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Research Article

ELASTODYNAMIC PROBLEM OF TWO COLLINEAR GRIFFITH CRACKS IN AN ELASTIC STRIP

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ABSTRACT
In this paper an elastodynamic problem of two collinear Griffith crack in an infinite orthotropic strip of finite thickness with stress free boundary is considered. An integral transform technique is employed to solve the problem. The asymptotic expressions for the stresses are derived. Approximate analytical expressions for dynamic stress intensity factors are obtained by retaining terms up to the order of h^{-4} for large h . Graphical plots of numerical results for different orthotropic materials are also presented.

Keywords: *Griffith Crack, Orthotropic Elastic Medium, Local Stress Field and Stress Intensity Factor (Sif)*

INTRODUCTION
Orthotropic materials such as composites are widely used in different branches of engineering. Since the ratio of strength to weight of such materials in many cases is higher than other conventional engineering materials, applications of the orthotropic materials have been widely expanded. There has been increasing interest in analytical solutions of elastodynamic crack problems in an anisotropic medium, particularly in an orthotropic medium, due to their importance and usefulness from the technological point of view. Atkinson (1965) considered the problem of steady-state propagation of a semi-infinite crack in an aelotropic material using the Cauchy-integral formula. Crack propagation problems in orthotropic elastic media have been considered by Kassir and Tse(1983), Danyluk and Singh (1984), Arcisz and Sih (1984), Piva(1986), Piva and Viola (1988) and many others. The analysis of the work done by Kassir and Tse (1983) has been extended by De and Patra (1992) to an orthotropic strip of finite thickness through an integral transform technique. Ang (1988) obtained the dynamic stress intensity factor around a crack in an anisotropic layer sandwiched between two anisotropic half-planes. Itou (1989) solved the dynamic problem of two coplanar Griffith cracks in an orthotropic layer sandwiched between two elastic half-planes by reducing the problem to a pair of dual integral equations. De and Patra (1993) have solved the elastodynamic problems of i) two equal collinear Griffith cracks, ii) an infinite row of parallel cracks and iii) a 'grid' of cracks of equal arms, propagating with constant speed in a stressed orthotropic medium through a complex variable approach. In recent years, some significant work have been done by several authors viz. Rizk (2006) investigated the analysis of the elastic homogeneous orthotropic semi-infinite plate with an internal crack and edge crack perpendicular to the boundary under thermal shock. Nobile and Carloni (2005) analytically investigated fracture behavior of composite materials, Matbuly (2006) obtained analytical solution for an interfacial crack subjected to dynamic anti-plane shear loading. Piva, Viola and Tornabene (2005) studied crack propagation in an orthotropic medium with coupled elastodynamic properties.

In the present work, the distribution of stress due to the steady-state propagation of two equal collinear Griffith crack in an infinite orthotropic strip of finite thickness 2h with stress free boundary is presented. It is assumed that cracks are propagating with constant speed c and without change in length along the positive x axis. An integral transform technique is used so that the problem has been reduced to the solution of a set of triple integral equations. These triple integral equations are reduced to a Fredholm integral equation of the second kind using finite Hilbert technique, which is finally solved by an iterative procedure. Approximate analytical expressions for dynamic stress intensity factors are derived by retaining terms up to the order of h^{-4}, for large h. Numerical calculations are carried out for Magnesium and Beryllium and corresponding graphical plots are presented.

The Elastodynamic Problem

We consider a problem of two collinear Griffith cracks of finite length in an infinite orthotropic strip with stress free boundary. The cracks, defined by the relation $a \le |X| \le 1, Y = \pm 0$ are propagating with constant speed c, without change in length along the positive x axis, where the coordinate axes x,y,z coincide with the axes of elastic symmetry of the material.

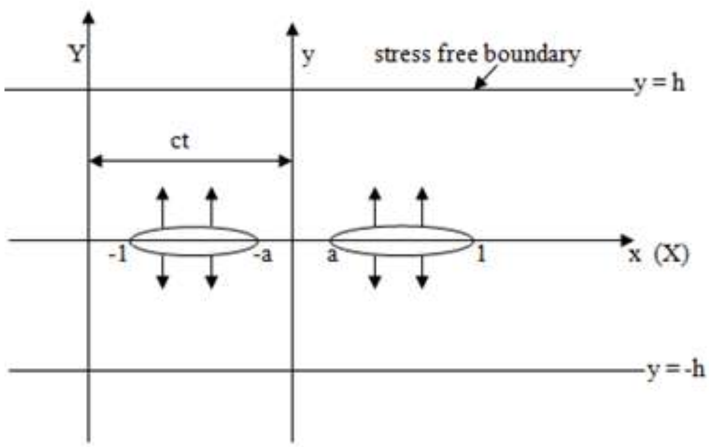

Figure 1: Two collinear Griffith cracks propagating with constant speed c along the x axis

