

ON SOFT RGB -CONTINUOUS FUNCTIONS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the concepts of soft rgb-continuity, soft rgb-open map, soft rgb-closed map and soft rgb-irresolute function and some of its properties are obtained.

Keywords: Soft rgb-Continuous Map, Soft rgb-Closed Map, Soft rgb-Open Map, Soft rgb-Irresolute, Soft T_{rgb} -Space

INTRODUCTION

Soft set theory was initiated by Molodtsov as a new method for vagueness. He showed in his paper that this theory can be applied to several areas such as smoothness of functions, game theory, Riemann-integration, operation research, etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of other theories developed for vagueness. Shabir and Naz introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also defined some concepts of soft sets on soft topological spaces such as soft interior, soft closure, soft spaces and soft separation axioms. Kharal *et al.*, introduced soft functions over classes of soft sets. Aras *et al.*, studied and discussed the properties of soft continuous mappings. The purpose of this paper is to introduce the concepts of soft rgb-continuity, soft rgb-open, soft rgb-closed and soft rgb-irresolute and to obtain some characterization of these mappings.

Preliminaries

Definition 1.1: Let X be an initial universal set and E be the set of parameters. Let $P(X)$ denote the power set of X and let $A \subseteq E$. A pair (F, A) is called a soft set over X , where F is a mapping given by $F: A \rightarrow P(X)$.

The collection of soft sets (F, A) over a universe X and the parameter set A is a family of soft sets denoted by $SS(X)_A$.

Definition 1.2: A soft set (F, A) over X is said to be a null soft set denoted by $\tilde{\phi}$ if for all $a \in A$, $F(a) = \phi$ (null set).

Definition 1.3: A soft set (F, A) over X is said to be an absolute soft set denoted by \tilde{X} if for all $a \in A$, $F(a) = X$. Clearly $\tilde{X}^c = \tilde{\phi}$ and $\tilde{\phi}^c = \tilde{X}$.

Definition 1.4: Let τ be a collection of soft sets over a universe X with a fixed set A of parameters, Then $\tau \subseteq SS(X)_A$ is called a soft topology on X with a fixed set A if,

- (i) ϕ_A, X_A belong to τ
- (ii) The union of any number of soft sets in τ belongs to τ
- (iii) The intersection of any two soft sets in τ belongs to τ

Definition 1.5: Let (X, τ, E) be a soft topological space over X . A soft set (F, E) over X is said to be soft closed, if its relative complement $(F, E)'$ belongs to τ .

Definition 1.6: Let (X, τ, E) be a soft topological space over X . Then the soft closure of a soft set (F, E) over X is denoted by $cl(F, E)$ is the intersection of all soft closed super sets of (F, E) .

Definition 1.7: Let (X, τ, E) be a soft topological space over X . Then the soft interior of a soft set (F, E) over X is denoted by $int(F, E)$ is the union of all soft open sets contained in (F, E) .

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Definition 1.8: Let (X, τ, E) be a soft topological space. A soft set (F, E) is called soft regular generalized b-closed set (briefly soft rgb-closed set) if $\text{bcl}(F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is soft regular open over X .

Definition 1.9: Let (X, τ, E) be a soft topological space. A soft set (F, E) is called a soft regular generalized b-open set (briefly soft rgb-open) if $(F, E)^c$ is soft rgb-closed over X .

Soft rgb-Continuous Functions

Definition 2.1: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be soft continuous if $f^{-1}((F, E))$ is soft closed in (X, τ, E) , for every soft closed set (F, E) in (Y, τ', E) .

Definition 2.2: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be

- (i) a soft open mapping if $f((G, E))$ is soft open over Y for all soft open sets (G, E) over X .
- (ii) a soft closed mapping if $f((F, E))$ is soft closed over Y for all soft closed sets (G, E) over X .
- (iii) a soft irresolute mapping if $f^{-1}(F, E)$ is soft closed over X for every soft closed set (F, E) over Y .

Definition 2.3: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be

- (i) soft rgb-continuous if $f^{-1}((F, E))$ is soft rgb-closed in (X, τ, E) , for every soft closed set (F, E) in (Y, τ', E) .
- (ii) soft rgb-irresolute if $f^{-1}((F, E))$ is soft rgb-closed in (X, τ, E) , for every soft rgb-closed set (F, E) in (Y, τ', E) .

