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***Research Article***

# A STOCHASTIC MODEL ON THE TIME TO RECRUITMENT FOR A SINGLE GRADE MANPOWER SYSTEM HAVING CORRELATED INTER-DECISION TIMES

**A. Amala Nancy, *A. Devi and A. Srinivasan**
*PG and Research Department of Mathematics,*
*Bishop Heber College, Trichy-17, India*
**Author for Correspondence*

**ABSTRACT**
In this paper, the problem of time to recruitment is studied for a single grade manpower system with attrition generated by correlated inter-policy decision times, using univariate policy of recruitment based on shock model approach. Two mathematical models are constructed using two different univariate policies of recruitment A different probabilistic analysis is made to derive the analytical result for variance of the time to recruitment for different cases on the distribution of the threshold for the cumulative loss of manpower in the organization for both the models.

***Keywords:*** *Single Grade Manpower System, Correlated Inter-Decision Times, Shock Model Approach, Univariate Policy of Recruitment and Variance of Time to Recruitment*

**INTRODUCTION**
A marketing organization is not free from attrition due to exodus of personnel when the management takes policy decisions regarding pay, perquisites and work targets. This attrition which leads to loss of manpower will adversely affect the smooth functioning of the organization if it turns its blind eye towards compensating this loss by recruitment. Frequent recruitment is not advisable as it involves more cost. In view of this situation and from the fact that the depletion of manpower and the inter-decision times are probabilistic, the organization requires an appropriate recruitment policy to plan for recruitment. Bartholomew and Frobes (1979), Bartholomew (1973) and Grinold *et al.*, (1977) have discussed manpower planning models using different kinds of wastage and different types of distributions by Markovian and renewal theoretic approach. The problem of time to recruitment for an organization having one grade is studied using shock model approach by several authors. Parthasarathy and Sathiyamoorthi (2003), Mariappan (2002), Kasturri (2007), Esther Clara (2012), have studied this problem by considering different conditions on the loss of manpower and threshold for cumulative loss of manpower when inter-decision times are exchangeable constantly correlated exponential random variables, using a univariate CUM policy of recruitment which states that recruitment is done whenever the cumulative loss of manpower crosses or exceeds a threshold. Poorni Perianayaki (2007), Esther Clara (2012) studied this problem using the univariate MAX policy of recruitment in which recruitment is done whenever the maximum loss of manpower crosses or exceeds a threshold. In all the above cited work, the analytical results for the moments of the time to recruitment are obtained through its distribution, using Laplace-Stieltjes transform. The method used in this paper is not only free from the conventional Laplace transform method, but it directly estimates the moments of the time to recruitment. Recently, Amala Nancy *et al.*, (2014) have obtained the variance of the time to recruitment for a single grade manpower system when the inter-decision times are independent and identically distributed random variables using a univariate Max policy of recruitment with a different probabilistic approach. The objective of the present paper is to extend the work of Amala Nancy *et al.*, (2014) for correlated inter-decision times.

***Model Description and Analysis for Model 1***
Consider an organization taking policy decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let $X_i$ be

independent and identically distributed continuous random variables representing the amount of depletion of manpower(loss of man hours) due to $i^{th}$ policy decision with distribution G(.) and probability density functions g(.); $S_i$ be the cumulative loss of manpower in the first i decisions; $U_i$ the time between $(i-1)^{th}$ and $i^{th}$ decisions, are exchangeable and constantly correlated exponential random variables with density function f(x) and mean u, $f_k^*(.)$ be the Laplace Stieltje's transform of $f_k(.)$,the probability density function of the waiting time $R_k$ upto k decisions and $\rho$ be the correlation between $U_i$ and $U_j$, $i\neq j$ and $v=u(1-\rho)$. Let T be a continuous random variable representing the threshold for the cumulative loss of manpower with distribution H(.) and probability density functions h(.).Let $\chi(A)$ be the indicator function of the event A. Let W be the time to recruitment for the organization with mean E(W) and variance V(W). The univariate CUM policy of recruitment employed in this model is stated as follows: Recruitment is done whenever the cumulative loss of man hours in the organization exceeds T.

