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***Research Article***

# PERFORMANCE MEASURES OF FM/FM/1 QUEUING SYSTEM WITH N-POLICY

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## ABSTRACT

In this paper we analyse the N-policy of FM/FM/1 Queuing system in Fuzzy environment which helps to control the queues in different situation. We have studied a N-policy queuing system with infinite capacity under uncertain arrival and service information. A mathematical Parametric Non-linear Programming (NLP) method is used to construct the membership function of the system characteristic of a N-policy queue with infinite capacity in which arrival rate, service rate, are fuzzy numbers. The α-cut and Zadeh's Extension Principle are used to transform a fuzzy queue into a family of conventional crisp queues. By means of membership functions of the system characteristics, a set of parametric non linear program is developed to calculate the lower and upper bound of the system characteristics function at α. Thus the membership functions of the system characteristics are constructed. Numerical example is also illustrated to check the validity of the proposed system.

***Keywords:*** *Infinite Capacity, N-Policy, Fuzzy Sets, α-cut, Membership Function and Zadeh's Extension Principle*

## INTRODUCTION

Queuing models with control operating policies are effective approaches for performance analysis of computer and telecommunication systems, manufacturing, production systems and inventory control. In N-policy the server is turned on when $N \geq 1$ and off when the system is empty. Queues with N-policy have been studied by many researchers like Kella, Lee *et al.*, Lee and Park, Pearn *et al.*, Wang *et al.*, Arumuganathan and Jeyakumar, Choudry and Madan. Rich and vast literature surveys are found for N-policy queues. Among the various paradigmatic changes in science and mathematics, one such change concerns the concept of uncertainity. Based on uncertainity Zadeh introduced a theory whose objects- fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree. The available literature shows that the simplification of fuzzy queuing system opens the road for creating realistic queuing models, which are potentially more useful than the commonly used crisp queues. Li and Lee investigated the analytical results for two typical fuzzy queues FM/FM/1/∞, M/F/1/∞ where F represents fuzzy time and FM represents fuzzified exponential distribution. Nagi and Lee proposed a procedure for using the α-cut and two variables simulation to analyse fuzzy queues. Unfortunately their approach provides only crisp solutions. Using parametric programming Kao *et al.*, constructed the membership functions of the system characteristics for fuzzy queues and applied them to simple fuzzy queues like F/M/1/∞, M/F/1/∞, FM/FM/1/∞ successfully. Chaun *et al.*, have obtained the membership function of system characteristics of a retail queuing model with fuzzy arrival, retail and service rate. Ritha and Sreelekha have analysed fuzzy N-policy queues with infinite capacity using Triangular membership function. With help of the available literature we analyse the N-policy of FM/FM/1 queues with infinite capacity by using α-cut and Zadeh's Extension Principle.

### *Model Description*

We consider the model FM/FM/1 in which arrival stream forms a poisson process and the actual number of customers in any arriving module are random variable. The server provides service to the customers with exponential service rate μ.The concept of N-policy FM/FM/1 queue was analyzed by Ritha and Sreelekha Menon and derived $N_s$, where $N_s$ is the expected number of customers in the system. We develop and analyse the membership function of the system characteristics of the expected number of

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customers in the system by α-cut and Zadeh's Extension Principle. And we also obtain for $N_q$, where $N_q$ is the expected number of customers in the queue. In N-policy M/M/1 queue with infinite capacity, it is assumed that arrival of customers follow a Poisson process with parameter λ. The serving times are according to exponential distribution with service rate μ. The server can serve only one customer at a time. Arrival of customers form a single waiting line at a server based on the order of their arrivals that is First-come First-served discipline. Let $N_s$ and $N_q$ denote the expected number of customers in the system and queue respectively. By Markov chain approach $N_s$ and $N_q$ are derived as

$$N_s = \frac{N-1}{2} + \frac{\lambda}{(\mu-\lambda)} \qquad N_q = \left[\frac{N-1}{2} + \frac{\lambda}{(\mu-\lambda)}\right] - 1$$