Definition 2.4: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be

- (i) soft rgb-open if $f((F, E))$ is soft rgb-open in (Y, τ', E) , for every soft open set (F, E) in (X, τ, E) .
- (ii) soft rgb-closed if $f((F, E))$ is soft rgb-closed in (Y, τ', E) , for every soft closed set (F, E) in (X, τ, E) .

Definition 2.5: A soft topological space (X, τ, E) is called soft T_{rgb} space if every soft rgb-closed set is soft b-closed over X .

Theorem 2.6: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft function and

- (a) if f is soft continuous, then f is soft rgb-continuous.
- (b) if f is soft semi-continuous, then f is soft rgb-continuous.
- (c) if f is soft pre-continuous, then f is soft rgb-continuous.
- (d) if f is soft α -continuous, then f is soft rgb-continuous.
- (e) if f is soft b-continuous, then f is soft rgb-continuous.
- (f) if f is soft g^* -continuous, then f is soft rgb-continuous.
- (g) if f is soft gp-continuous, then f is soft rgb-continuous.
- (h) if f is soft gb-continuous, then f is soft rgb-continuous.
- (i) if f is soft $g\alpha$ -continuous, then f is soft rgb-continuous.
- (j) if f is soft $g\alpha$ b-continuous, then f is soft rgb-continuous.
- (k) if f is soft gpr-continuous, then f is soft rgb-continuous.
- (l) if f is soft sgb-continuous, then f is soft rgb-continuous.

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- (m) if f is soft sg-continuous, then f is soft rgb-continuous.
- (n) if f is soft gs-continuous, then f is soft rgb-continuous.
- (o) if f is soft rg-continuous, then f is soft rgb-continuous.

Proof: (a) Let (X, τ, E) be a soft topological space, $f; (X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft continuous function and (F, E) be a soft closed set over Y . Since f is soft continuous, $f^{-1}((F, E))$ is soft closed over X . But every soft closed set is soft rgb-closed set over X , $f^{-1}((F, E))$ is a soft rgb-closed set over X . Hence $f; (X, \tau, E) \rightarrow (Y, \tau', E)$ is soft rgb-continuous.

The other proofs are similar to the above.

Remark 2.7: The converse of the above theorem need not be true as shown in the example.

Example 2.8: Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ and $E = \{e_1, e_2\}$. Then $\tau = \{\tilde{\phi}, \tilde{X}, (F_5, E), (F_{14}, E)\}$ is a soft topology over X and $\tau' = \{\tilde{\phi}, \tilde{Y}, (G_1, E), (G_2, E), (G_3, E)\}$ is a soft topology over Y , where (F_5, E) and (F_{14}, E) are soft sets over X and (G_1, E) , (G_2, E) and (G_3, E) are soft sets over Y , defined as follows:

$$\begin{aligned} F_2(e_1) &= \phi, F_2(e_2) = \{x_1\} & F_3(e_1) &= \phi, F_3(e_2) = \{x_2\} \\ F_4(e_1) &= \phi, F_4(e_2) = \{x_1, x_2\} & F_5(e_1) &= \{x_1\}, F_5(e_2) = \phi \\ F_6(e_1) &= \{x_1\}, F_6(e_2) = \{x_1\} & F_7(e_1) &= \{x_1\}, F_7(e_2) = \{x_2\} \\ F_8(e_1) &= \{x_1\}, F_8(e_2) = \{x_1, x_2\} & F_9(e_1) &= \{x_2\}, F_9(e_2) = \phi \\ F_{10}(e_1) &= \{x_2\}, F_{10}(e_2) = \{x_1\} & F_{11}(e_1) &= \{x_2\}, F_{11}(e_2) = \{x_2\} \\ F_{12}(e_1) &= \{x_2\}, F_{12}(e_2) = \{x_1, x_2\} & F_{13}(e_1) &= \{x_1, x_2\}, F_{13}(e_2) = \phi \\ F_{14}(e_1) &= \{x_1, x_2\}, F_{14}(e_2) = \{x_1\} & F_{15}(e_1) &= \{x_1, x_2\}, F_{15}(e_2) = \{x_2\} \text{ and} \\ G_1(e_1) &= Y, G_1(e_2) = \{y_2\} & G_2(e_1) &= \{y_1\}, G_2(e_2) = \{y_2\} \end{aligned}$$

