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Research Article

RADIATION, HEAT AND MASS TRANSFER EFFECTS ON MAGNETOHYDRODYNAMIC UNSTEADY FREE CONVECTIVE WALTER'S MEMORY FLOW PAST A VERTICAL PLATE WITH CHEMICAL REACTION THROUGH A POROUS MEDIUM

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ABSTRACT
An analytical study is performed to study the influence of radiation and mass transfer on unsteady hydromagnetic free convective memory flow of viscous, incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat absorbing sink with chemical reaction taking into an account. Approximate solutions have been derived for the mean velocity, mean temperature and mean concentration using multi-parameter perturbation technique and these are presented in graphical form. The effects of different physical parameters such as magnetic parameter, Grashof number, modified Grashof number, Prandtl number, Schmidt number, Eckert number, Radiation parameter; Chemical reaction parameter and heat sink strength parameter are discussed.

Keywords: *Radiation, Memory Fluid Flow, Suction, MHD, Viscous Dissipation, Heat Sink and Chemical Reaction*

INTRODUCTION
The most common type of body force, which acts on a fluid, is due to gravity, so that the body force can be defined as in magnitude and direction by the acceleration due to gravity. Sometimes, electromagnetic effects are important. The electric and magnetic fields themselves must obey a set of physical laws, which are expressed by Maxwell's equations. The solution of such problems requires the simultaneous solution of the equations of fluid mechanics and electromagnetism. One special case of this type of coupling is known as magnetohydrodynamic.

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon in this case is relatively complex than that in pure thermal/solutal convection process. Processes involving heat and mass transfer in porous media are often encountered in the chemical industry and formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing and environmental pollution, in reservoir engineering in connection with thermal recovery process, in the study of dynamics of hot and salty springs of a sea and designing of chemical processing equipment. Underground spreading of chemical waste and other pollutants, grain storage, evaporation cooling, and solidification are a few other application areas where combined thermosolutal convection in porous media are observed. For some industrial applications such as glass production and furnace design and in space technology applications, such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant.

However, the exhaustive volume of work devoted to this area is amply documented by the most recent books by Beard and Walters (1964) Elastico-viscous boundary layer flows, two dimensional flows near a stagnation point. Trevisan and Bejan (1985) have studied the problem of combined heat and mass transfer by free convection in a porous medium. They studied the natural convection phenomenon occurring inside a porous layer with both heat and mass transfer from the side and derived the natural circulation by a combination of buoyancy effects due to both temperature and concentration variations. Kafoussias (1992) discussed the effects of mass transfer on free convective flow of a viscous fluid past a vertical isothermal cone surface. He obtained the effects of the buoyancy parameter and Schmidt number on the flow field. The problem of convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream is investigated by Vajravelu and Hadjinicolaou (1997). Bestman and Adjepong (1998) analyzed unsteady hydromagnetic free convection flow with radiative heat transfer in a rotating fluid. Ingham and Pop (1998, 2002), Vafai (2000), and Pop and Ingham (2001) studied the problem of transient flow of a fluid past a moving semi-infinite vertical porous plate. However, many problem areas which are important in applications, as well as in theory still persist. Abd *et al.,* (2003) carried out the finite difference method for the problem of radiation effects on MHD unsteady free-convection flow over vertical plate with variable surface temperature. The problem of flow of a micropolar fluid past a moving semi infinite vertical porous plate with mixed radiative convection is studied by Kim and Fedorov (2003). Abel and Mahesha (2008) have investigated the effects of thermal conductivity, non-uniform heat source and viscous dissipation in the presence of thermal radiation on the flow and heat transfer in viscoelastic fluid over a stretching sheet, which is subjected to an external magnetic field. Numerical study of transient free convective mass transfer in a Walters-B viscoelastic flow with wall suction was analyzed by Chang *et al.,* (2011).

The study of electrically conducting viscous fluid that flows through convergent or divergent channels under the influence of an external magnetic field not only is fascinating theoretically but also finds applications in mathematical modeling of several industrial and biological systems. A possible practical application of the theory we envisage is in the field of industrial metal casting, the control of molten metal flows.

Moreover, the magnetohydrodynamic (MHD) rotating fluids in the presence of a magnetic field are encountered in many important problems in geophysics, astrophysics, and cosmical and geophysical fluid dynamics. It can provide explanations for the observed maintenance and secular variations of the geomagnetic field. It is also relevant in the solar physics involved in the sunspot development, the solar cycle, and the structure of rotating magnetic stars. The effect of the Coriolis force due to the Earth's rotation is found to be significant as compared to the inertial and viscous forces in the equations of motion. The Coriolis and electromagnetic forces are of comparable magnitude, the former having a strong effect on the hydromagnetic flow in the Earth's liquid core, which plays an important role in the mean geomagnetic field. Several investigations are carried out on the problem of hydrodynamic flow of a viscous incompressible fluid in rotating medium considering various variations in the problem. Alagoa *et al.,* (1999) studied radiative and free convection effects on MHD flow through porous medium between infinite parallel plates with time-dependent suction. Nield and Bejan (1999) convection in porous media. Chowdhury and Islam (2000) were studied the MHD free convection flow of visco-elastic fluid past an infinite vertical porous plate. Seddeek (2001) discussed the problem of thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature-dependent viscosity. Abel *et al.,* (2008) investigated the effects of viscous and ohmic dissipation in MHD flow of viscoelastic boundary layer flow. Mustafa *et al.*,(2008) obtained the analytical solution of unsteady MHD memory flow with oscillatory suction, variable free stream and heat source. Gireesh *et al.,* (2009) analyzed the effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Gireesh Kumar and Satyanarayana (2011) mass transfer effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Kesavaiah *et al.,* (2011) investigated effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Srinathuni Lavanya and Chenna Kesavaiah (2014) Radiation and Soret effects to MHD flow in vertical surface with chemical reaction and heat generation through a porous medium.