Since the problem is considered in absence of body force, the equations restricted to motion in the xy-plane are given below

$$C_{11}\frac{\partial^2 U}{\partial X^2} + \mu_{12}\frac{\partial^2 U}{\partial Y^2} + (C_{12} + \mu_{12})\frac{\partial^2 V}{\partial X \partial Y} = \rho\frac{\partial^2 U}{\partial t^2} \tag{2.1}$$

$$C_{22}\frac{\partial^2 V}{\partial Y^2} + \mu_{12}\frac{\partial^2 V}{\partial X^2} + (C_{12} + \mu_{12})\frac{\partial^2 U}{\partial X \partial Y} = \rho\frac{\partial^2 V}{\partial t^2} \tag{2.2}$$

where $U = U(X,Y,t), V = V(X,Y,t)$ are displacement components in X and Y directions, t is time, ρ is the density of the material and μ_{12}, C_{ij} are elastic coefficients.

Applying Galilean transformation $x = X - ct, y = Y, t = t$, and assuming $U = u(x,y), V = v(x,y)$, the system of equation (2.1) and (2.2) reduce to

$$\frac{\partial^2 u}{\partial x^2} + 2n\frac{\partial^2 v}{\partial x \partial y} + m\frac{\partial^2 u}{\partial y^2} = 0, \tag{2.3}$$

$$\frac{\partial^2 v}{\partial x^2} + 2n_1\frac{\partial^2 u}{\partial x \partial y} + m_1\frac{\partial^2 v}{\partial y^2} = 0. \tag{2.4}$$

where

$$2n = \frac{C_{12} + \mu_{12}}{C_{11}(1 - M_1^2)}, 2n_1 = \frac{C_{12} + \mu_{12}}{\mu_{12}(1 - M_2^2)}, \tag{2.5}$$

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$$m=\frac{\mu_{12}}{C_{11}(1-M_1^2)},m_1=\frac{C_{22}}{\mu_{12}(1-M_2^2)}, \tag{2.6}$$

in which $M_j=\frac{c}{v_j}(j=1,2)$ and $v_1=\left(\frac{C_{11}}{\rho}\right)^{\frac{1}{2}},v_2=\left(\frac{\mu_{12}}{\rho}\right)^{\frac{1}{2}}$ are assumed to be less than 1 (subsonic propagation).
As the problem under consideration is symmetrical with respect to the x-axis, it is sufficient to consider the half-strip $0\le y\le h$.
The Boundary conditions of the problem are given by

$$\sigma_y(x,0)=p_0f(x),a<|x|<1 \tag{2.7a}$$
$$v(x,0)=\ 0,0<|x|<a,|x|>1 \tag{2.7b}$$
$$\tau_{xy}(x,0)=0,-\infty<x<\infty \tag{2.7c}$$

where $f(x)$ is a prescribed function.

As the boundary $y=h$ is assumed to be stress free, we have

$$\tau_{xy}(x,h)=0,-\infty<x<\infty \tag{2.7d}$$
$$\sigma_y(x,h)=0,-\infty<x<\infty \tag{2.7e}$$

where x,y are moving axis attached with the crack.
Solution of the Problem
An integral solution of equations (2.3) and (2.4) can be written as

$$u(x,y)=\int_0^\infty A(s,y)sin(sx)\,ds,$$
$$v(x,y)=\int_0^\infty B(s,y)cos(sx)\,ds, \tag{3.1}$$

where A and B are arbitrary functions. Substituting (3.1) into equations (2.3) and (2.4), the following differential equations are obtained,

$$-\mu_{12}\frac{\partial^2 A}{\partial y^2}+(C_{12}+\mu_{12})s\frac{\partial B}{\partial y}+(C_{11}-c^2\rho)s^2A=0, \tag{3.2}$$
$$-C_{22}\frac{\partial^2 B}{\partial y^2}-(C_{12}+\mu_{12})s\frac{\partial A}{\partial y}+(\mu_{12}-c^2\rho)s^2B=0. \tag{3.3}$$