$$G_3(e_1) = \{y_1, y_3\}, G_3(e_2) = \{y_2\}.$$

Then soft closed sets over X are $\tilde{\phi}, \tilde{X}, (F_3, E), (F_{12}, E)$ and soft closed sets over Y are $\tilde{\phi}, \tilde{Y}, (G_4, E), (G_5, E)$, where

$$\begin{aligned} F_3(e_1) &= \phi, F_3(e_2) = \{x_2\} & F_{12}(e_1) &= \{x_2\}, F_{12}(e_2) = \{x_1, x_2\} \text{ and} \\ G_4(e_1) &= \phi, G_4(e_2) = \{y_1, y_3\} & G_5(e_1) &= \{y_2, y_3\}, G_5(e_2) = \{y_1, y_3\} & G_6(e_1) &= \{y_2\}, G_6(e_2) = \{y_1, y_3\} \end{aligned}$$

If we define the mapping $f; (X, \tau, E) \rightarrow (Y, \tau', E)$ as $f(x_1) = y_2$ and $f(x_2) = y_1$, then

- (a) f is soft rgb-continuous mapping, but not a soft continuous map, since $f^{-1}((G_5, E)) = \{\{x_1\}, e_1, \{x_2\}, e_2\}$ is not soft closed over X .
- (b) f is a soft rgb-continuous map, but not a soft semi-continuous map since $f^{-1}((G_6, E)) = \{\{x_1\}, e_1, \{x_2\}, e_2\}$ is not soft semi-closed over X .
- (c) f is a soft rgb-continuous map, but not a soft pre-continuous map since $f^{-1}((G_5, E)) = \{\{x_1\}, e_1, \{x_2\}, e_2\}$ is not soft pre-closed over X .
- (d) f is a soft rgb-continuous map, but not a soft α -continuous map since $f^{-1}((G_6, E)) = \{\{x_1\}, e_1, \{x_2\}, e_2\}$ is not soft α -closed over X .

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(e) f is soft rgb-continuous mapping, but not a soft b-continuous map, since $f^{-1}((G_5, E)) = \{\{x_1\}, e_1, \{x_2\}, e_2\}$ is not soft b-closed over X .

Example 2.9: Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ and $E = \{e_1, e_2\}$. Then $\tau = \{\tilde{\phi}, \tilde{X}, (F_5, E), (F_{14}, E)\}$ is a soft topology over X and $\tau' = \{\tilde{\phi}, \tilde{Y}, (G_1, E), (G_2, E), (G_3, E)\}$ is a soft topology over Y , where (F_5, E) and (F_{14}, E) are soft sets over X and (G_1, E) , (G_2, E) and (G_3, E) are soft sets over Y defined as follows:

$$\begin{aligned} F_2(e_1) &= \phi, F_2(e_2) = \{x_1\} & F_3(e_1) &= \phi, F_3(e_2) = \{x_2\} \\ F_4(e_1) &= \phi, F_4(e_2) = \{x_1, x_2\} & F_5(e_1) &= \{x_1\}, F_5(e_2) = \phi \\ F_6(e_1) &= \{x_1\}, F_6(e_2) = \{x_1\} & F_7(e_1) &= \{x_1\}, F_7(e_2) = \{x_2\} \\ F_8(e_1) &= \{x_1\}, F_8(e_2) = \{x_1, x_2\} & F_9(e_1) &= \{x_2\}, F_9(e_2) = \phi \\ F_{10}(e_1) &= \{x_2\}, F_{10}(e_2) = \{x_1\} & F_{11}(e_1) &= \{x_2\}, F_{11}(e_2) = \{x_2\} \\ F_{12}(e_1) &= \{x_2\}, F_{12}(e_2) = \{x_1, x_2\} & F_{13}(e_1) &= \{x_1, x_2\}, F_{13}(e_2) = \phi \\ F_{14}(e_1) &= \{x_1, x_2\}, F_{14}(e_2) = \{x_1\} & F_{15}(e_1) &= \{x_1, x_2\}, F_{15}(e_2) = \{x_2\} \text{ and} \\ G_1(e_1) &= \{y_1\}, G_1(e_2) = \{y_2\} & G_2(e_1) &= \{y_1, y_2\}, G_2(e_2) = \{y_1\}, \end{aligned}$$

$$G_3(e_1) = \{y_1\}, G_3(e_2) = \phi$$

Then (X, τ, E) and (Y, τ', E) are soft topological spaces.

