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Research Article

n-fold Ring $\mathbf{A = (A, \theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_{n+1})}$

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ABSTRACT
In this Article an attempt is made to put the Concept of n-fold Ring $A = (A, \theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_{n+1})$.

Keywords:*n-fold Ring, semi group, binary operation, Algebraic Extension, Code,* $zero\ element\ of\ n\ fold\ ring$*, n- fold ring* $A = (A, \theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_{n+1})$*with unity, Abelian n-fold ring, Splitting field of F[x], Zero divisor in n- fold ring, n- fold integral domain, n- fold ideal, n-fold ring homomorphism , n-fold principle ideal ring, Kernel of n fold ring homomorphism, n- fold division ring, n-fold field, n- fold celing polynomial ring, n-fold near ring, n- fold hemi ring, n- fold non associative ring, n-fold near ring*

INTRODUCTION

Herstein cotes in (1992)
Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by*, such that
1 a, b ∈ G implies that a*b ∈ G
2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$
Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a .$$

Definition : A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively , such that for all a,b,c in R :
1 a+b is in R.
2 a+b = b+a.
3 (a+b)+c = a+(b+c).
4 There is an element 0 in R such that a+0 = a,∀ a ∈ R
5 There exist an element –a in R such that a + (-a) = 0.
6 a*b is in R
7 a*(b*c) = (a*b)*c.
8 a *(b+c) = a * b + a * c and (b+c) * a = b*a + c*a.
It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R ; if there is such we shall describe R as a ring with unit element.
If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

MATTER (DISCUSSION)

n- foldRing

Let A be a non-empty set and$\theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_{n+1}$ be binary operations on A. Then $A = (A, \theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_{n+1})$ is said to be n fold ring if
1) (A, θ_1) is an abelian group
2) (A, θ_2) is semi group , (A, θ_3) is semi group , ……., (A, θ_{n+1}) is semi group
3) $\theta_2$is distributive over θ_1 , θ_3 is distributive over θ_1 , ……, θ_{n+1} is distributive over θ_1 .
Exa01.

$$A = \{a_0 G_0 + a_1 G_1 + \cdots + a_{n-1} G_{n-1} + a_n G_n \ /\ a_i \in F \ and\ n \in N \ \& G_i \in C(P)\}$$

Where $(P) = class\ of\ algebriac\ structures$, P : Set of Codes in Universe.

Exa02 .
$A = \{a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \ / \ a_i \in F \ and \ n \in N\}$,
$\{x_1, \ldots, x_{n-1}, x_n\}$ are in Algebraic Extension of F.
Exa03.
$A = \{a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \ / \ a_i \in F \ and \ n \in N\}$,
$\{x_1, \ldots, x_{n-1}, x_n\}$ are in Splitting field of F[x].
Exa04.
$A = \{a_0G_0 + a_1G_1 + \cdots + a_{n-1}G_{n-1} + a_nG_n \ /a_i \in F \ and \ n \in N \ \&G_i \in C(P)\}$Where $C(P) = class \ of \ algebriac \ structures$ of Splitting field of F[x].

zero element of n fold ring

Identityof A with respect to θ_1 is known as zero element of n fold ring and it is denoted by 0.

n-fold Ring $A = (A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$with Unity

$A = (A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$ is n fold ring is said to be n fold ring with unity if multiplicative identity of A is common with respect to all binary operations $\theta_2, \theta_3, \ldots \ldots, \theta_{n+1}$ and it is denoted by 1.

Commutative / Abelian n-fold Ring $A = (A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$

An n fold ring $A = (A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$ is said to be commutative or abelian if A is commutative with respect to all binary operation i.e. $\theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1}$.

Zero Divisor in n- fold Ring= $(A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$

An element $0 \neq a \in A$is said to be zero divisor of n fold ring $A = (A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$ if $\exists \ b \neq 0 \in A$ such that $ab = 0$ with respect to any binary operation $\theta_2, \theta_3, \ldots \ldots, \theta_{n+1}$.

n-foldIntegral Domain

n fold commutative ring with unity is said to be n fold integral domain if it has no zero divisor.

n-foldIdeal

Let U be any subset of n fold ring A is said to be n fold ideal
if 1. U is n fold sub ringAnd 2 . $\forall \ u \in U \ and \ r \in A \rightarrow ur \ \& \ ru$are both in U with respect to binary operation $\theta_2, \theta_3, \ldots \ldots, \theta_{n+1}$.