***Main Results***

By the recruitment policy, recruitment is done whenever the cumulative loss of manpower exceeds the threshold T. When the first decision is taken, recruitment would not have been done for $U_1$ units of time. If the loss of manpower $X_1(=S_1)$ due to the first policy decision is greater than T, then recruitment is done and in this case $W=U_1=R_1$. However, if $S_1\leq T$, the non recruitment period will continue till the next policy decision is taken. If the cumulative $S_2$ of the loss of manpower in the first two decisions exceeds T, then recruitment is done and $W=U_1+U_2=R_2$. If $S_2\leq T$, then the non recruitment period will continue till the next policy decision is taken and depending on $S_3>T$ or $S_2\leq T$, recruitment is done or the non-recruitment period continues and so on. Hence

$$W=\sum_{i=0}^{\infty} R_{i+1}\chi(S_i\leq T<S_{i+1}) \tag{1}$$

From (1) and using the definition of $R_{i+1,}$ we get

$$E(W)=u\sum_{i=0}^{\infty}(i+1)P(S_i\leq T<S_{i+1}) \tag{2}$$

For the correlated inter-decision times it is found that (Esther Clara(2012))

$$E(R_{i+1}{}^2)=u^2\left[\rho^2 i(i+1)+(i+1)(i+2)\right] \tag{3}$$

Therefore from (1) and (3), we get

$$E(W^2)=\sum_{i=0}^{\infty}u^2\left[\rho^2 i(i+1)+(i+1)(i+2)\right]P(S_i\leq T<S_{i+1}) \tag{4}$$

By the law of total probability, we get

$$P(S_i\leq T<S_{i+1})=\int_0^{\infty}\int_0^{t}\overline{G}(t-x)\left(e_{S_i,T}(x,t)\right)dxdt \tag{5}$$

Using (5) in (2) and (4), we get

$$E(W)=u\sum_{i=0}^{\infty}(i+1)\int_0^{\infty}\int_0^{t}\overline{G}(t-x)\left(e_{S_i,T}(x,t)\right)dxdt \tag{6}$$

and

$$E(W^2)=\sum_{i=0}^{\infty}u^2\left[\rho^2 i(i+1)+(i+1)(i+2)\right]\int_0^{\infty}\int_0^{t}\overline{G}(t-x)\left(e_{S_i,T}(x,t)\right)dxdt \tag{7}$$

Equation (6) together with (7) gives the mean and variance of the time to recruitment for the present model which are obtained by using a different probabilistic analysis.

***Special Case***

We now obtain explicit analytical expressions for E(W) and Var(W) by assuming G(x)=1-exp(-αx) and considering several cases on different distributions for the threshold.

*Case (i)*

Suppose the distribution of the threshold is exponential with parameter 'θ'. From (6), (7) and on simplification we get

h(t)= $\theta e^{-\theta t}$

$$E(W)=\left(\frac{v}{1-\rho}\right)\left(\frac{\alpha+\theta}{\theta}\right) \quad (8)$$

$$E(W^2)=2\left(\frac{v}{1-\rho}\right)^2\left\{\frac{\left[(\alpha+\theta)+\rho^2\alpha\right](\alpha+\theta)}{\theta^2}\right\} \quad (9)$$

From (8) and (9), we get

$$V(W)=\left(\frac{v}{1-\rho}\right)^2\left(\frac{\left[(\alpha+\theta)+2\rho^2\alpha\right](\alpha+\theta)}{\theta^2}\right) \quad (10)$$

(8) and (10) give the mean and variance of the time to recruitment for case(i)

*Case (ii)*

Suppose the distribution of the threshold is an extended exponential with scale parameter 'θ' and shape parameter 2. In this case, from Gupta and Kundu (2001)

$$h(t)=2\theta e^{-\theta t}\left(1-e^{-\theta t}\right)$$

Therefore from (6) and (7) and on simplification, we get

$$E(W)=\left(\frac{v}{1-\rho}\right)\left\{2\left(\frac{\alpha+\theta}{\theta}\right)-\left(\frac{\alpha+2\theta}{2\theta}\right)\right\} \quad (11)$$

$$E(W^2)=4\left(\frac{v}{1-\rho}\right)^2\left\{\frac{\left[(\alpha+\theta)+\rho^2\alpha\right](\alpha+\theta)}{\theta^2}\right\}-2u^2\left\{\frac{\left[(\alpha+2\theta)+\rho^2\alpha\right](\alpha+2\theta)}{4\theta^2}\right\} \quad (12)$$