If we define the mapping $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ as

$f(x_1) = y_2, f(x_2) = y_1$, then f is soft rgb-continuous, but not soft g^* -continuous since for the soft closed set $(H, E) = \{\phi, e_1, \{y_2\}, e_2\}$, $f^1((H, E)) = \{\phi, e_1, \{x_1\}, e_2\}$ is not soft g^* -closed over X .

Example 2.10: Let $X = \{h_1, h_2, h_3, h_4\}$, $Y = \{y_1, y_2, y_3, y_4\}$ and $E = \{e_1, e_2, e_3\}$. Then $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E)\}$ is a soft topology over X and $\tau' = \{\tilde{\phi}, \tilde{Y}, (G_1, E), (G_2, E), (G_3, E)\}$ is a soft topology over Y , where (F_1, E) , (F_2, E) , (F_3, E) , (F_4, E) , (F_5, E) , (F_6, E) , (F_7, E) and (F_8, E) are soft sets over X and (G_1, E) , (G_2, E) and (G_3, E) are soft sets over Y , which are defined as follows:

$$\begin{aligned} F_1(e_1) &= \{h_1\}, F_1(e_2) = \{h_2, h_3\}, F_1(e_3) = \{h_1, h_4\} & F_2(e_1) &= \{h_2, h_4\}, F_2(e_2) = \{h_1, h_3, h_4\}, F_2(e_3) = \{h_2, h_3, h_4\} \\ F_3(e_1) &= \phi, F_3(e_2) = \{h_3\}, F_3(e_3) = \{h_1\} & F_4(e_1) &= \{h_1, h_2, h_4\}, F_4(e_2) = X, F_4(e_3) = X \\ F_5(e_1) &= \{h_2, h_4\}, F_5(e_2) = \{h_1, h_3, h_4\}, F_5(e_3) = X & F_6(e_1) &= \phi, F_6(e_2) = \{h_3\}, F_6(e_3) = \{h_4\} \\ F_7(e_1) &= \phi, F_7(e_2) = \{h_3\}, F_7(e_3) = \{h_1, h_4\} & F_8(e_1) &= \phi, F_8(e_2) = \{h_3\}, F_8(e_3) = \phi \text{ and} \\ G_1(e_1) &= \{y_1, y_2, y_4\}, G_1(e_2) = \{y_1, y_3\}, G_1(e_3) = \{y_2, y_4\} \\ G_2(e_1) &= Y, G_2(e_2) = \{y_1, y_3, y_4\}, G_2(e_3) = \{y_1, y_2, y_4\} & G_3(e_1) &= \{y_2\}, G_3(e_2) = \phi, G_3(e_3) = \phi \end{aligned}$$

Then, (X, τ, E) and (Y, τ', E) are soft topological spaces.

Soft closed sets over Y are $\tilde{\phi}, \tilde{X}, (G_4, E), (G_5, E)$ and (G_6, E) , where,

$$\begin{aligned} G_4(e_1) &= \{y_3\}, G_4(e_2) = \{y_2, y_4\}, G_4(e_3) = \{y_1, y_3\} & G_5(e_1) &= \phi, G_5(e_2) = \{y_2\}, G_5(e_3) = \{y_3\} \\ G_6(e_1) &= \{y_1, y_3, y_4\}, G_6(e_2) = Y, G_6(e_3) = Y. \end{aligned}$$

If we define the mapping $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ as $f(h_1) = y_3, f(h_2) = y_4, f(h_3) = y_2$ and $f(h_4) = y_1$. Then,

- f is soft rgb-continuous, but not soft gp-continuous since $f^1((G_4, E)) = (F_1, E)$ is not soft gp-closed.
- f is soft rgb-continuous, but not soft gb-continuous, since $f^1((G_5, E)) = (F_3, E)$ is not soft

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gp-closed.