Solution of (3.2) and (3.3) are given by

$$A(s,y)=A_1(s)ch(\gamma_1 sy)+A_2(s)ch(\gamma_2 sy)+C_1(s)sh(\gamma_1 sy)+C_2(s)sh(\gamma_2 sy), \tag{3.4}$$
$$B(s,y)=B_1(s)sh(\gamma_1 sy)+B_2(s)sh(\gamma_2 sy)+D_1(s)ch(\gamma_1 sy)+D_2(s)ch(\gamma_2 sy), \tag{3.5}$$

where $A_j(s),C_j(s)\ (j=1,2)$e are arbitrary functions and $B_j(s),D_j(s)$ are related to $A_j(s),C_j(s)$ by

$$\left.\begin{aligned}B_j(s)&=-\frac{\alpha_j}{\gamma_j}A_j(s),(j=1,2)\\ D_j(s)&=-\frac{\alpha_j}{\gamma_j}C_j(s),(j=1,2)\end{aligned}\right\}. \tag{3.6}$$

with

$$\alpha_j = \frac{C_{12} + 2\mu_{12} - c^2\rho - \gamma_j^2\mu_{12}}{C_{12} + \mu_{12}} \tag{3.7}$$

where γ_1^2 and γ_2^2 are positive roots of the equation

$$\mu_{12}(C_{12} + 2\mu_{12})\gamma^4 + [(C_{12} + \mu_{12})^2 - (C_{12+}2\mu_{12})(C_{12} + 2\mu_{12} - c^2\rho) - \mu_{12}(\mu_{12} - c^2\rho)]\gamma^2 + (C_{12} + 2\mu_{12} - c^2\rho)(\mu_{12} - c^2\rho) = 0. \tag{3.8}$$

From boundary condition (2.7c) we have,

$$C_2(s) = -\frac{\beta_1\gamma_2}{\beta_2\gamma_1}C_1(s) \tag{3.9a}$$

where

$$\beta_j(s) = \alpha_{j+}\gamma_j^2 \ (j = 1,2). \tag{3.9b}$$

Boundary conditions (2.7d) and (2.7c) in conjunction with (3.9) yield

$$\left.\begin{aligned} A_1(s) &= \delta_1(sh)c_1(s) \\ A_2(s) &= \delta_2(sh)c_2(s) \end{aligned}\right\} \tag{3.10}$$

where

$$\delta_1(sh) = \frac{(C_{12} - \alpha_2 C_{22})\left(\frac{\gamma_1^2 + \alpha_1}{\gamma_1}\right)ch(\gamma_2 sh)ch(\gamma_1 sh) - (C_{12} - \alpha_1 C_{22})\left(\frac{\gamma_2^2 + \alpha_2}{\gamma_2}\right)sh(\gamma_1 sh)sh(\gamma_2 sh) - \beta_1\frac{(\gamma_2^2 + \alpha_2)}{\beta_2\gamma_1}(C_{12} - \alpha_2 C_{22})}{(C_{12} - \alpha_1 C_{22})\left(\frac{\gamma_2^2 + \alpha_2}{\gamma_2}\right)ch(\gamma_1 sh)sh(\gamma_2 sh) - (C_{12} - \alpha_2 C_{22})\left(\frac{\gamma_1^2 + \alpha_1}{\gamma_2}\right)ch(\gamma_2 sh)sh(\gamma_1 sh)}, \tag{3.11}$$

$$\delta_2(sh) = \frac{\frac{\beta_1\gamma_2}{\beta_2\gamma_1}(C_{12} - \alpha_2 C_{22})\left(\frac{\gamma_1^2 + \alpha_1}{\gamma_1}\right)sh(\gamma_1 sh)sh(\gamma_2 sh) - \frac{\beta_1\gamma_2}{\beta_2\gamma_1}(C_{12} - \alpha_1 C_{22})\left(\frac{\gamma_2^2 + \alpha_2}{\gamma_2}\right)ch(\gamma_1 sh)ch(\gamma_2 sh) + (C_{12} - \alpha_1 C_{22})\left(\frac{\gamma_1^2 + \alpha_1}{\gamma_1}\right)}{(C_{12} - \alpha_1 C_{22})\left(\frac{\gamma_2^2 + \alpha_2}{\gamma_2}\right)ch(\gamma_1 sh)sh(\gamma_2 sh) - (C_{12} - \alpha_2 C_{22})\left(\frac{\gamma_1^2 + \alpha_1}{\gamma_2}\right)ch(\gamma_2 sh)sh(\gamma_1 sh)}. \tag{3.12}$$

Again the boundary conditions (2.7a) and (2.7b), together with the relation (3.9) yield the following triple integral equations for $c_1(s)$

$$\int_0^\infty sc_1(s)\,[1 + M(sh)]cos(sx)ds = \frac{-p_0 f(x)}{D}, a < x < 1 \tag{3.13}$$

$$\int_0^\infty c_1(s)\,cos(sx)ds = 0, 0 \le x < a, x > 1 \tag{3.14}$$

where D is a constant given by

$$D = C_{12} - \alpha_1 C_{22} - \frac{\gamma_2\beta_1}{\gamma_1\beta_2}(C_{12} - \alpha_2 C_{22}). \tag{3.15}$$

Here, we note that $M(sh) \to 0$ as $h \to \infty$.