Viscoelastic flows arise in numerous processes in chemical engineering systems. Such flows possess both viscous and elastic properties and can exhibit normal stresses and relaxation effects. An extensive range of mathematical models has been developed to simulate the diverse hydrodynamic behavior of these non-Newtonian fluids. Rivlin–Ericksen second order model by Metzner and White (1965). Kafousias and

Raptis (1981) have discussed the mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction of injection. Ji *et al.*, (1990) studied the Von Karman Oldroyd-B viscoelastic flow from a rotating disk using the Galerkin method with B-spline test functions. An eloquent exposition of viscoelastic fluid models has been presented by Joseph (1990). Khan *et al.*, (2003) also investigated the effect of work done by deformation in Walter's liquid B but with uniform heat source. Hayat *et al.*, (2008) also investigated the effects of work done by deformation in second grade fluid with partial slip condition, in this no account of heat source has been taken into consideration. The Oldroyd model (1950). The mixture of polymethyl mehacrylate and pyridine at $25^{\circ}C$ containing 30.5g of polymer per liter behaves very nearly as the Walter's liquid model-B, (1960, 1962). Siddappa and Khapate (1975) studied the second order Rivlin–Ericksen viscoelastic boundary layer flow along a stretching surface. Rochelle and Peddieson (1980) used an implicit difference scheme to analyze the steady boundary-layer flow of a nonlinear Maxwell viscoelastic fluid past a parabola and a paraboloid. Raptis and Tziyanidis (1981) have studied the flow of a Walter's liquid B model in the presence of constant heat flux between the fluid and the plate and taking into account the influence of the memory fluid on the energy equation. Rao and Finlayson (1990) used an adaptive finite element technique to analyze viscoelastic flow of a Maxwell fluid. MHD free convection flow of an elasto viscous fluid past an infinite vertical plate was analyzed by Samria *et al.*, (1990). Renardy (1997) the upper convicted Maxwell model and the Walters-B model. Both steady and unsteady flows have been investigated at length in a diverse range of geometries using a wide spectrum of analytical and computational methods. Rao (1999) Johnson–Seagalman model, Thermo solutal instability of Walter's (model-B) visco-elastic rotating fluid permeated with suspended particles and variable gravity field in porous medium was studied by Sharma and Rana (2001). Sharma *et al.*, (2002) have analyses the Rayleigh-Taylor instability of Walter'B elastic-viscous fluid through porous medium. Sharma and Chaudhary (2003) effect of variable suction on transient free convective viscous incompressible flow past a vertical plate with periodic temperature variations in slip flow regime. Ramanamurthy *et al.*, (2007) have discussed the MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana (2009). Rajesh (2011) Heat source and mass transfer effects on MHD flow of an elasto-viscous fluid through a porous medium. Nabil *et al.*, (2012) Numerical study of viscous dissipation effect on free convection heat and mass transfer of MHD non-Newtonian fluid flow through a porous medium. Rita and Sajal (2013) free convective MHD Flow of a Non-Newtonian fluid past an infinite vertical plate with constant suction and heat sink. Rita and Paban (2014) Effects of MHD visco-elastic fluid flow past a moving plate with Double Diffusive convection in presence of heat generation. Pillai *et al.*, (2014) investigated the effects of work done by deformation in viscoelastic fluid in porous media with uniform heat source.

The present study is to study the radiation, heat and mass transfer effects on unsteady hydromagnetic free convective memory flow of viscous, incompressible and electrically conducting fluid flow an infinite vertical plate in the presence of chemical reaction taking into an account. Our main interest is to observe how various parameters affect the flow past an infinite vertical porous plate.

Therefore, the main idea of the present work is to make a mathematical modeling of this phenomenon and the out purpose is to find the relation between the different parameters and the external forces with the solutions of the problem.

Formulation of the Problem

Consider unsteady hydromagnetic free convective flow of viscous, incompressible and electrically conducting and radiating fluid past an infinite vertical porous plate in the presence of constant suction and heat absorbing sink with chemical reaction. Consider the infinite vertical plate embedded an infinite mass of the fluid. Initially the temperature and concentration of both being assumed at T_∞ and C_∞. At time $t > 0$, the plate temperature and concentration are raised to T_∞ and C_∞, and a periodic temperature and/concentration are assumed to be superimposed on this mean constant temperature/ concentration of the plate. Let the $x-$axis be taken in the vertically upward direction along the infinite vertical plate and $y-$axis is normal to it. The magnetic field of uniform strength is applied and induced magnetic field is neglected. Boussineq's approximation, the problem is governed by the following set of equations.

$$\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)+\nu\frac{\partial^2 u}{\partial y^2}+B_1\left(\frac{\partial^3 u}{\partial y^2\partial t}+v\frac{\partial^3 u}{\partial y^3}\right)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\kappa\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\left(\frac{\partial q_r}{\partial y}\right)+S\left(T-T_\infty\right)+\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kr'\left(C-C_\infty\right) \tag{4}$$

From (1) we have

$$v=-v_0 \tag{5}$$

On disregarding the Joulean heat dissipation, the boundary conditions of the problem are:

$$\begin{aligned} &u=0, v=-v_0, T=T_w+\varepsilon\left(T_w-T_\infty\right)e^{i\omega t}, C=C_w+\varepsilon\left(C_w-C_\infty\right)e^{i\omega t} \quad at \quad y=0 \\ &u\to 0, \qquad T\to T_\infty, \qquad C\to C_\infty \qquad as \quad y\to\infty \end{aligned} \tag{6}$$

Introducing the non-dimensional quantities and parameters:

$$\begin{aligned} &u^*=\frac{u}{v_0},\ y^*=\frac{v_0 y}{\nu},\ t^*=\frac{t v_0^2}{4\nu},\ T^*=\frac{T-T_\infty}{T_w-T_\infty},\ C^*=\frac{C-C_\infty}{C_w-C_\infty},\ K=\frac{K_0}{\rho C_p} \\ &\Pr=\frac{\nu}{\kappa},\ Sc=\frac{\nu}{D},\ S^*=\frac{S\nu}{v_0^2},\ Ec=\frac{v_0^2}{C_p\left(T_w-T_\infty\right)},\ Kr=\frac{Kr'\nu}{v_0^2},\ Rm=\frac{B_1 v_0^2}{\nu^2} \\ &M=\frac{\sigma B_0^2\nu}{\rho v_0^2},\ Gr=\frac{\nu\beta g\left(T_w-T_\infty\right)}{v_0^3},\ Gc=\frac{\nu\beta^* g\left(C_w-C_\infty\right)}{v_0^3},\ R=\frac{4I\nu}{\rho C_p v_0^2} \end{aligned} \tag{7}$$

where Gr is the thermal Grashof number, Gc is modified Grashof Number, $\Pr$ is Prandtl Number, M is the magnetic field, Sc is Schmdit number, Kr is Chemical Reaction, K is Porous Permeability, S is Heat source parameter, R is the radiation parameter respectively.