(c) f is soft rgb -continuous, but not soft $g\alpha$ -continuous, since $f^{-1}((G_6, E)) = (F_4, E)$ is not soft $g\alpha$ -closed.

(d) f is soft rgb -continuous, but not soft $g\alpha$ b-continuous, since $f^{-1}((G_4, E)) = (F_1, E)$ is not soft $g\alpha$ b-closed.

Example 2.11: Let $X = \{a, b, c\}$, $Y = \{y_1, y_2, y_3\}$ and $E = \{e_1, e_2\}$. Then $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ is a soft topology over X and $\tau' = \{\tilde{\phi}, \tilde{Y}, (G_1, E), (G_2, E), (G_3, E)\}$ is a soft topology over Y , where (F_1, E) , (F_2, E) and (F_3, E) are soft sets over X and (G_1, E) , (G_2, E) and (G_3, E) are soft sets over Y , which are defined as follows:

$$F_1(e_1) = \{a\}, \quad F_1(e_2) = \{b\} \quad F_2(e_1) = \{b\}, \quad F_2(e_2) = \{b\}$$

$$F_3(e_1) = \{a, b\}, \quad F_3(e_2) = \{a, b\} \text{ and}$$

$$G_1(e_1) = Y, \quad G_1(e_2) = \{y_2, y_1\} \quad G_2(e_1) = \{y_1\}, \quad G_2(e_2) = \{y_2\}$$

$$G_3(e_1) = \{y_1, y_2\}, \quad G_3(e_2) = \{y_2, y_1\}$$

Then, (X, τ, E) and (Y, τ', E) are soft topological spaces.

Soft closed sets over Y are $\tilde{\phi}$, \tilde{X} , (G_4, E) , (G_5, E) and (G_6, E) , where

$$G_4(e_1) = \phi, \quad G_4(e_2) = \{y_3\} \quad G_5(e_1) = \{y_2, y_3\} \quad G_5(e_2) = \{y_1, y_3\},$$

$$G_6(e_1) = \{y_3\}, \quad G_6(e_2) = \{y_3\}$$

If we define the mapping $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ as $f(a) = y_2$, $f(b) = y_3$ and $f(c) = y_1$. Then, f is soft rgb -continuous, but not soft gpr -continuous since $f^{-1}((G_6, E)) = (F_2, E)$ is not soft gpr -closed.

Example 2.12: From example 2.10, f is soft rgb -continuous, but not soft sgb -continuous, since $f^{-1}((G_5, E)) = (F_3, E)$ is not soft sgb -closed.

Example 2.13: From example 2.10, f is soft rgb -continuous, but not soft sg -continuous, since $f^{-1}((G_6, E)) = (F_4, E)$ is not soft sg -closed.

Example 2.14: From example 2.10, f is soft rgb -continuous, but not soft gs -continuous, since $f^{-1}((G_4, E)) = (F_1, E)$ is not soft gs -closed.

Example 2.15: Let $X = \{x_1, x_2, x_3, x_4\}$, $Y = \{y_1, y_2, y_3\}$ and $E = \{e_1, e_2\}$. Then $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E)\}$ is a soft topology over X and $\tau' = \{\tilde{\phi}, \tilde{Y}, (G_1, E), (G_2, E), (G_3, E)\}$ is a soft topology over Y , where (F_1, E) , (F_2, E) , (F_3, E) , (F_4, E) , (F_5, E) , (F_6, E) , (F_7, E) , (F_8, E) , (F_9, E) , (F_{10}, E) and (F_{11}, E) are soft sets over X and (G_1, E) , (G_2, E) and (G_3, E) are soft sets over Y , which are defined as follows:

$$F_1(e_1) = \{x_1\}, \quad F_1(e_2) = \{x_1\} \quad F_2(e_1) = \{x_2\}, \quad F_2(e_2) = \{x_2\}$$

$$F_3(e_1) = \{x_1, x_2\}, \quad F_3(e_2) = \{x_1, x_2\} \quad F_4(e_1) = \{x_1, x_2, x_3\}, \quad F_4(e_2) = \{x_1, x_3\},$$

$$F_5(e_1) = \{x_1, x_2, x_4\}, \quad F_5(e_2) = \{x_1, x_2, x_3\}, \quad F_6(e_1) = \{x_2\} \quad F_6(e_2) = \phi$$