Assuming $c_1(s) = \frac{1}{s}\int_a^1 h(t^2)sin(st)dt$, (3.16)

we found that the equation (3.16) is identically satisfied only if

$$\int_a^1 h(x^2)dx = 0 \tag{3.17}$$

and equation (3.13) leads to the following Fredhlom integral equation

$$h(x^2) + \int_a^1 h(t^2)k(x^2, t)dt = F(x^2), a < x < 1 \tag{3.18a}$$

where, $k(x^2, t) = -\frac{4}{\pi^2}\sqrt{\frac{x^2 - a^2}{1 - x^2}}\int_a^1 \sqrt{\frac{1 - y^2}{y^2 - a^2}} \times \frac{y}{y^2 - x^2}k_1(y, t)dy$ (3.18b)

with $k_1(y, t) = \int_0^\infty M(sh)cos(sy)sin(st)ds$

and $F(x^2) = \frac{4p_0 f(x)}{\pi^2 D}\sqrt{\frac{x^2-a^2}{1-x^2}}\int_a^1 \sqrt{\frac{1-y^2}{y^2-a^2}} \times \frac{ydy}{y^2-x^2} + \frac{k''}{\sqrt{(x^2-a^2)(1-x^2)}}$ (3.18c)

with k'' being an arbitrary constant determine by using conditions (3.17).

Now if we take $h \gg 1$ and set $\xi = sh$, then equation (3.18b) leads to

$$k(x^2,t) = \frac{2}{\pi}\sqrt{\frac{x^2-a^2}{1-x^2}}\left[I_0 t h^{-2} + I_1 t h^{-4}\left(t^2+3x^2-\frac{3}{2}k^2\right)\right] + O(h^{-6}) \qquad (3.19a)$$

where

$$I_j = \frac{(-1)^j}{(2j+1)!}\int_0^\infty \xi^{2j+1} M(\xi) d\xi, (j = 0,1)$$

and $k^2 = 1 - a^2$. (3.19b)

Integrating (3.18a) with respect to x from a to 1,
we have

$$\int_a^1 h(x^2)\,dx + \int_a^1\left[\int_a^1 h(t^2)k(x^2,t)dt\right]dx$$
$$= \int_a^1\left[\frac{4p_0 f(x)}{\pi^2 D}\sqrt{\frac{x^2-a^2}{1-x^2}}\left\{\sqrt{\frac{1-y^2}{y^2-a^2}} \times \frac{ydy}{y^2-x^2}\right\}dx + \int_a^1 \frac{k''}{\sqrt{(x^2-a^2)(1-x^2)}}dx\right]. \qquad (3.19c)$$

Particular case: Taking $f(x) = 1$ and utilizing condition (3.17) and (3.19a), the above equation becomes

$$k'' = \frac{2p_0}{\pi D}\left(\frac{E}{F} - a^2\right) + \frac{1}{F}\int_a^1 h(t^2)\,k(t)dt$$

where

$$k(t) = \frac{2}{\pi}\left[\frac{I_0 t}{h^2}(E - a^2F) + \frac{I_1 t}{h^4}\left\{\left(t^2 - \frac{3}{2}k^2\right)(E - a^2F) - a^2(E + F + 2E)\right\}\right]\left[\frac{I_0 t}{h^2}\right.$$
$$\left. + \frac{I_1 t}{h^4}\left(t^2 + 3x^2 - \frac{3}{2}k^2\right)\right] + O(h^{-6}), \qquad (3.20a)$$

E and F are known as elliptic integrals of 1st and 2nd kind respectively.

Now, we consider the problem of propagation of two collinear Griffith Cracks in an orthotropic strip of sufficiently large thickness($i.e., h \gg 1$), then for large h, the integral equation (3.20a) reduces to the form

$$h(x^2) + \int_a^1 h(t^2)L(x^2,t)dt = S(x^2), a < x < 1 \qquad (3.21a)$$

where

$$L(x^2,t) = \frac{2t}{\pi\sqrt{(x^2-a^2)(1-x^2)}}\left[\frac{I_0}{h^2}\left(x^2 - \frac{E}{F}\right)\right.$$
$$\left. + \frac{I_1}{h^4}\left\{\left(t^2 - \frac{3}{2}k^2\right)\left(x^2 - \frac{E}{F}\right) + 3x^2(x^2 - a^2) + \frac{Ea^2}{F} + a^2 - \frac{2E}{F}\right\}\right] + O(h^{-6}) \qquad (3.21b)$$

and

$$S(x^2) = \frac{2p_0\left(\frac{E}{F} - x^2\right)}{\pi D\sqrt{(x^2-a^2)(1-x^2)}}. \qquad (3.21c)$$