The radiative heat flux q_r is given by equation (5) in the spirit of Cogly *et al.*, (1968)

$$\frac{\partial q_r}{\partial y}=4\left(T-T_\infty\right)I$$

where $I=\int_0^\infty K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T}d\lambda$, $K_{\lambda w}-$ is the absorption coefficient at the wall and $e_{b\lambda}-$ is Planck's function, I is absorption coefficient

The equations (2), (3) and (4) reduce to following non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=Gr\,T+Gc\,C+\frac{\partial^2 u}{\partial y^2}+R_m\left(\frac{\partial^3 u}{\partial y^2\partial t}-\frac{\partial^3 u}{\partial y^3}\right)-M\,u \tag{8}$$

$$\frac{\Pr}{4}\frac{\partial T}{\partial t}-\Pr\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial y^2}+\left(S-R\right)\Pr T+Ec\Pr\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

$$\frac{Sc}{4}\frac{\partial C}{\partial t}-Sc\frac{\partial C}{\partial y}=\frac{\partial^2 C}{\partial y^2}-KrSc\,C \qquad (10)$$

(After dropping the asterisks)
The corresponding boundary conditions are

$$\begin{aligned} &u=0, T=1+\varepsilon e^{i\omega t}, C=1+\varepsilon e^{i\omega t} \quad && at \quad y=0 \\ &u\to 0, \quad T\to 0, \quad C\to 0 && as \; y\to\infty \end{aligned} \qquad (11)$$

Solution of the Problem

To solve equations (8), (9) and (10), we assume ω to be very small and the velocity, temperature and concentration in the neighborhood of the plate as

$$\begin{aligned} u &= u_0(y)+\varepsilon e^{i\omega t}u_1(y) \; +0\left(\varepsilon^2\right) \\ T &= T_0(y)+\varepsilon e^{i\omega t}T_1(y) \; +0\left(\varepsilon^2\right) \\ C &= C_0(y)+\varepsilon e^{nt}C_1(y)+0\left(\varepsilon^2\right) \end{aligned} \qquad (12)$$

Where u_0, T_0, and C_0 are mean velocity, mean temperature and mean concentration respectively.

Using (12) in equations (8), (9) and (10), equating harmonic and non-harmonic terms for mean velocity, mean temperature and mean concentration, after neglecting coefficient of ε^2, we get

$$R_m u_0''' + u_0'' + u_0' - Mu_0 = -Gr\,T_0 - Gc\,C_0 \qquad (13)$$

$$T_0'' + \Pr T_0' + (S-R)\Pr T_0 = -\Pr Ec\,u_0'^2 \qquad (14)$$

$$C_0'' + Sc\; C_0' - Sc\,Kr\,C_0 = 0 \qquad (15)$$

The equation (13) is third order differential equation due to presence of elasticity. Therefore u_0 is expanded using (Beard and Walters rule, 1964; Chowdhary and Islam, 2000)

$$u_0 = u_{00} + R_m u_{01} \qquad (16)$$

Zero-Order of R_m

$$u_{00}'' + u_{00}' - Mu_{00} = -Gr\,T_0 - Gc\,C_0 \qquad (17)$$

First-Order of R_m

$$u_{01}'' + u_{01}' - Mu_{01} = u_{00}''' \qquad (18)$$

Using multi parameter perturbation technique and assuming $Ec << 1$, we write

$$\begin{aligned} u_{00} &= u_{000}(y) + Ec\,u_{001}(y) \\ u_{01} &= u_{011}(y) + Ec\,u_{012}(y) \\ T_{00} &= T_{00}(y) + Ec\,T_{01}(y) \\ C_{00} &= C_{00}(y) + Ec\,C_{01}(y) \end{aligned} \qquad (19)$$

Using equations (19) in equations (14), (15), (17) and (18) and equating the coefficient of Ec^0 and Ec', we get the following sets of differential equations

Zero order of Ec

$$u_{000}'' + u_{000}' - Mu_{000} = -Gr\,T_{00} - Gc\,C_{00} \qquad (20)$$

$$u_{011}'' + u_{000}' - Mu_{011} = -u_{000}''' \qquad (21)$$

$$T_{00}'' + \Pr T_{00}' + (S-R)\Pr T_{00} = 0 \qquad (22)$$

$$C_{00}'' + Sc\; C_{00}' - Sc\,Kr\,C_{00} = 0 \qquad (23)$$

First order of Ec

$$u''_{001}+u'_{001}-Mu_{001}=-Gr\,T_{01}-Gc\,C_{01} \tag{24}$$

$$u''_{012}+u'_{012}-Mu_{012}=-u'''_{001} \tag{25}$$

$$T''_{01}+\Pr T'_{01}+(S-R)\Pr T_{01}=-\Pr Ecu'^{2}_{000} \tag{26}$$

$$C''_{01}+Sc\,C'_{01}-Sc\,Kr\,C_{01}=0 \tag{27}$$

Here primes denote differentiation with respect to y

The respective boundary conditions are

$$\begin{aligned} &u_{000}=u_{001}=u_{011}=u_{012}=0,\ T_{00}=1,\ T_{01}=0,\ C_{00}=1,\quad C_{01}=0 \qquad y=0\\ &u_{000}\to u_{001}\to u_{011}\to u_{012}\to 0, T_{00}\to T_{01}\to 0, C_{00}\to C_{00}\to 0 \quad y\to\infty \end{aligned} \tag{28}$$

Solving these differential equations from (20) – (27) using boundary conditions (28), then making use of equations (19) and finally with the help of (16), we obtain mean velocity u_0, mean temperature T_0 and mean concentration C_0 as follows.