n-foldRing Homomorphism

Let $A = (A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$
$B = (B, \psi_1, \psi_2, \psi_3, \ldots \ldots, \psi_{n+1})$be any two n- fold rings.
A mapping H from A to B if

$$\begin{array}{ll} H(a\theta_1 b) = H(a)\psi_1 H(b), & \forall \ a, b \in A \\ H(a\theta_2 b) = H(a)\psi_2 H(b), & \forall \ a, b \in A \\ H(a\theta_3 b) = H(a)\psi_3 H(b), & \forall \ a, b \in A \\ \downarrow \quad \downarrow\downarrow \quad \downarrow \quad \downarrow & \\ \downarrow \quad \downarrow\downarrow \quad \downarrow \quad \downarrow & \\ H(a\theta_{n+1} b) = H(a)\psi_{n+1} H(b), & \forall \ a, b \in A \end{array}$$

n-fold Principle Ideal Ring

Let A be a n fold ring is said to be principle ideal ring if every ideal of n fold R is principle ideal.

Kernel of n Fold Ring Homomorphism

$H : A \rightarrow B$be a n- fold ring homomorphism

$$K(H) = \{ a \in A \ / \ H(a) = 0 = zero \ element \ of \ B \}$$

n-foldDivision Ring

A n fold ring is said to be n fold division ring if it is not n fold commutative ring and inverse of each non zeroelement with respect to each binary operation exist.

n-foldField

A commutative n fold division ring is known as n fold field.

n-foldCelingPolynomial Ring

Let F = fieldand= $\{x \ / \ x = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \ / \ n \in N \ \&a_i \in F, x \ is \ in \ determinate\}$, $A = (A, \theta_1, \theta_2, \theta_3, \ldots \ldots, \theta_{n+1})$ is a n fold celing polynomial ring if

1. (A,θ_1) is an abelian group
2. $(A,\theta_2)$is semi group.
3. $(A,\theta_3)$is semi group.

↓↓ ↓ ↓

$(A,\theta_{n+1})$is semi group.

4. $\theta_2,\theta_3,\theta_{n+1}$ is distributive over θ_1.

n-foldHemi Ring

Let A be any non-empty setand $\theta_1,\theta_2,\dots\dots,\theta_{n+1}$ are binary operations on A .
$then\ A=(A,\theta_1,\theta_2,\theta_3,\dots\dots,\theta_{n+1})$is said to be hemi ring if

1. $(A,\theta_1)$is monoid.
2. $(A,\theta_2)$is monoid.
3. $(A,\theta_3)$is monoid.

↓ ↓↓ ↓ ↓

$(A,\theta_{n+1})$is monoid.

4. $\theta_2,\theta_3,\theta_{n+1}$ is distributive over θ_1.

Exa:-$A=\{\,a_0+a_1x_1+\dots+a_nx_n\ /\ a_i\in R\,\}$ is a n fold hemi ring if R is hemi ring.

n-foldNear Ring

Let A be any non-empty set and$\theta_1,\theta_2,\dots\dots,\theta_{n+1}$ are binary operations on A $then\ A=(A,\theta_1,\theta_2,\theta_3,\dots\dots,\theta_{n+1})$ is said to be n fold near ring if

1. $(A,\theta_1)$is not necessary abelian group.
2. $(A,\theta_2)$is semi group.
3. (A,θ_3) is semi group

↓↓ ↓ ↓

(A,θ_{n+1}) is semi group.

4. $\theta_2,\theta_3,\theta_{n+1}$ is distributive over θ_1.

Exa:-$A=\{\,a_0+a_1x_1+\dots+a_nx_n\ /\ a_i\in R\,\}$ is a n fold near ring if R is near ring.

n-foldNon Associative Ring

Let A be any non-empty set.And $\theta_1,\theta_2,\dots\dots,\theta_{n+1}$ are binary operations on A $then\ A=(A,\theta_1,\theta_2,\theta_3,\dots\dots,\theta_{n+1})$ is said to be n fold non associative ring if

1. (A,θ_1) is abelian group.
2. $\theta_2,\theta_3,\theta_{n+1}$ is distributive over θ_1.

Exa:-$A=\{\,a_0+a_1x_1+\dots+a_nx_n\ /\ a_i\in R\,\}$ is a n fold non associative ring if R is non associative ring.

CONCLUSION

From the above discussion we come to conclusion that n-fold Ring ,$zero\ element\ of\ n\ fold\ ring$, n-fold ring $A=(A,\theta_1,\theta_2,\theta_3,\dots\dots,\theta_{n+1})$with unity, Abelian n-fold ring , Splitting field of F[x], Zero divisor in n- fold ring , n- fold integral domain, n- fold ideal , n-fold ring homomorphism, n-fold principle ideal ring, Kernel of n fold ring homomorphism, n- fold division ring, n- fold field, n- fold celing polynomial ring, n-fold near ring, n- fold hemi ring, n- fold non associative ring, n-fold near ring are defined.

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