As $h \gg 1$ and $|L(x^2,t)| < 1$ the solution of integral equation (3.21a) is given by

$$h(x^2) = h_0(x^2) + \frac{1}{h^2}h_1(x^2) + \frac{1}{h^4}h_2(x^2) + O(h^{-6}) \qquad (3.22)$$

where

$$h_0(x^2) = \frac{-2p_0\left(x^2 - \frac{E}{F}\right)}{\pi D\sqrt{(x^2-a^2)(1-x^2)}}, \qquad (3.23a)$$

$$h_1(x^2) = \frac{p_0 I_0 C_0\left(x^2 - \frac{E}{F}\right)}{\pi D\sqrt{(x^2-a^2)(1-x^2)}}, \qquad (3.23b)$$

$$h_2(x^2) = \frac{-p_0 C_0}{2\pi D\sqrt{(x^2-a^2)(1-x^2)}}\left[I_0^2\left(x^2 - \frac{E}{F}\right)C_0 - 2I_1(3x^4 + C_1x^2 + C_2)\right], \qquad (3.23c)$$

with

$$C_0 = 1 + a^2 - 2\frac{E}{F}, C_1 = \frac{k^4}{4C_0} - (1+a^2), C_2 = a^2 + \frac{E}{F}\left(C_1 - \frac{k^4}{4C_0}\right). \qquad (3.23d)$$

Again for large value of h, asymptotic expansion of $\delta_1(sh)$ and $\delta_2(sh)$ are given by

$$\delta_1(sh) \sim -1 + \nu_1 e^{-(\gamma_1+\gamma_2)sh}$$

$$\delta_2(sh) \sim \nu_2 - \nu_3 e^{-\gamma_2 sh} \qquad (3.24a)$$

where

$$\nu_1 = \frac{4\beta_1\alpha_1\gamma_1}{\beta_2(\alpha_1\gamma_2 - \alpha_2\gamma_1)}, \nu_2 = \frac{\beta_1\gamma_2}{\beta_2\gamma_1}, \nu_3 = \frac{2\beta_1\gamma_2(\alpha_1\gamma_2 + \alpha_2\gamma_1)}{\beta_2\gamma_1(\alpha_1\gamma_2 - \alpha_2\gamma_1)}. \qquad (3.24b)$$

Using the above asymptotic expansion of $\delta_1(sh)$ and$\delta_2(sh)$, the stresses at any point of the solid are given by

$$\sigma_x = (C_{11} - \alpha_1 C_{12})\left(J_1 + \nu_1 J_1^{(1)}\right) + (\alpha_2 C_{12} - C_{11})\left(\nu_2 J_2 + \nu_3 J_2^{(1)}\right),$$

$$\sigma_y = (C_{12} - \alpha_1 C_{22})\left(J_1 + \nu_1 J_1^{(1)}\right) + (\alpha_2 C_{22} - C_{12})\left(\nu_2 J_2 + \nu_3 J_2^{(1)}\right),$$

$$\tau_{xy} = \mu_{12}\left[\frac{\beta_1}{\gamma_1}\left(J_1^{(2)} - J_2^{(2)} + \nu_1 J_1^{(3)}\right) - \frac{\nu_3\beta_2}{\gamma_2}J_2^{(3)}\right],$$

where

$$J_i = -\frac{1}{2}\int_a^1 h(t^2)\left[\frac{t+x}{\gamma_i^2y^2+(t+x)^2} + \frac{t-x}{\gamma_i^2y^2+(t-x)^2}\right]dt\ (i=1,2),$$

$$J_i^{(1)} = \frac{1}{4}\int_a^1 h(t^2)\left[\frac{t+x}{\{(\gamma_i+\gamma_2)h - \gamma_i y\}^2 + (t+x)^2} + \frac{t+x}{\{(\gamma_i+\gamma_2)h + \gamma_i y\}^2 + (t+x)^2} + \frac{t-x}{\{(\gamma_i+\gamma_2)h - \gamma_i y\}^2 + (t-x)^2} + \frac{t-x}{\{(\gamma_i+\gamma_2)h + \gamma_i y\}^2 + (t-x)^2}\right]dt\ (i=1,2),$$

$$J_i^{(2)} = \frac{1}{2}\int_a^1 h(t^2)\left[\frac{t+x}{\gamma_i^2y^2+(t+x)^2} + \frac{t-x}{\gamma_i^2y^2+(t-x)^2}\right]dt\ (i=1,2),$$

$$J_i^{(3)} = \frac{1}{4}\int_a^1 h(t^2)\left[\frac{(\gamma_i+\gamma_2)h - \gamma_i y}{\{(\gamma_i+\gamma_2)h - \gamma_i y\}^2 + (t+x)^2} - \frac{(\gamma_i+\gamma_2)h - \gamma_i y}{\{(\gamma_i+\gamma_2)h - \gamma_i y\}^2 + (t-x)^2} + \frac{(\gamma_i+\gamma_2)h + \gamma_i y}{\{(\gamma_i+\gamma_2)h + \gamma_i y\}^2 + (t-x)^2} - \frac{(\gamma_i+\gamma_2)h + \gamma_i y}{\{(\gamma_i+\gamma_2)h + \gamma_i y\}^2 + (t+x)^2}\right]dt\ (i=1,2).$$