***FUZZY N-POLICY QUEUES**:* We consider an N-policy queuing model with infinite capacity in which arriving customers follow a Poisson process with a fuzzy arrival rate $\overline{\lambda}$ and service times are exponential with a fuzzy service rate $\overline{\mu}$, where $\overline{\lambda}$ and $\overline{\mu}$ are imprecise and uncertain. Let $\phi_{\overline{\lambda}}(p)\, and\ \phi_{\overline{\mu}}(q)$ denote the membership function of $\overline{\lambda}$ and $\overline{\mu}$ respectively. Then we have the following fuzzy sets as,

$$\overline{\lambda} = \{(p, \phi_{\overline{\lambda}}(p) / p \in P\} \qquad \overline{\mu} = \{(q, \phi_{\overline{\mu}}(q) / q \in Q\}$$

where P, Q are the crisp universal sets of the arrival rate, service rate for busy period. Let $f(p,q)$ denote the system characteristic of interest. Since $\overline{\lambda}\ and\ \overline{\mu}$ are fuzzy numbers. Then $f(\overline{\lambda}, \overline{\mu})$ is also fuzzy numbers. By Zadeh's Extension principle, the membership function of the system characteristics $f(\overline{\lambda}, \overline{\mu})$ is defined as $\phi_{f(\overline{\lambda},\overline{\mu})}(Z) = \sup_{p\in P, q\in Q} \min\{\phi_{\overline{\lambda}}(p), \phi_{\overline{\mu}}(q) / Z = f(p,q)\}$ where *f(p, q)* is given by

$f(p,q) = \left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]$. Then the membership function of the ECS is given by,

$$\phi_{ECS}(z) = \phi_{f(\overline{\lambda},\overline{\mu})}(Z) = \sup_{p\in P, q\in Q} \min\{\phi_{\overline{\lambda}}(p), \phi_{\overline{\mu}}(q) / Z = \left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]\} \text{----(1)}$$

The membership function in the equation (1) is not in the usual forms thus making it very difficult to imagine its shapes. For this we approach the problem using the mathematical programming techniques. Parametric NLP are developed to find α cut of $f(\overline{\lambda}, \overline{\mu})$ based on the extension principle.

***PARAMETRIC NON LINEAR PROGRAMMING:*** To construct the membership function $\phi_{ECS}(z)$, it is required to determine the α cuts of ECS. For that the α cuts of $\overline{\lambda}, \overline{\mu}$ are represented by crisp intervals as follows.

$$\lambda(\alpha) = \left[p_\alpha^L \quad p_\alpha^U\right] = \left(\min\{p \in P / \phi_{\overline{\lambda}}(p) \geq \alpha\}, \max\{p \in P / \phi_{\overline{\lambda}}(p) \geq \alpha\}\right) \text{-------(2a)}$$

$$\mu(\alpha) = \left[q_\alpha^L \quad q_\alpha^U\right] = \left(\min\{q \in Q / \phi_{\overline{\mu}}(q) \geq \alpha\}, \max\{q \in Q / \phi_{\overline{\mu}}(q) \geq \alpha\}\right) \text{------(2b)}$$

Further, the bounds of these intervals can be described as functions of α and can be obtained as,

$$p_\alpha^L = \min \phi_{\overline{\lambda}}^{-1}(\alpha),\ p_\alpha^U = \min \phi_{\overline{\lambda}}^{-1}(\alpha),\ q_\alpha^L = \min \phi_{\overline{\mu}}^{-1}(\alpha),\ q_\alpha^L = \min \phi_{\overline{\mu}}^{-1}(\alpha)$$

Therefore by making use of the α-cuts for ECS we construct the membership functions of (1) which is parameterized by α. To derive the membership function of ECS it is suffice to find the left and right shape function of $\phi_{ECS}(Z)$. This can be achieved by following the Zadeh's extension principle for $\phi_{ECS}(Z)$ which is the minimum of $\phi_{\overline{\lambda}}(p), \phi_{\overline{\mu}}(p)$. Now to derive $\phi_{ECS}(Z) = \alpha$ such that at least one of the following cases to be hold which satisfies $\phi_{ECS}(Z) = \alpha$.