$$\begin{aligned} u_0 &= A_1 e^{m_6 y}+A_2 e^{m_2 y}+A_3 e^{m_8 y}+Ec\{A_8 e^{m_{12} y}+A_9 e^{2m_8 y}+A_{10}e^{2m_6 y}+A_{11}e^{2m_2 y}+A_{12}e^{(m_6+m_8)y}\\ &+A_{13}e^{(m_2+m_6)y}+A_{14}e^{(m_2+m_8)y}+A_{15}e^{m_{14} y}\}+Rm\big[A_4 e^{m_8 y}+A_5 e^{m_6 y}+A_6 e^{m_2 y}+A_7 e^{m_{10} y}\\ &+Ec\{A_{16}e^{m_{14} y}+A_{17}e^{m_{12} y}+A_{18}e^{2m_8 y}+A_{19}e^{2m_6 y}+A_{20}e^{2m_2 y}+A_{21}e^{(m_6+m_8)y}+A_{22}e^{(m_2+m_6)y}\\ &+A_{23}e^{(m_2+m_8)y}+A_{24}e^{m_{16} y}\}\big] \end{aligned}$$

$$T_0=e^{m_6 y}+Ec\left\{B_1 e^{2m_8 y}+B_2 e^{2m_6 y}+B_3 e^{2m_2 y}+B_4 e^{(m_6+m_8)y}+B_5 e^{(m_2+m_6)y}+B_6 e^{(m_2+m_8)y}+B_7 e^{m_{12} y}\right\}$$

$$C_0=e^{m_2 y}$$

RESULTS AND DISCUSSION

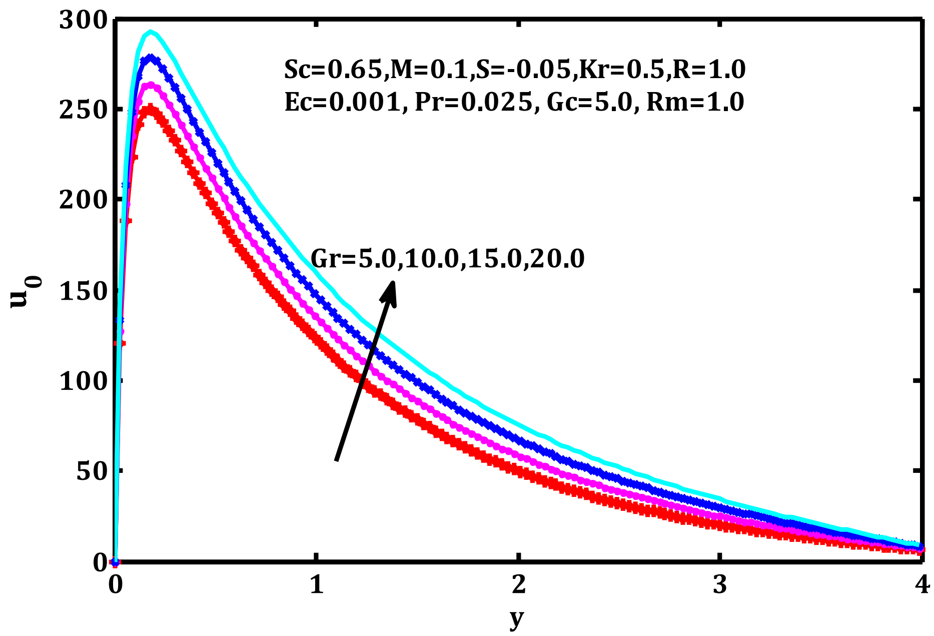

Figure 1: Mean velocity profiles for different values of Gr

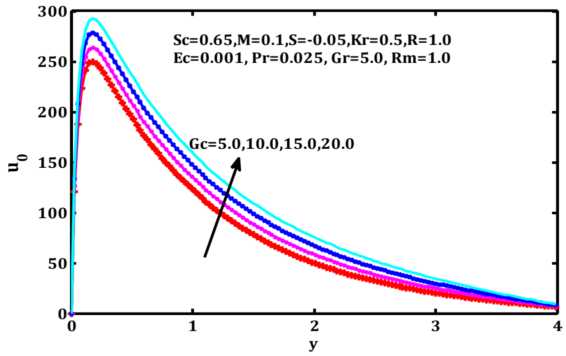

Figure 2: Mean velocity profiles for different values of Gc

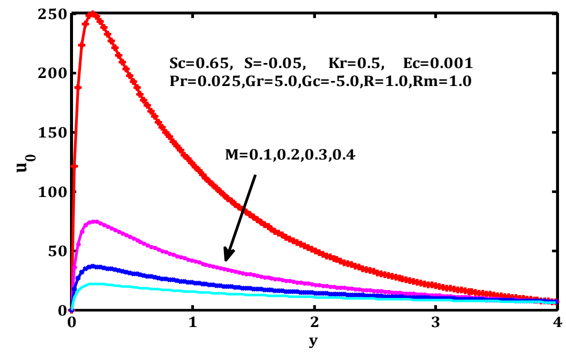

Figure 3: Mean velocity profiles for different values of M

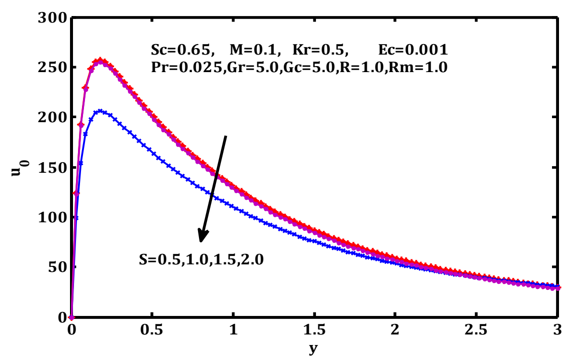

Figure 4: Mean velocity profiles for different values of S

The problem of radiation and mass transfer on unsteady hydromagnetic free convective memory flow of incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat absorbing sink with chemical reaction has been formulated, analysed and solved by using multi-parameter perturbation technique. Approximate solutions have been derived for the mean velocity, mean temperature and mean concentration. The effects of the flow parameters such as Hartmann number (M), suction parameter (S), thermal Grashof number for (Gr) and Solutal Grashof number (Gc), Schmidt number (Sc), Prandtl number (Pr) and Eckert number (Ec). An insight into the effects of these parameters of the flow field can be obtained by the study of the mean velocity components, mean temperature and mean concentration distributions. The components of the velocity $u_0(y)$ mean temperature $T_0(y)$ and mean concentration $C_0(y)$ have been plotted against the dimension y for several sets of the values of the parameters.