From (11) and (12), we get

$$V(W)=\left(\frac{4u^2\rho^2\alpha(\alpha+\theta)}{\theta^2}\right)-\left(\frac{2u^2\rho^2\alpha(\alpha+2\theta)}{4\theta^2}\right)-\left(3u^2\left(\frac{\alpha+2\theta}{2\theta}\right)^2\right)+\left(\frac{2u(\alpha+\theta)(\alpha+2\theta)}{\theta^2}\right) \quad (13)$$

(11) and (13) give the mean and variance of the time to recruitment for case(ii).

*Case (iii)*

Suppose the distribution of the threshold has SCBZ property with parameter '$\theta_1$' and '$\theta_2$'. In this case, from Rao *et al.*, (1990)

h(t)= $p(\theta_1+\mu)e^{-(\theta_1+\mu)t}+q\theta_2 e^{-\theta_2 t}$

Therefore from (6) and (7) and on simplification, we get

$$E(W)=\left(\frac{v}{1-\rho}\right)\left\{p\left(\frac{\alpha+\theta_1+\mu}{\theta_1+\mu}\right)+q\left(\frac{\alpha+\theta_2}{\theta_2}\right)\right\} \quad (14)$$

$$E(W^2)=\left(\frac{v}{1-\rho}\right)^2\left[2p\left\{\frac{\left[(\alpha+\theta_1+\mu)+\rho^2\alpha\right](\alpha+\theta_1+\mu)}{(\theta_1+\mu)^2}\right\}+2q\left\{\frac{\left[(\alpha+\theta_2)+\rho^2\alpha\right](\alpha+\theta_2)}{\theta_2^{\,2}}\right\}\right] \quad (15)$$

From (14) and (15), we get

$$V(W)=\left(\frac{v}{1-\rho}\right)^2\left[2p\left\{\frac{\left[(\alpha+\theta_1+\mu)+\rho^2\alpha\right](\alpha+\theta_1+\mu)}{(\theta_1+\mu)^2}\right\}+2q\left\{\frac{\left[(\alpha+\theta_2)+\rho^2\alpha\right](\alpha+\theta_2)}{\theta_2^{\,2}}\right\}\right]$$
$$-\left[\left(\frac{v}{1-\rho}\right)\left\{p\left(\frac{\alpha+\theta_1+\mu}{\theta_1+\mu}\right)+q\left(\frac{\alpha+\theta_2}{\theta_2}\right)\right\}\right]^2 \quad (16)$$

(14) and (16) give the mean and variance of the time to recruitment for case (iii).

***Model Description and Analysis for Model 2***

All the assumptions and notations are same as in model 1 expect the policy of recruitment. The univariate MAX policy of recruitment employed in model 2 is stated as follows: Recruitment is done whenever the maximum loss of man hours in the organization exceeds T.

***Main Results***

By the recruitment policy, recruitment is done whenever the maximum loss of manpower exceeds the threshold T. When the first decision is taken, recruitment would not have been done for $U_1$ units of time. If the loss of manpower $X_1(=Z_1)$ due to the first policy decision is greater than T, then recruitment is done and in this case $W=U_1=R_1$. However, if $Z_1\leq T$, the non recruitment period will continue till the next policy decision is taken. If the maximum $Z_2$ of the loss of manpower in the first two decisions exceeds T, then recruitment is done and $W= U_1 + U_2=R_2$. If $Z_2\leq T$, then the non recruitment period will continue till the next policy decision is taken and depending on $Z_3 >T$ or $Z_3\leq T$, recruitment is done or the non recruitment period continues and so on. Hence