Thus,

Case (i) $\phi_{\bar{\lambda}}(p) = \alpha$, $\phi_{\bar{\mu}}(q) \geq \alpha$

Case (ii) $\phi_{\bar{\lambda}}(p) \geq \alpha$, $\phi_{\bar{\lambda}}(q) = \alpha$

This can be accomplished by using parametric NLP techniques. The NLP techniques to find the lower and upper bounds of α cut of $\phi_{\overline{ECS}}(Z)$

for case (i) $[ECS]_{\alpha}^{L_1} = \min\left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]$

$[ECS]_{\alpha}^{U_1} = \max\left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]$ ------(3a)

for case (ii) $[ECS]_{\alpha}^{L_2} = \min\left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]$

$[ECS]_{\alpha}^{U_2} = \max\left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]$ ------(3b)

As $\lambda(\alpha), \mu(\alpha)$ given in equations (2a)&(2b) $p \in \lambda(\alpha), q \in \mu(\alpha)$ can be replaced by $p \in [p_{\alpha}^{L} \quad p_{\alpha}^{U}]$, $q \in [q_{\alpha}^{L} \quad q_{\alpha}^{U}]$ which are given by the α cuts and in turn they form a nested structure with respect to α which are expressed in (3a) & (3b).

Hence for given $0 < \alpha_2 < \alpha_1 < 1$, we have, $[p_{\alpha_1}^{L} \quad p_{\alpha_1}^{U}] \subseteq [p_{\alpha_2}^{L} \quad p_{\alpha_2}^{U}]$ , $[q_{\alpha_1}^{L} \quad q_{\alpha_1}^{U}] \subseteq [q_{\alpha_2}^{L} \quad q_{\alpha_2}^{U}]$

Thus equation (3a) has the unique smallest element and equation (3b) has the unique largest element. Now to find the membership function of $\phi_{\overline{ECS}}(Z)$ which is equivalent to find the lower bound of $[ECS]_{\alpha}^{L}$ and upper bound of $[ECS]_{\alpha}^{U}$ is written as

$[ECS]_{\alpha}^{L} = \min\left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]$ Such that $p_{\alpha}^{L} \leq p \leq p_{\alpha}^{U}$, $q_{\alpha}^{L} \leq q \leq q_{\alpha}^{U}$ ------- (4a)

$[ECS]_{\alpha}^{U} = \max\left[\frac{N-1}{2} + \frac{p}{(q-p)}\right]$, Such that $p_{\alpha}^{L} \leq p \leq p_{\alpha}^{U}$, $q_{\alpha}^{L} \leq q \leq q_{\alpha}^{U}$ ------- (4b)

That is at least any one of p, q must hit in the boundaries of their α cut which satisfies $\phi_{ECS}(Z) = \alpha$

By applying the results of Zimmerman [15] and convexity property we obtain,

$[ECS]_{\alpha_1}^{L} \geq [ECS]_{\alpha_2}^{L}$ and $[ECS]_{\alpha_1}^{U} \leq [ECS]_{\alpha_2}^{U}$ where $0 < \alpha_2 < \alpha_1 < 1$.In both $[ECS]_{\alpha}^{L}$ and $[ECS]_{\alpha}^{U}$ are invertible with respect to α then the left shape function $L(Z) = \left[[ECS]_{\alpha}^{L}\right]^{-1}$ and right shape function $R(Z) = \left[[ECS]_{\alpha}^{U}\right]^{-1}$ can be derived, such that

$$\phi_{\overline{ECS}}(Z) = \begin{cases} L(Z) & [ECS]_{\alpha=0}^{L} \leq Z \leq [ECS]_{\alpha=1}^{L} \\ 1 & [ECS]_{\alpha=1}^{L} \leq Z \leq [ECS]_{\alpha=1}^{U} \\ R(Z) & [ECS]_{\alpha=1}^{U} \leq Z \leq [ECS]_{\alpha=0}^{U} \end{cases}$$