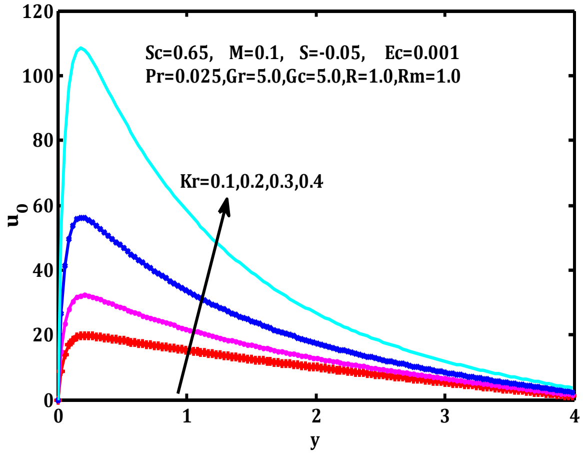

Figure 5: Mean velocity profiles for different values of Kr

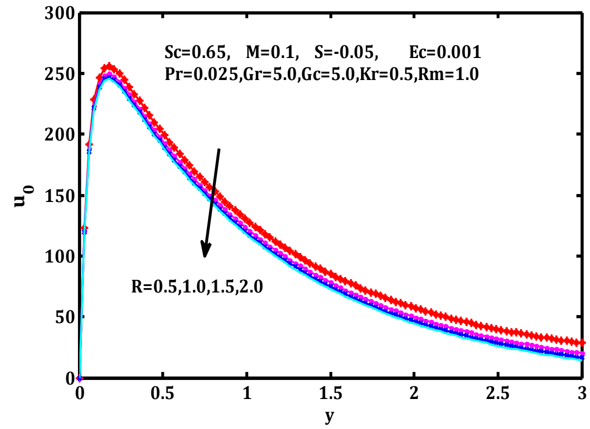

Figure 6: Mean velocity profiles for different values of R

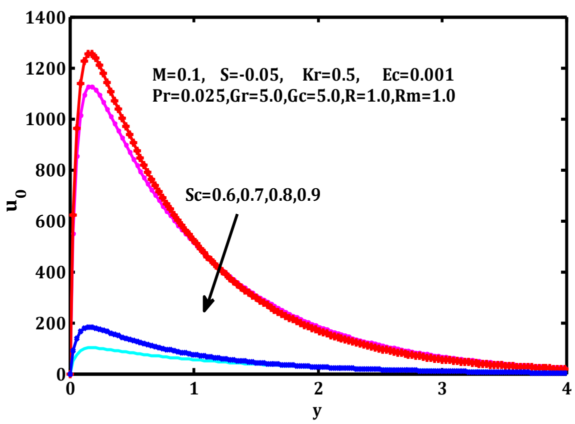

Figure 7: Mean velocity profiles for different values of Sc

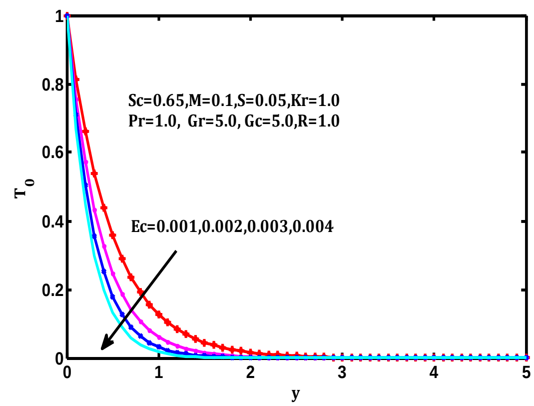

Figure 8: Mean velocity profiles for different values of Ec

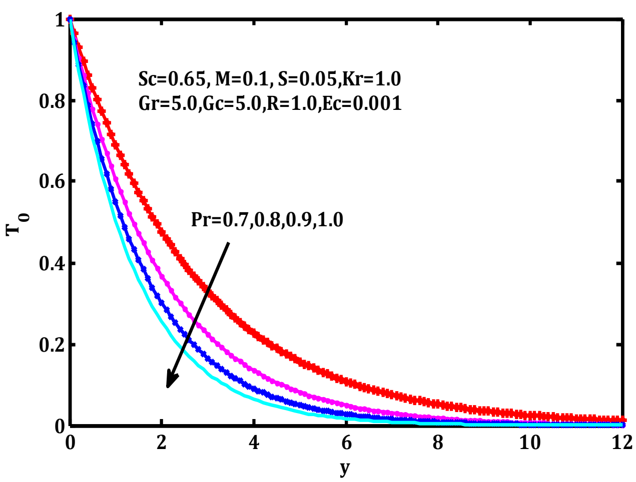

Figure 9: Mean velocity profiles for different values of Pr

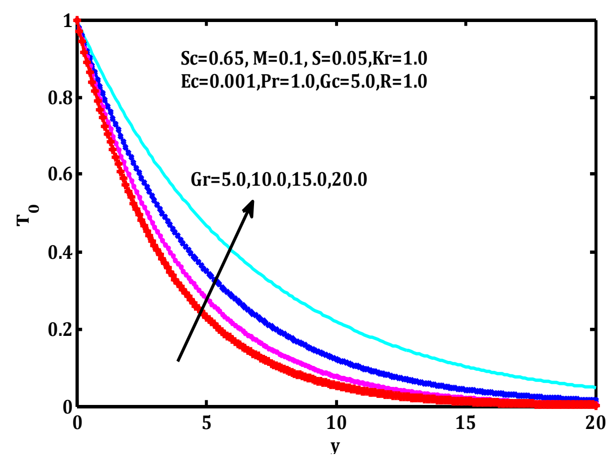

Figure 10: Mean velocity profiles for different values of Gr

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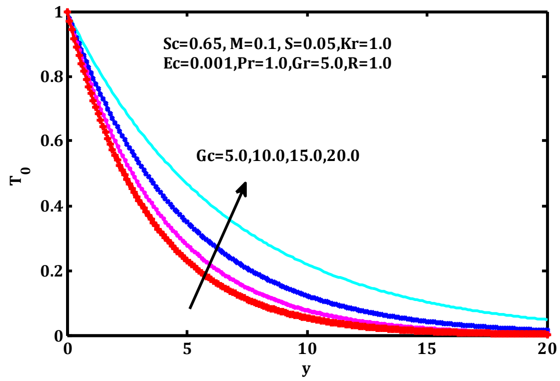