$$F_7(e_1) = \{x_1, x_2\}, \quad F_7(e_2) = \{x_1\}, \quad F_8(e_1) = \{x_1, x_2, x_3\}, \quad F_8(e_2) = \{x_1, x_2, x_3\}$$

$$F_9(e_1) = X, \quad F_9(e_2) = \{x_1, x_2, x_3\}, \quad F_{10}(e_1) = \{x_1, x_2\}, \quad F_{10}(e_2) = \{x_1, x_2, x_3\}$$

$$F_{11}(e_1) = \{x_1, x_2\}, \quad F_{11}(e_2) = \{x_1, x_3\} \text{ and}$$

$$G_1(e_1) = \{y_2, y_3\}, \quad G_1(e_2) = Y \quad G_2(e_1) = \{y_2\}, \quad G_2(e_2) = \{y_1\}$$

$$G_3(e_1) = \{y_2\}, \quad G_3(e_2) = \phi$$

Then, (X, τ, E) and (Y, τ', E) are soft topological spaces. Soft closed sets over Y are $\tilde{\phi}$, \tilde{X} , (G_4, E) , (G_5, E) and (G_6, E) , where,

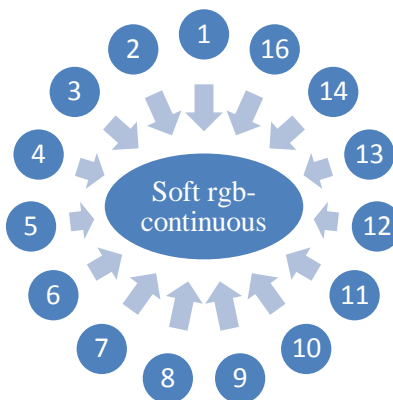
$$G_4(e_1) = \{y_1\}, \quad G_4(e_2) = \phi \quad G_5(e_1) = \{y_1, y_3\}, \quad G_5(e_2) = \{y_2, y_3\}$$

$$G_6(e_1) = \{y_1, y_3\}, \quad G_6(e_2) = Y$$

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If we define the mapping $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ as $f(x_1) = y_2, f(x_2) = y_1, f(x_3) = y_3$ and $f(x_4) = y_3$. Then, f is soft rgb-continuous, but not soft rg-continuous since $f^{-1}((G_4, E)) = (F_6, E)$ is not soft rg-closed.

The above results we have the following diagram:



1. Soft continuous 2. Soft pre-continuous 3. Soft semi- continuous 4. Soft α - continuous
5. Soft g^* - continuous 6. Soft gb- continuous 7. Soft gp- continuous 8. Soft gs- continuous 9. soft sg- continuous
10. Soft ga - continuous 11. Soft rg- continuous 12. Soft sgb- continuous
13. Soft ggb- continuous 14. Soft gpr- continuous 15. Soft b- continuous

Theorem 2.16: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces and $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is be a soft function, then the following conditions are equivalent

- (i) f is soft rgb-continuous
- (ii) The inverse image of each soft open set over Y is soft rgb-open over X .
- (iii) $f(\text{rgb-cl}(F, E)) \subseteq \text{cl}(f((F, E)))$ for each soft set (F, E) over X .
- (iv) For each soft set (G, E) over Y $\text{rgb-cl}(f^{-1}((G, E))) \subseteq f^{-1}(\text{cl}(G, E))$.

Proof: (i) \rightarrow (ii): Let f be soft rgb-continuous and (G, E) be a soft open set over Y . Then $Y - (G, E)$ is soft closed over Y . Therefore, by assumption, $f^{-1}(Y - (G, E))$ is soft rgb-closed over X . That is $X - f^{-1}((G, E))$ is soft rgb-closed over X . Hence $f^{-1}(G, E)$ soft rgb-open over X .

(ii) \rightarrow (i): Let (F, E) be any soft closed set over Y . Therefore $Y - (F, E)$ is soft open over Y and hence by hypothesis, $f^{-1}(Y - (F, E)) = X - f^{-1}((F, E))$ is soft rgb-open over X . Therefore $f^{-1}(F, E)$ is soft rgb-closed over X and consequently f is soft rgb-continuous.