In many cases, the value of $\{(ECS)_{\alpha}^{L} \; (ECS)_{\alpha}^{U} / \alpha \in [0 \quad 1]\}$ cannot be solved analytically, consequently a closed form membership function of Expected number of customers in the system cannot be obtained. The numerical solutions for $[ECS]_{\alpha}^{L}$ and $[ECS]_{\alpha}^{U}$ at different levels of α can be collected which approximate the shape of $L(Z)$ and $R(Z)$ that is the set of intervals $\{(ECS)_{\alpha}^{L} \; (ECS)_{\alpha}^{U} / \alpha \in [0 \quad 1]\}$ will estimate the shapes.

*Numerical Examples* (i). Expected Number of Customers in the System: For a FM/FM/1 queuing system, the corresponding parameters such as the arrival rate λ, service rate μ for busy period are fuzzy numbers. By letting $\bar{\lambda} = [0.02 \quad 0.03 \quad 0.04 \quad 0.05]$, $\bar{\mu} = [0.04 \quad 0.05 \quad 0.06 \quad 0.07]$.N=1001. Then the expected number of customers in the system(ECS) is given by $ECS = \left[\frac{N-1}{2} + \frac{p}{q-p}\right]$

where N is the number of customers and p, q are the fuzzy variable corresponding to $\bar{\lambda}, \bar{\mu}$ respectively. Thus, $[p_\alpha^L \quad p_\alpha^U] = [0.02 + \alpha \quad 0.05 - \alpha]$, $[q_\alpha^L \quad q_\alpha^U] = [0.04 + \alpha \quad 0.07 - \alpha]$

By substituting the above values, the effect of parameters on the expected number of customers in the system (ECS) is tabulated and their graphical representation is also shown below.

**Table 1: The α- cuts for the performance measure of ECS**

| a | ECS L | ECS U |
|---|---|---|
| 0 | 501 | 502.5 |
| 0.1 | 506 | 497.5 |
| 0.2 | 511 | 492.5 |
| 0.3 | 516 | 487.5 |
| 0.4 | 521 | 482.5 |
| 0.5 | 526 | 477.5 |
| 0.6 | 531 | 472.5 |
| 0.7 | 536 | 467.5 |
| 0.8 | 541 | 462.5 |
| 0.9 | 546 | 457.5 |
| 1 | 551 | 452.5 |

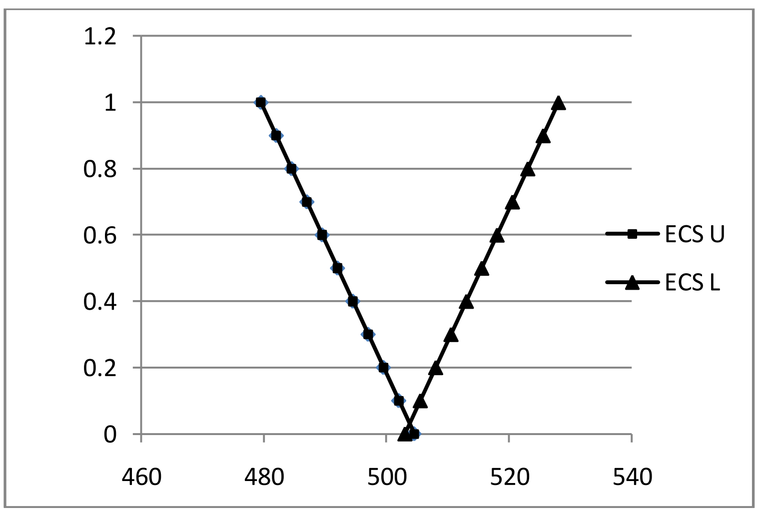


**Figure 1: The membership function of ECS**

**(ii)** Expected Number of Customers in the Queue: By letting $\bar{\lambda} = [0.04 \quad 0.06 \quad 0.08 \quad 0.10]$, $\bar{\mu} = [0.06 \quad 0.08 \quad 0.10 \quad 0.12]$, N=1001. Then the expected number of customers in the queue (ECQ) is given by $ECQ = \left[\frac{N-1}{2} + \frac{p}{q-p}\right] - 1$ where N is the number of customers and p, q are fuzzy variable corresponding to $\bar{\lambda}, \bar{\mu}$ respectively.

