0.183793
			0.1	0.287264
0.025	0.2	100	0.01	0.166608
			0.1	0.269650
0.025	0.4	1	0.01	0.477373
			0.1	0.578098
0.025	0.4	10	0.01	0.181885
			0.1	0.285666
0.025	0.4	100	0.01	0.135556
			0.1	0.235888
0.05	0.2	10	0.01	0.195295
			0.1	0.300082

Table 3: Position of inner expanding surface $\bar{\eta}$ at different value of $\bar{\alpha}$, *h* and G_1

$\bar{\alpha}$	k_p	G_1	*h*	$\bar{\eta}$
0	0	-	0.01	0.909622
			0.1	0.853079
0	0.2	10	0.01	0.916010
			0.1	0.860021
0.025	0	-	0.01	0.902854
			0.1	0.845492
0.025	0.2	1	0.01	0.855875
			0.1	0.792908
0.025	0.2	10	0.01	0.910958
			0.1	0.854204
0.025	0.2	100	0.01	0.917131
			0.1	0.861101
0.025	0.4	10	0.01	0.915722
			0.1	0.857532
0.05	0.2	10	0.01	0.906430
			0.1	0.848873

Table 4: Variation of initial dimensionless compressibility l_1 for different values of $\bar{\alpha}$, k_p and G_1

$\bar{\alpha}$	k_p	G_1	l_1
0	0	-	0.714285
		1	0.606060
0	0.2	10	0.739090
		100	0.755757
0	0.4	1	0.483871
		10	0.756048
		100	0.801129
0.025	0	-	0.697278
0.025	0.2	1	0.596143
		10	0.725429
		100	0.741249
0.025	0.4	1	0.479075
		10	0.745585
		100	0.789523
0.05	0	-	0.681818
0.05	0.2	1	0.586907
		10	0.712488
		100	0.727815
0.05	0.4	1	0.474509
		10	0.735700
		100	0.738456

The effects of increasing the parameter of gravitation *h* are
(i) to decrease the shock-Mach number *M* (see table 1);
(ii) to increase the value of β, i.e. to decrease the shock strength (see table 2);
(iii) to increase the distance of inner expanding surface from the shock front (see table 3); and

(iv) to increase the reduced pressure $\frac{p}{p_2}$ and reduced density $\frac{\rho}{\rho_2}$ and to decrease the reduced velocity $\frac{u}{u_2}$ at any point in the flow-field behind the shock (see figures 1 to 6).

Nomenclature

a	speed of the sound in the mixture
a^*	characteristic parameter
b	characteristic parameter
$\bar{b}$	internal volume of the molecules of the gas
C	constant
C_p	specific heat of the gas at constant pressure
C_{sp}	specific heat of solid particles
C_v	specific heat of the gas at constant volume
C_{vm}	specific heat of the mixture at constant volume process
E	total energy
E_c	constant
g	non-dimensional fluid density variable
G	ratio of the density of solid particles to the species density of the gas
G^*	gravitational constant
h	parameter of gravitation
K_p	mass fraction of the solid particles in mixture
l	compressibility of the mixture
m	mass of heavy nucleus at the centre
M	shock-Mach number
M_e	effective shock-Mach number
M_m	total mass of the mixture
M_{sp}	total mass of the solid particles
p	pressure of the mixture
p_g	partial pressure of the gas in the mixture
P	non-dimensional fluid pressure variable
r	radial distance
$\bar{r}$	value of r at inner expanding surface
R	shock radius
$\dot{R}$	shock velocity
R^*	gas constant
s	non-negative number
S	constant entropy
t	time
T	temperature of the gas
U_m	internal energy per unit mass
u	flow velocity
v	non-dimensional fluid velocity variable
V_m	total volume of the mixture
V_{sp}	volumetric extension of the solid particles

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Z volume fraction of the solid particles in the mixture

α constant
$\bar{\alpha}$ parameter of non-idealness of gas in the mixture
β density ratio across the shock
β' ratio of specific heat
Γ ratio of specific heats of the mixture
γ ratio of specific heats of the gas
δ^* constant
ρ density of the mixture
$\bar{\rho}_g$ partial density of the gas in the mixture
ρ_{sp} density of the solid particle
ρ_g density of the gas
η dimensionless independent variable
$\bar{\eta}$ value of η at the inner expanding surface