Figure 11: Mean velocity profiles for different values of Gc

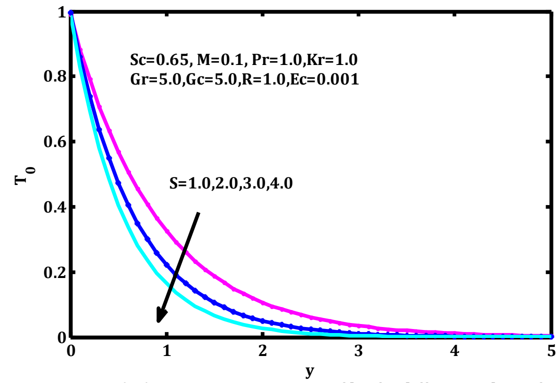

Figure 12: Mean velocity profiles for different values of S

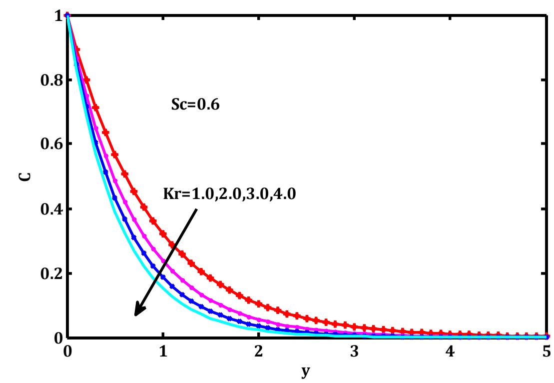

Figure 13: Mean velocity profiles for different values of Kr

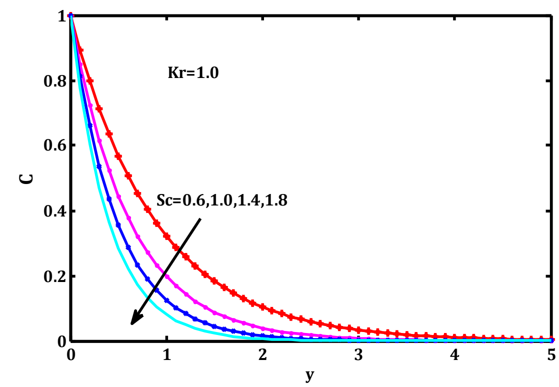

Figure 14: Mean velocity profiles for different values of Sc

Mean velocity profiles shows from figures (1) – (7). Figures (1) - (4) represent the mean velocity profiles due to variations in thermal Grashof number (Gr), Solutal Grashof number (Gc), Magnetic parameter (M) and Sink strength parameter (S). It is observed that the mean velocity increases with increase of thermal Grashof number and solutal Grashof number. It also observed that mean velocity decrease with increase in magnetic parameter and sink strength parameter, this is an indication that the force which tends to oppose the fluid flow increases with increase in the magnetic field parameter. Figures (5) - (7) reveals the mean velocity profiles due to variations in chemical reaction parameter (Kr), radiation parameter (R) and Schmidt number (Sc). It is noticed that whenever radiation and Schmidt number increases the mean velocity decrease. Also, from the figures, it can be concluded that the Newtonian fluid shows a rising trend as compared to visco-elastic fluid for both kind of surface systems. Further, slightly away from the plate the dispersion in the velocity profiles is considerable as compared to the initial stage. The reverse effect observed in chemical reaction parameter (Kr) in mean velocity.

Mean temperature profiles shows from figures (8) - (12). These figures reveals the mean temperature profiles due to variations in Eckert number (Ec), Prandtl number $(\Pr)$, thermal Grashof number (Gr), Solutal Grashof number (Gc) and Sink strength parameter (S). It is noticed that whenever Prandtl number, sink strength parameter and Eckert number increases the mean temperature decrease. It is also

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observed that the increases in Eckert number, Prandtl number and Sink strength parameter causes the decrease in mean temperature; while the mean temperature profile due to variations in Thermal Grashof number (Gr), Solutal Grashof number (Gc). It is noticed that whenever thermal Grashof number and solutal Grashof number increases the mean temperature also increase.

Mean concentration profiles shown in figure (12) and (13). From these figures it is observed that the increases in chemical reaction parameter (Kr) and Schmidt number (Sc) causes the decrease in mean concentration.

Conclusion

- The results indicate that as the radiation and magnetic parameters increase, the value of the velocity decreases. This conclusion meets the logic of the magnetic field exerting a retarding force on the free convection flow. Moreover, it is noted that there is a fall in the temperature due to the heat created by the viscous dissipation, free convection and heat source.
- An increase in the Grashof number, leads to a rise in the magnitude of fluid velocity due to enhancement in buoyancy force. The peak value of the velocity an increase rapidly near the porous plate as buoyancy force for heat transfer increases and then decays the free stream velocity.
- An increase in the chemical reaction parameter tends to increase the velocity and decrease the species concentration. The hydrodynamic and the concentration boundary layer become thin as the reaction parameter increases.
- An increase in Prandtl number leads to decrease in the thermal boundary layer and in general lower average temperature within the boundary layer region being the smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Prandtl number. Hence for smaller Prandtl number, the rate of heat transfer is reduced. This problem has many scientific and engineering applications such as:
- Flow of blood through the arteries.
- Soil mechanics, water purification, and powder metallurgy.
- Study of the interaction of the geomagnetic field with in the geothermal region.
- The petroleum engineer concerned with the movement of oil, gas and water through the reservoir of an oil or gas field.

It is hoped that the present work will serve as a vehicle for understanding more complex problems involving the various physical effects investigated in the present problem.

REFERENCES

Abd El-Naby MA, Elbarbary EME and Abdelazem NY (2003). Finite difference solution of radiation effects on MHD unsteady free convection flow over vertical plate with variable surface temperature, *Journal of Applied Mathematics* **2** 65.

Abel MS and Mahesha N (2008): Heat transfer in MHD viscoelastic fluid over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation, *Applied Mathematical Modelling* **32**(10)1965-1983.

Abel MS, Sanjayanand Emmanuel and Nandeppanavar Mahantesh M (2008). Viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations, *Communication Nonlinear Science and Numerical Simulation* **13**(9) 1808–1821.

Alagoa KD, Tay G and Abbey TM (1999). Radiative and free convection effects of a MHD flow through porous medium between infinite parallel plates with time dependent suction, *Astrophysics and Space Science* **260** 455-468.

Beard DM and Walters K (1964). Elastico-viscous boundary layer flows, two dimensional flows near a stagnation point. *Proceedings of the Cambridge Philosophical Society* **60** 667-674.