Thus, $\left[p_{\alpha}^{L} \quad p_{\alpha}^{U}\right]=\left[0.04+\alpha \quad 0.10-\alpha\right]$, $\left[q_{\alpha}^{L} \quad q_{\alpha}^{U}\right]=\left[0.06+\alpha \quad 0.12-\alpha\right]$

By substituting the above values, the effect of parameters on the expected number of customers in the Queue (ECQ) is tabulated and their graphical representation is also shown.

**Table 2: The α- cuts for the performance measure of ECQ**

| a | ECS L | ECS U |
|---|---|---|
| 0 | 503 | 504.5 |
| 0.1 | 505.5 | 502 |
| 0.2 | 508 | 499.5 |
| 0.3 | 510.5 | 497 |
| 0.4 | 513 | 494.5 |
| 0.5 | 515.5 | 492 |
| 0.6 | 518 | 489.5 |
| 0.7 | 520.5 | 487 |
| 0.8 | 523 | 484.5 |
| 0.9 | 525.5 | 482 |
| 1 | 528 | 479.5 |

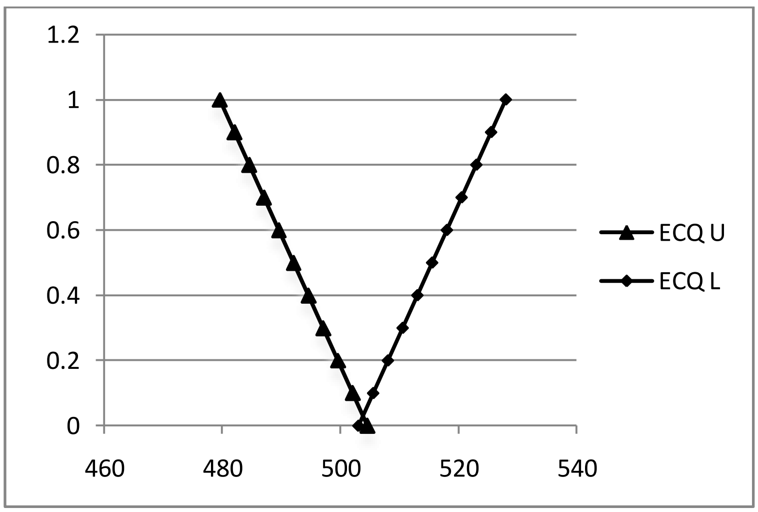


**Figure 2: The membership function for ECQ**

Crisp intervals of fuzzy ECS and ECQ of different possibilities of α levels are presented in Table 1 and Table 2. Further figure 1 and figure 2 depict the rough shape of ECS and ECQ of different α values. By performing α cuts of of ECS and ECQ at eleven distinct α levels 0, 0.1, 0.2, …….1.0 we note that the above information is very useful for designing the fuzzy queuing system.

**CONCLUSION**

In this paper a system of performance measure of fuzzy N-Policy queuing system has been studied. The fuzzy queuing model has more applicability in the real environments than the crisp systems. This paper applies the concept of α-cut and Zadhe's extension principle to FM/FM/1 and there by deriving the membership function of the Expected number of customers in the System and Queue for the model. By noting the expected number of customers we find that it is more meaningful to express ECS as a membership function rather than by a crisp value. It is a well established fact in the literature that in practical situation, the arrival rate and service rate are not known exactly. The benefit and significance of a fuzzy performance measure include maintaining the fuzziness of input information completely. Thus it can be concluded that the fuzzy queuing systems are more realistic and much more useful than the commonly used crisp queues for decision makers.

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