Subscripts
0 reference state
1 ahead of the shock
2 behind the shock

Superscripts
$'$ differentiation with respect to η

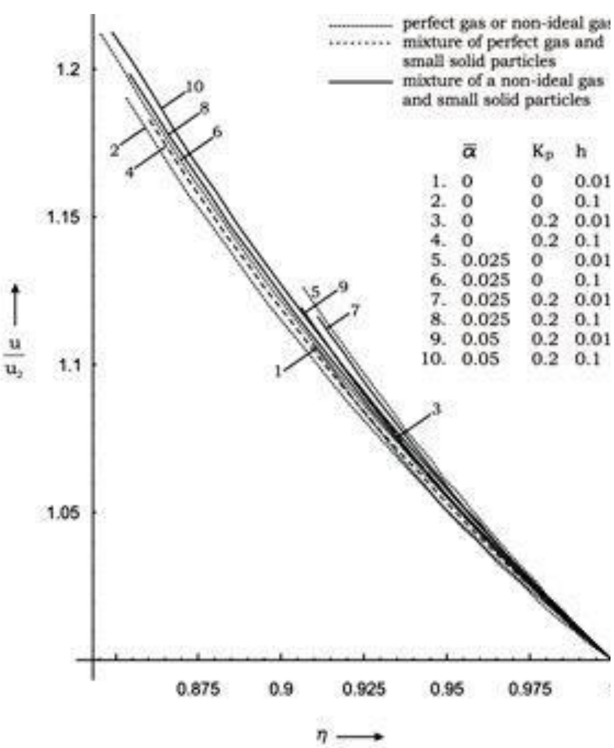

Figure 1: Variation of redused velocity $\frac{u}{u_2}$ in the flow-field behanid the shock front for G_1 = 10

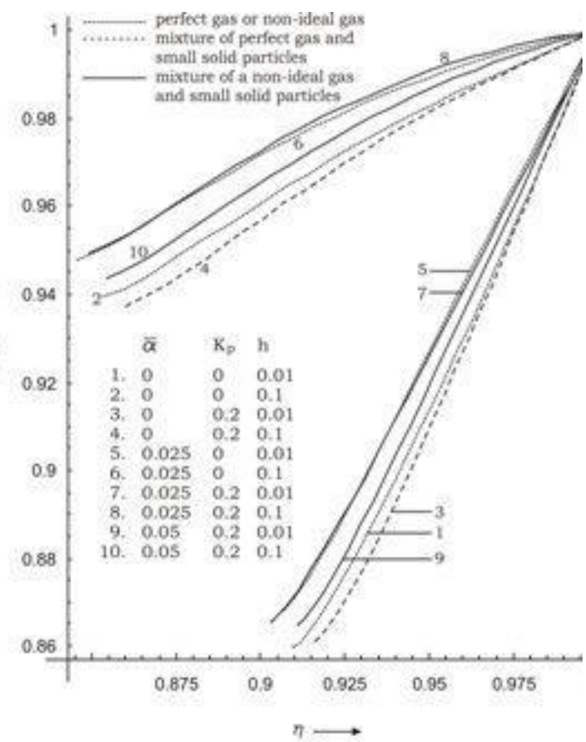

Figure 2: Variation of reduced pressure $\frac{p}{p_2}$ in the flow-field behind the shock front for G_1= 10

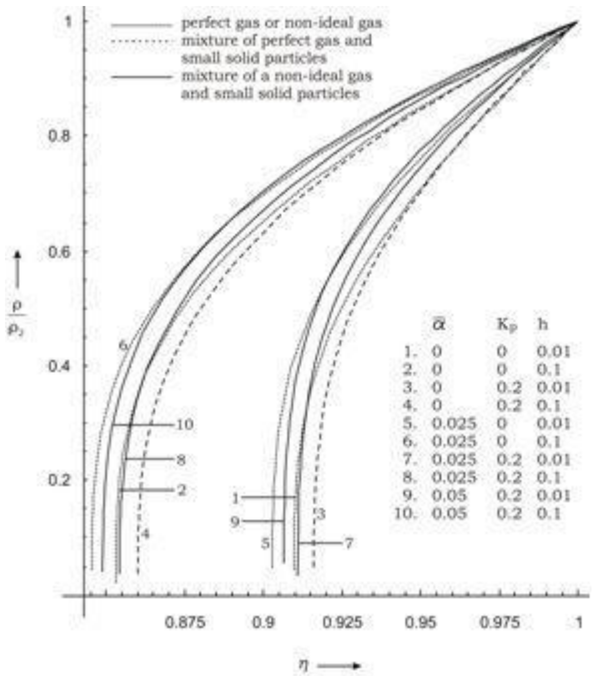

Figure 3: Variation of reduced density $\frac{\rho}{\rho_2}$ in the flow-field behind the shock front for G_1= 10