(i) \rightarrow (iii): Let (F, E) be a soft set over X . Since $(F, E) \subseteq f^{-1}(f(F, E))$ and $f((F, E)) \subseteq \text{cl}(f((F, E)))$,

we have $(F, E) \subseteq f^{-1}(f((F, E))) \subseteq f^{-1}(\text{cl}(f((F, E))))$. Therefore by assumption, $f^{-1}(\text{cl}(f((F, E))))$ is soft rgb-closed over X .

Hence, $\text{rgb-cl}(F, E) \subseteq f^{-1}(\text{cl}(f(F, E)))$.

Thus $f(\text{rgb-cl}(F, E)) \subseteq f(f^{-1}(\text{cl}(f((F, E))))) \subseteq \text{cl}(f((F, E)))$.

(iii) \rightarrow (iv): Let (G, E) be a soft set over Y and $f^{-1}(G, E) = (F, E)$. So by our assumption $f(\text{rgb-cl}(F, E)) = f(\text{rgb-cl}(f^{-1}((G, E))))$

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$$\begin{aligned} &\cong cl(f(f^{-1}((G, E)))) \\ &\cong cl(G, E) \end{aligned}$$

Therefore $rgb-cl(f^{-1}((G, E))) \cong f^{-1}(f(rgb-cl(f^{-1}((G, E)))) \cong f^{-1}(cl(G, E))$.

(iv) \rightarrow (i): Let (F, E) be a soft closed set over Y . Then by our assumption,

$rgb-cl(f^{-1}((F, E))) \cong f^{-1}(cl(F, E)) = f^{-1}((F, E))$. Therefore $f^{-1}((F, E))$ is soft rgb -closed. Hence f is soft rgb -continuous.

Theorem 2.17: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces and $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft function. If f is soft rgb -continuous then $f^{-1}(int(G, E)) \cong rgb-int(f^{-1}((G, E)))$ for every soft set (G, E) over Y .

Proof: Let $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ be soft rgb -continuous and (G, E) be any soft set over Y . Then $int(G, E)$ is a soft open set over Y . Therefore by our assumption, $f^{-1}(int(G, E))$ is soft rgb -open over X . Since $f^{-1}(int(G, E)) \cong f^{-1}((G, E))$ and $rgb-int(f^{-1}((G, E)))$ is the largest soft rgb -open set contained in $f^{-1}((G, E))$, $f^{-1}(int(G, E)) \cong rgb-int(f^{-1}((G, E)))$. Hence the proof.

Theorem 2.18: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is soft rgb -closed if $rgb-cl(f((F, E))) \cong f(cl(F, E)) \forall$ soft set (F, E) over X .

Proof: The proof is obvious.

Theorem 2.19: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces and $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft function. Then f is soft rgb -open if $f(int(F, E)) \cong rgb-int(f((F, E))) \forall$ soft set (F, E) over X .

Proof: Let $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ be a soft rgb -open mapping and (F, E) be any soft set over X . Since $int(F, E)$ is soft open over X and f is soft rgb -open, $f(int(F, E))$ is a soft rgb -open set over Y . Also, $f(int(F, E)) \cong f((F, E))$. Therefore, $f(int(F, E)) \cong rgb-int(f((F, E)))$.

Theorem 2.20: Every soft rgb -irresolute function is soft rgb -continuous.

Proof: The proof is obvious.

Theorem 2.21: Let f be a soft rgb -continuous function from a soft topological space (X, τ, E) to a soft T_{rgb} space (Y, τ, E) then f is soft rgb -irresolute.

Proof: Let (F, E) be a soft rgb -closed set over Y . Since Y is T_{rgb} , (F, E) is soft closed over Y . Therefore, by assumption, $f^{-1}((F, E))$ is soft rgb -closed over X and consequently f is soft rgb -irresolute.

Theorem 2.22: Let (X, τ, E) and (Y, τ', E) be two soft topological spaces and $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is be a soft function, then the following conditions are equivalent

- (i) f is soft rgb -irresolute
- (ii) The inverse image of each soft rgb -open set over Y is soft rgb -open over X .
- (iii) $f(rgb-cl(F, E)) \cong rgb-cl(f((F, E)))$ for each soft set (F, E) over X .
- (iv) For each soft set (G, E) over Y $rgb-cl(f^{-1}((G, E))) \cong f^{-1}(rgb-cl(G, E))$.