$$W=\sum_{i=0}^{\infty} R_{i+1}\chi(Z_i \leq T < Z_{i+1}) \quad (17)$$

Proceeding as in the analysis for model 1, we get

$$E(W)=u\sum_{i=0}^{\infty}(i+1)P(Z_i \leq T < Z_{i+1}) \quad (18)$$

and

$$E(W^2)=\sum_{i=0}^{\infty}u^2\left[\rho^2 i(i+1)+(i+1)(i+2)\right]P(Z_i \leq T < Z_{i+1}) \quad (19)$$

where

$$P(Z_i \leq T < Z_{i+1})=\int_0^{\infty}\left[G(t)\right]^i h(t)dt\int_0^{\infty}\overline{G}(t)h(t)dt \quad (20)$$

Using (20) in (18) and (19) on simplification we get

$$E(W)=u(AB) \quad (21)$$

and

$$E(W^2)=2u^2A[(\rho^2+1)C-\rho^2B] \quad (22)$$

where $A=\int_0^{\infty}\overline{G}(t)h(t)dt$, $B=\int_0^{\infty}\left[\overline{G}(t)\right]^{-2}h(t)dt$ and $C=\int_0^{\infty}\left[\overline{G}(t)\right]^{-3}h(t)dt$ (23)

Equation (21) together with (22) give the mean and variance of the time to recruitment for the present model and these results are new in the context of using univariate MAX policy of recruitment with random threshold.

***Special Case***

We now obtain explicit analytical expressions for E(W) and V(W) by assuming G(x)= 1-exp(-αx) and considering several cases on different distributions for the threshold.

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*Case (i)*

Suppose the distribution of the threshold is Erlang with parameters 'θ' and 'k'.

In this case, equation (21) together with (22) give the mean and variance of the time to recruitment for model 2, where

$$A=\left[\frac{\theta k}{\alpha+\theta k}\right]^{k},\ B=\left[\frac{\theta k}{\theta k-2\alpha}\right]^{k} \text{ and } C\left[\frac{\theta k}{\theta k-3\alpha}\right]^{k}$$

Note:

The results for the performance measures using the present method for exponential threshold can be deduced for case(i) of model 2,by taking k=1.

*Case (ii)*

Suppose the distribution of the threshold has SCBZ property with parameter '$\theta_1$' and '$\theta_2$'.

In this case, equation (21) together with (22) give the mean and variance of the time to recruitment for model 2, where

$$A=\left[\frac{p(\theta_1+\mu)}{\alpha+\theta_1+\mu}+\frac{q\theta_2}{\alpha+\theta_2}\right],\ B=\left[\frac{p(\theta_1+\mu)}{\theta_1+\mu-2\alpha}+\frac{q\theta_2}{\theta_2-2\alpha}\right] \text{ and } C=\left[\frac{p(\theta_1+\mu)}{\theta_1+\mu-3\alpha}+\frac{q\theta_2}{\theta_2-3\alpha}\right]$$

*Case (iii)*

Suppose the distribution of the threshold is an extended exponential with scale parameter 'θ' and shape parameter 2.

In this case, equation (21) together with (22) give the mean and variance of the time to recruitment for model 2, where

$$A=\left[\frac{2\theta^2}{(\alpha+\theta)(\alpha+2\theta)}\right],\ B=\left[\frac{\theta^2}{(\theta-2\alpha)(\theta-\alpha)}\right] \text{ and } C=\left[\frac{2\theta^2}{(\theta-3\alpha)(2\theta-3\alpha)}\right]$$

**CONCLUSION**

From the results for the performance measures, we note that the mean and variance of the time to recruitment for both the models increase with $\rho$ when other parameters fixed. Hence, the results obtained are better from the organization's point of view when the inter-decision times are correlated. In the context of attrition, the model developed in this paper can be utilized to plan for adequate provision of manpower in the organization. The goodness of fit for the distribution assumed in this paper can be tested by collecting relevant data. Further, the observations on the performance measures given in this paper will be useful to enhance the facilitation of the assessment of the manpower profile in future manpower development prediction, not only on industry but also in a broader domain.

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