Stress Intensity Factor:

The stress intensity factor at the outer tip $x = 1$ of the crack is given by

$$K_0 = \lim_{x\to 1+}[2(x-1)]^{1/2}\sigma_y(x,0)$$

$$= \frac{p_0}{\sqrt{1-a^2}}\left[\left(\frac{E}{F} - 1\right)\left\{1 - \frac{I_0C_0}{2h^2} + \frac{I_0^2C_0^2}{4h^4}\right\} + \frac{I_1C_0}{2h^4}(3 + C_1 + C_2)\right] + O(h^{-6}).$$

The stress intensity factor at the inner tip $x = a$ of the crack is given by

$$K_1 = \lim_{x \to a-} [2(a-x)]^{1/2}\, \sigma_y(x,0)$$
$$= \frac{p_0}{\sqrt{[a(a-1)]}}\left[\left(a^2 - \frac{E}{F}\right)\left\{1 - \frac{I_0 C_0}{2h^2} + \frac{I_0^2 C_0^2}{4h^4}\right\} - \frac{I_1 C_0}{2h^4}(3a^4 + C_1 a^2 + C_2)\right] + O(h^{-6}).$$

RESULTS AND DISCUSSION

As a particular case of the problem, numerical results have been carried out for two orthotropic materials viz. Beryllium and Magnesium. The values of elastic constant of the materials have been taken from the Mukherjee and Das (2007), Das (2002) and Garg (1981), which is given as follows:

Materials	**C_{11} 10^4mpa**	**C_{22} 10^4mpa**	**C_{12} 10^4mpa**	**C_{66} 10^4mpa**
Beryllium	3.148	3.649	0.888	1.124
Magnesium	0.575	0.601	0.195	0.167

The stress intensity factors at the crack tips K_0 and K_1 have been plotted against the different values of crack speed. Keeping the value of a fixed (a = 0.5), stress intensity factors at the inner and outer tip of the crack have been plotted against crack speed for different thickness of the strip i.e. h = 50, 70, 100. Figure 2 and 3 shows the plots of variation of K_0 and K_1 with crack speed for Beryllium. Figure 4 and 5 shows the same plots for Magnesium.

It is observed from Figure [2-5] that nature of curves for inner and outer tip of the crack is similar with a small linear shifting.

From Figure 2 and 4, it is observed that curve of variation of stress intensity factor K_0 at the outer tip x = 1 with crack speed decrease with decrease in h.