Proof: (i) \rightarrow (ii): Let f be soft rgb -irresolute and (G, E) be a soft rgb -open set over Y . Then $Y - (G, E)$ is soft rgb -closed over Y . Therefore, by assumption, $f^{-1}(Y - (G, E))$ is soft rgb -closed over X . That is $X - f^{-1}((G, E))$ is soft rgb -closed over X . Hence $f^{-1}(G, E)$ soft rgb -open over X .

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(ii) \rightarrow (i): Let (F, E) be any soft rgb-closed set over Y . Therefore $Y - (F, E)$ is soft rgb-open over Y and hence by hypothesis, $f^{-1}(Y - (F, E)) = X - f^{-1}((F, E))$ is soft rgb-open over X . Therefore $f^{-1}(F, E)$ is soft rgb-closed over X and consequently f is soft rgb-irresolute.

(i) \rightarrow (iii): Let (F, E) be any soft set over X . Since $(F, E) \subseteq f^{-1}(f(F, E))$ and $f((F, E)) \subseteq \text{rgb-cl}(f((F, E)))$,

we have $(F, E) \subseteq f^{-1}(f((F, E))) \subseteq f^{-1}(\text{rgb-cl}(f((F, E))))$. Therefore by assumption, $f^{-1}(\text{rgb-cl}(f((F, E))))$ is soft rgb-closed over X .

Hence, $\text{rgb-cl}(F, E) \subseteq f^{-1}(\text{rgb-cl}(f(F, E)))$.

Thus $f(\text{rgb-cl}(F, E)) \subseteq f(f^{-1}(\text{rgb-cl}(f(F, E)))) \subseteq \text{rgb-cl}(f(F, E))$.

(iii) \rightarrow (iv): Let (G, E) be any soft set over Y and $f^{-1}(G, E) = (F, E)$. So by our assumption

$$f(\text{rgb-cl}(F, E)) = f(\text{rgb-cl}(f^{-1}((G, E)))) \subseteq \text{rgb-cl}(f(f^{-1}((G, E)))) \\ \subseteq \text{rgb-cl}(G, E)$$

Therefore $\text{rgb-cl}(f^{-1}((G, E))) \subseteq f^{-1}(f(\text{rgb-cl}(f^{-1}((G, E)))) \subseteq f^{-1}(\text{rgb-cl}(G, E))$.

(iv) \rightarrow (i): Let (F, E) be a soft rgb-closed set over Y . Then by our assumption, $\text{rgb-cl}(f^{-1}((F, E))) \subseteq f^{-1}(\text{rgb-cl}(F, E)) = f^{-1}((F, E))$. Therefore $f^{-1}((F, E))$ is soft rgb-closed. Hence f is soft rgb-irresolute.

Theorem 2.23: Let $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ and $g : (Y, \tau', E) \rightarrow (Z, \tau^*, E)$ be two soft functions. Then

(i) $g \circ f : (X, \tau, E) \rightarrow (Z, \tau^*, E)$ is soft rgb-continuous, if f is soft rgb-continuous and g is soft continuous.

(ii) $g \circ f : (X, \tau, E) \rightarrow (Z, \tau^*, E)$ is soft rgb-irresolute, if f and g are soft rgb-irresolute functions.

(iii) $g \circ f : (X, \tau, E) \rightarrow (Z, \tau^*, E)$ is a soft rgb-continuous function, if f is a soft rgb-irresolute function and g is a soft rgb-continuous function.

Proof: The proof is straightforward.

Theorem 2.24: If $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is a soft closed function and $g : (Y, \tau', E) \rightarrow (Z, \tau^*, E)$ is soft rgb-closed, then $g \circ f$ is soft rgb-closed function.

Proof: The proof is obvious.

Theorem 2.25: Let $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ and $g : (Y, \tau', E) \rightarrow (Z, \tau^*, E)$ be two soft maps such that $g \circ f$ is a soft rgb-closed map,

(i) If f is soft continuous and surjective, then g is a soft rgb-closed map.

(ii) If g is soft rgb-irresolute and injective, then f is soft rgb-closed map.

Proof: The proof is obvious.

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