Bestman AR and Adjepong SK (1998). Unsteady hydromagnetic free convection flow with radiative transfer in a rotating fluid, *Astrophysics and Space Science* **143** 73-80.

Chang TB, Mehmood A, Beg OA, Narahari M, Islam MN and Ameen F (2011). Numerical study of transient free convective mass transfer in a Walters-B viscoelastic flow with wall suction, *Communications in Nonlinear Science and Numerical Simulation* **16** 216-225.

Chowdhury MK and Islam MN (2000). MHD free convection flow of visco-elastic fluid past an infinite vertical porous plate, *Heat and Mass Transfer* **36** 439.

Cogly AC, Vincentry WC and Gilles SE (1968). A Differential approximation for radiative transfer in a non-gray gas near equilibrium, *AIAA Journal* **6** 551-555.

Gireesh Kumar J and Satyanarayana PV (2011). Mass transfer effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink, *International Journal of Applied Mathematics and Mechanics* **7**(19) 97-109.

Gireesh Kumar J, Satya Narayana PV and Ramakrishna S (2009). Effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink, *Ultra Science* **21**(3) 639-650.

Hayat T, Javed T and Abbas Z (2008). Slip flow and heat transfer of a second grade fluid past a stretching sheet through a porous space, *International Journal of Heat and Mass Transfer* **51** 4528–4534.

Ingham D and Pop I (1998, 2002). Transport phenomena in porous media, *Pergamon, Oxford* **I, II**.

Ji Z, Rajagopal KR and Szeri AZ (1990). Multiplicity of solutions in Von Karman flows of viscoelastic fluids, *Journal of Non-Newtonian Fluid Mechanics* **36** 1–25.

Joseph DD (1990). *Fluid Dynamics of Viscoelastic Liquids* (New York: Springer-Verlag).

Kafousias NG and Raptis AA (1981). Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection, *Rev. Roum. Sci. Tech. Mec. Apl*. **26** 11-22.

Kafoussias NG (1992). The effects of mass transfer on free convective flow of a viscous and incompressible fluid past a vertical isothermal cone surface, *International Journal of Engineering Science* **30** 273.

Kesavaiah D Ch, Satya Narayana PV and Venkataramana S (2011). Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction, *International Journal of Applied Mathematics and Mechanics* **7**(1) 52-69.

Khan SK, Abel MS and Sonth RM (2003). Viscoelastic MHD flow heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work, *Heat and Mass Transfer* **40** 47–57.

Kim YJ and Fedorov AG (2003). Transient mixed radiative convection flow of a micropolar fluid past a moving semi-infinite vertical porous plate, *International Journal of Heat and Mass Transfer* **46** 1751.

Metzner AB and White JL (1965). Flow behaviour of visco - elastic fluids in the inlet region of a channel, *AIChE Journal* **11**(6) 989–995.

Mustafa S, Rafiuddin and Ramana Murthy MV (2008). Unsteady MHD memory flow with oscillatory suction, variable free stream and heat source, *ARPN Journal of Engineering and Applied Sciences* **3**(3) 17-24.

Nabil TM Eldabe, Sallam N Sallam and Mohamed Y Abou-zeid (2012). Numerical study of viscous dissipation effect on free convection heat and mass transfer of MHD non-Newtonian fluid flow through a porous medium, *Journal of the Egyptian Mathematical Society* 139-151.

Nield DA and Bejan A (1999). Convection in Porous Media, second edition (Springer, New York).

Oldroyd JG (1950). On the formulation of theological equations of state, *Proceedings of Royal Society (London) Series A* **200** 451–523.

Pillai KMC, Sai KS, Swamy NS, Nataraja HR, Tiwari SB and Rao BN (2004). Heat transfer in a viscoelastic boundary layer flow through a porous medium, *Computational Mechanics* **34** 27–37.

Pop I and Ingham D (2001). Convective heat transfer: mathematical and computational modeling of viscous fluids and porous media, *Pergamon, Oxford.*

Rajesh (2011). Heat source and mass transfer effects on MHD flow of an elasto-viscous fluid through a porous medium, *Annals of Faculty Engineering Hunedoara – International Journal of Engineering, Tome IX , Fascicule* **2** 205 – 212.

Ramana Murthy MV, Noushima Humera G, Rafiuddin and Chenna Krishan Reddy M (2007). MHD unsteady free convective Walter's memory flow with constant suction and heat sink, *ARPN Journal of Engineering and Applied Sciences* **2**(5) 12-16.

Rao IJ (1999). Flow of a Johnson–Segalman fluid between rotating coaxial cylinders with and without suction, *International Journal of Nonlinear Mechanics* **34**(1) 63–70.

Rao Rekha R and Finlayson Bruce A (1990). On the quality of viscoelastic flow solutions: an adaptive refinement study of a Newtonian and a Maxwell fluid, *International Journal for Numerical Methods in Fluids* **11**(5) 571–585.

Raptis AA and Tziyanidis GJ (1981). Viscoelastic flow past an infinite plate with suction and constant heat flux, *Journal of Physics* **14**(9) 129.

Renardy M (1997). High Wiesenberger number boundary layers for the upper convected Maxwell fluid, *Journal of Non-Newtonian Fluid Mechanics* **68** 1125–1132.

Rita Choudhury and Pabandhar (2014). Effects of MHD Visco-Elastic Fluid Flow Past a Moving Plate with Double Diffusive Convection in Presence of Heat Generation, *Wseas Transactions on Fluid Mechanics* **9** 89-101.

Rita Choudhury and Sajal Kumar Das (2013): Visco-Elastic MHD Fluid Flow Over a Vertical Plate with Dufour and Soret Effects, *International Journal of Scientific & Engineering Research* **4**(7) 11-17.

Rochelle SG and Peddieson J (1980). Viscoelastic boundary-layer flow past a parabola and a paraboloid, *International Journal of Engineering Science* **18**(6) 869–874.

Samria NK, Prasad R and Reddy MUS (1990). MHD free convection flow of an elasto-viscous fluid past an infinite vertical plate, *Astrophysics Physics and Space Science* **181** 125-134.

Seddeek MA (2001). Thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature-dependent viscosity, *Canadian Journal of Physics* **79** 725.