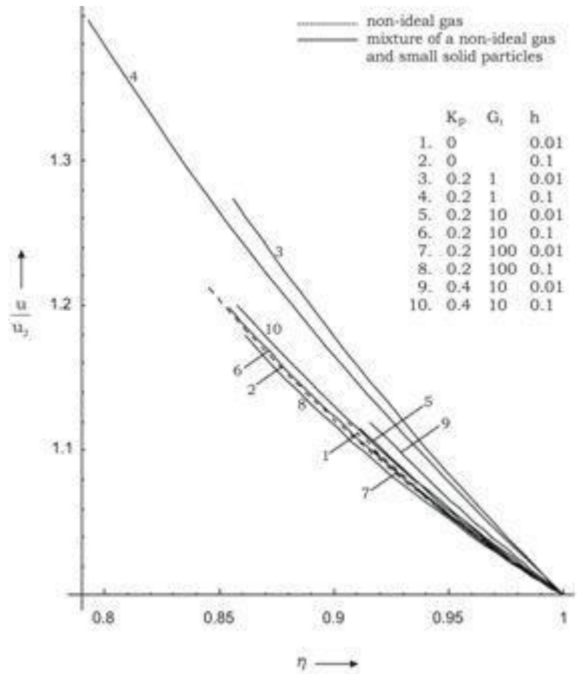

Figure 4: Variation of reduced velocity $\frac{u}{u_2}$ in the flow-field behind the shock front for $\bar{\alpha}$ =0.025

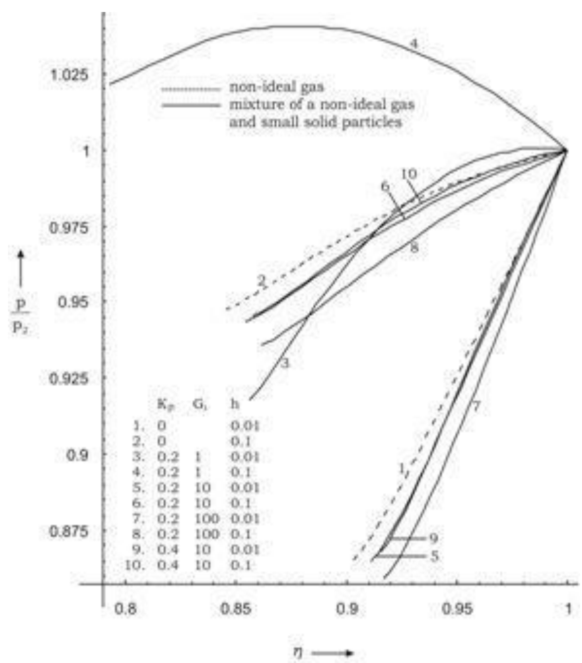

Figure 5: Variation of reduced pressure $\frac{p}{p_2}$ in the flow-field behind the shock front for $\bar{\alpha}$ =0.025

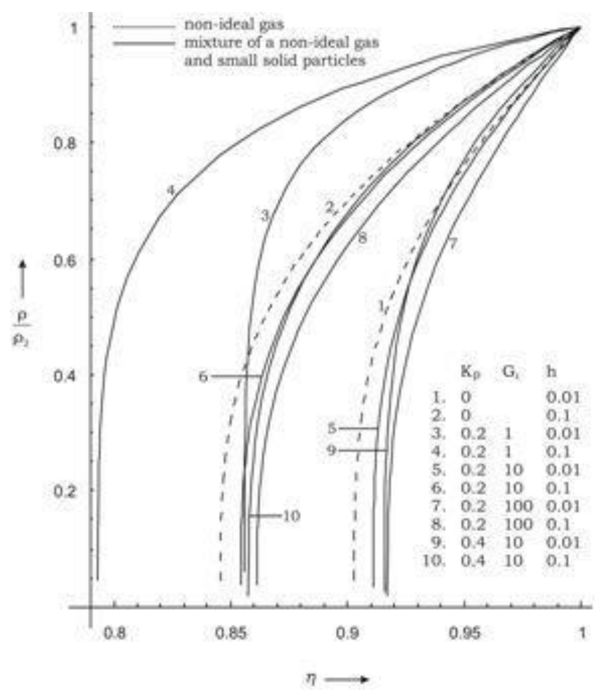

Figure 6: Variation of reduced density $\frac{\rho}{\rho_2}$ in the flow-field behind the shock front for $\bar{\alpha}$= 0.025

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