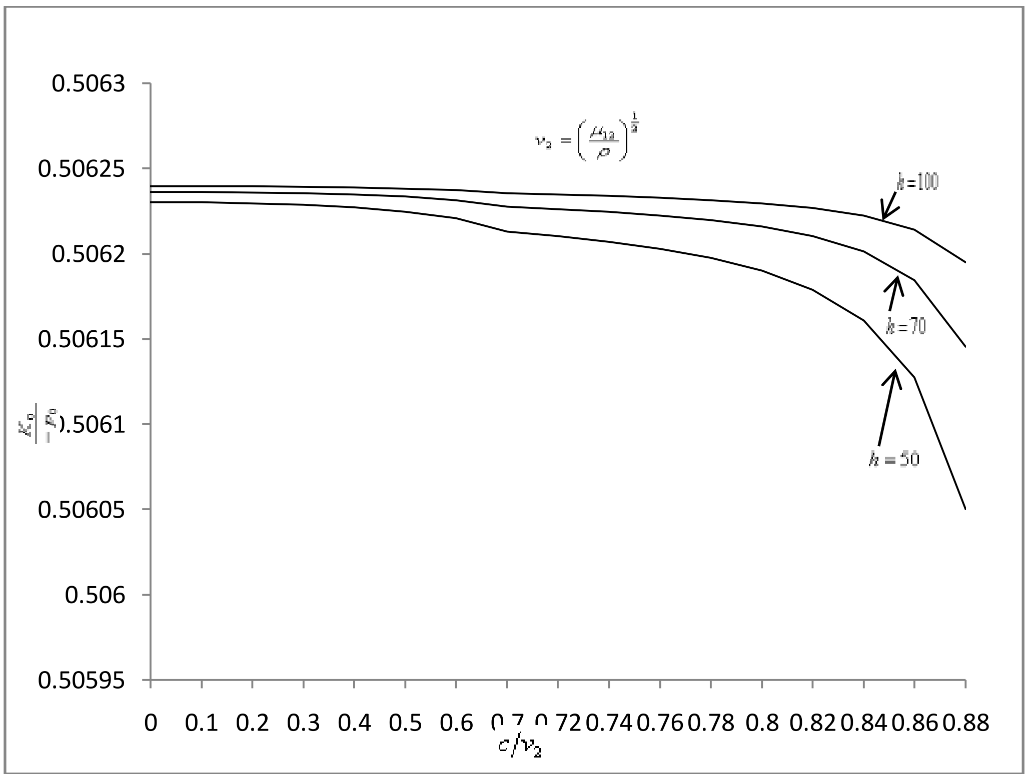

Figure 2: Variation of K_0 with crack speed for Beryllium

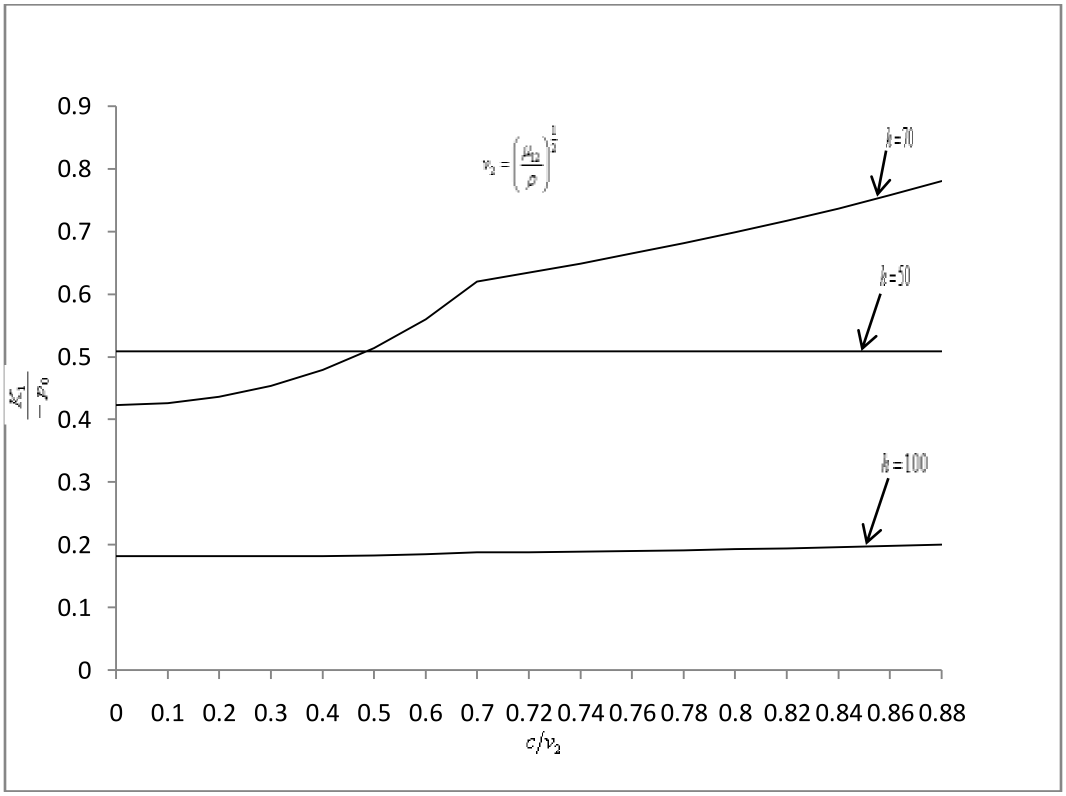

Figure 3: Variation of K_1 with crack speed for Beryllium

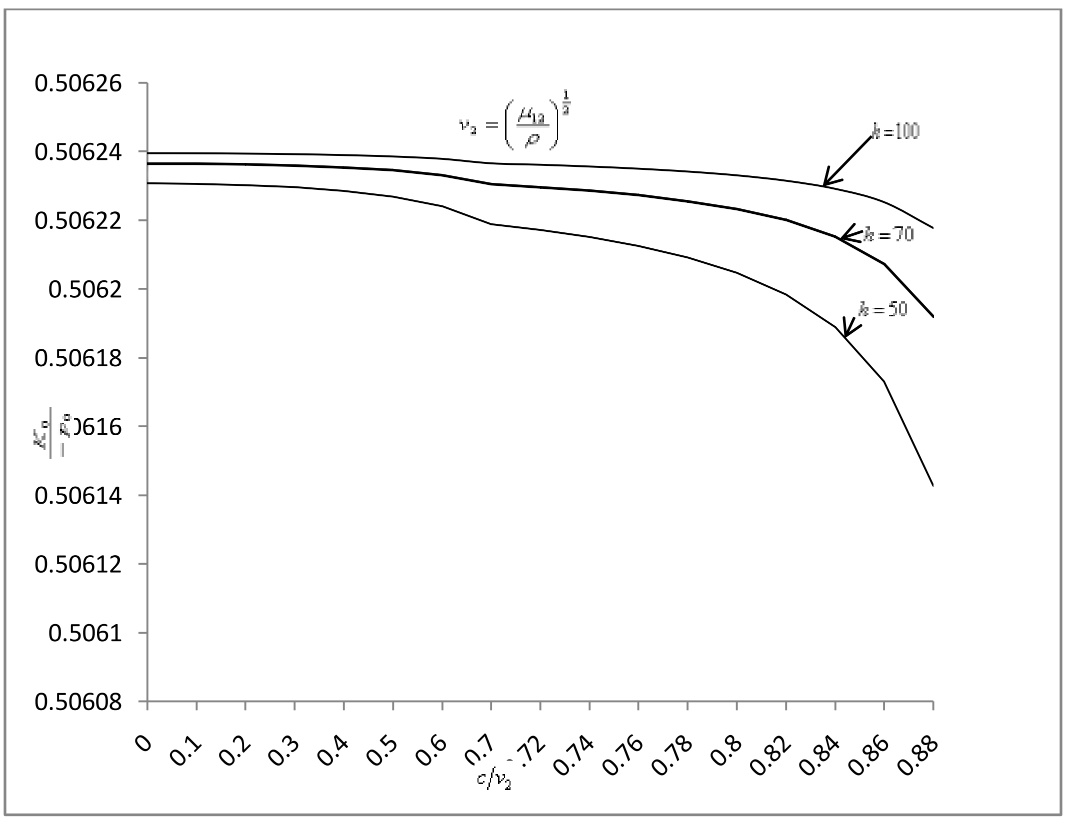

Figure 4: Variation of K_0 with crack speed for Magnesium

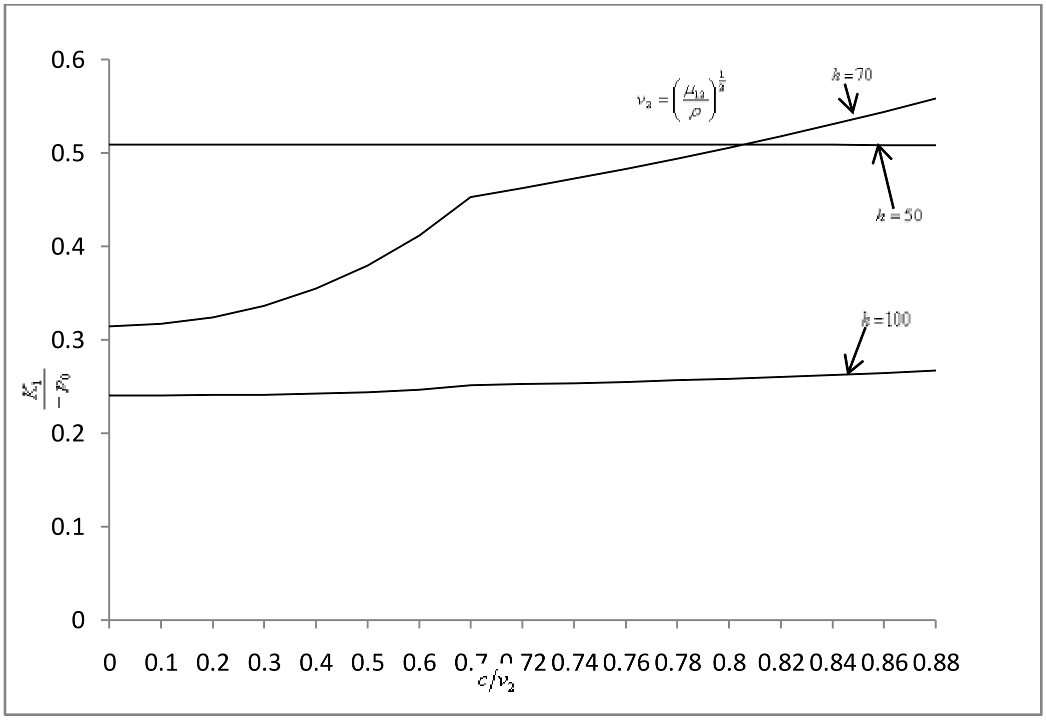

Figure 5: Variation of K_1 with crack speed for Magnesium

Conclusions

In this work, an elastodynamic problem of two collinear Griffith cracks in an infinite orthotropic strip of finite thickness with stress free boundary is considered. Approximate analytical expressions for dynamic stress intensity factors are obtained for numerical calculations.

From Figure 2 and 4, it is observed that curve of variation of stress factor K_0 at the outer tip $x = 1$ with crack speed decreases with decrease in h.

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