Sharma PK and Chaudhary RC (2003). Effect of variable suction on transient free convective viscous incompressible flow past a vertical plate with periodic temperature variations in slip flow regime, *Emirates Journal for Engineering Research* **8**(2) 33-38.

Sharma RC, Kumar P and Sharma S (2002). Rayleigh-Taylor instability of Walter' B elastico-viscous fluid through porous medium, *International Journal of Applied Mechanics and Engineering* **7** 433-444.

Sharma V and Rana GC (2001). Thermosolutal instability of Walters' (model B') visco-elastic rotating fluid permeated with suspended particles and variable gravity field in porous medium, *International Journal of Applied Mechanics and Engineering* **6** 843-860.

Siddappa B and Khapate BS (1975). Rivlin–Ericksen fluid flow past a stretching plate, *Rev Roum. Sci. Tech. Mech. Appl.* **21** 497.

Srinathuni Lavanya and D Chenna Kesavaiah (2014). Radiation and Soret Effects to MHD Flow in Vertical Surface with Chemical reaction and Heat generation through a Porous Medium, *International Journal of Computational Engineering Research* **4**(7) 62-73

Sudheer Babu M and Satya Narayana PV (2009). Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field, *JP Journal of Heat and Mass Transfer* **3** 219-234.

Trevisan OV and Bejan A (1985). Natural convection phenomena occurring inside a porous layer with both heat and mass transfer from the side, *International Journal of Heat Mass Transfer* **28** 1597.

Vafai K (2000). *Handbook of Porous Media* (Marcel Dekker, New York).

Vajravelu K and Hadjinicolaou A (1997). Convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream, *International Journal of Engineering Science* **35**(12) 1237.
Walter K (1960). The motion of an elastic-viscous liquid contained between concentric spheres, *Quarterly Journal of Mechanics and Applied Mathematics* **13**(3) 325-333.
Walter K (1960). The motion of an elastic-viscous liquid contained between coaxial cylinders (II), *Quarterly Journal of Mechanics and Applied Mathematics* **13**(4) 444-461.
Walters K (1962). Non-Newtonian effects in some elastico-viscous liquids whose behavior at small rates of shear is characterized by a general linear equation of state, *Quarterly Journal of Mechanics and Applied Mathematics* **15**(1) 63-76.

APPENDIX

$$m_2 = m_4 = -\left(\frac{Sc + \sqrt{Sc^2 + 4Kr\,Sc}}{2}\right),\; m_6 = m_{12} = -\left(\frac{\Pr + \sqrt{\Pr^2 + 4(S-R)\Pr}}{2}\right)$$

$$m_8 = m_{10} = m_{14} = m_{16} = -\left(\frac{1+\sqrt{1+4M}}{2}\right)$$

$$A_1 = -\frac{Gr}{m_6^2 + m_6 - M},\; A_2 = -\frac{Gc}{m_2^2 + m_2 - M},\; A_3 = -(A_1 + A_2),\; A_4 = -\frac{A_3 m_8^3}{m_8^2 + m_8 - M}$$

$$A_5 = -\frac{A_1 m_6^3}{m_6^2 + m_6 - M},\; A_6 = -\frac{A_2 m_2^3}{m_2^2 + m_2 - M},\; A_7 = -(A_4 + A_5 + A_6)$$

$$A_8 = -\frac{GrB_7}{m_{12}^2 + m_{12} - M},\; A_9 = -\frac{GrB_1}{4m_8^2 + 2m_8 - M},\; A_{10} = -\frac{GrB_2}{4m_6^2 + 2m_6 - M}$$

$$A_{11} = -\frac{GrB_3}{4m_2^2 + 2m_2 - M},\; A_{12} = -\frac{GrB_4}{(m_6+m_8)^2 + (m_6+m_8) - M}\; A_{13} = -\frac{GrB_5}{(m_2+m_6)^2 + (m_2+m_6) - M},$$

$$A_{14} = -\frac{GrB_6}{(m_2+m_8)^2 + (m_2+m_8) - M}$$

$$A_{15} = -(A_8 + A_9 + A_{10} + A_{11} + A_{12} + A_{13} + A_{14}),\; A_{16} = -\frac{A_{15} m_{14}^3}{m_{14}^2 + m_{14} - M}$$

$$A_{17} = -\frac{A_8 m_{12}^3}{m_{12}^2 + m_{12} - M},\; A_{18} = -\frac{8A_9 m_8^3}{4m_8^2 + m_8 - M},\; A_{19} = -\frac{8A_{10} m_6^3}{4m_6^2 + m_6 - M}\; A_{20} = -\frac{8A_{11} m_2^3}{4m_2^2 + m_2 - M},$$

$$A_{21} = -\frac{A_{12}(m_6+m_8)^3}{(m_6+m_8)^2 + (m_6+m_8) - M} \qquad A_{22} = -\frac{A_{13}(m_2+m_6)^3}{(m_2+m_6)^2 + (m_2+m_6) - M},$$

$$A_{23} = -\frac{A_{14}(m_2+m_8)^3}{(m_2+m_8)^2 + (m_2+m_8) - M}\; A_{24} = -(A_{16} + A_{17} + A_{18} + A_{19} + A_{20} + A_{21} + A_{22} + A_{23})$$

$$B_1 = -\frac{\Pr Ec m_8^2 A_3^2}{4m_8^2 + 2\Pr m_8 - (S-R)\Pr}, \qquad B_2 = -\frac{\Pr Ec m_6^2 A_1^2}{4m_6^2 + 2\Pr m_6 - (S-R)\Pr}$$

$$B_3 = -\frac{\Pr Ec m_2^2 A_2^2}{4m_2^2 + 2\Pr m_2 - (S-R)\Pr}\; B_4 = -\frac{2\Pr Ec m_8 A_3 m_6 A_1}{(m_6+m_8)^2 + \Pr(m_6+m_8) - (S-R)\Pr}$$

$$B_5 = -\frac{2\Pr Ecm_2 A_2 m_6 A_1}{(m_2+m_6)^2 + \Pr(m_2+m_6) - (S-R)\Pr}$$

$$B_6 = -\frac{2\Pr Ecm_2 A_2 m_8 A_3}{(m_2+m_8)^2 + \Pr(m_2+m_8) - (S-R)\Pr}, \quad B_7 = -(B_1 + B_2 + B_3 + B_4 + B_5 + B_6)$$