International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2014 Vol. 4 (2) April-June, pp. 90-92/Durge
Research Article

INNER PRODUCT SPACE $A = \{ a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \,/\, a_i \in F \text{ and } n \in N \}$

***Manohar Durge**
ANC, Anandwan Warora
**Author for Correspondence*

ABSTRACT

This piece of work consist of $(A, \oplus)$ is an abelian group, $(A, \oplus, \circledast)$ is a vector space, $A = \{ a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \,/\, a_i \in F \text{ and } n \in N \}$, is a modified inner product space.

***Keywords**: Binary Operation, Abelian Group, Vector Space, Inner Product, Field.*

INTRODUCTION

Herstein cotes in (1)

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that

1. a, b ∈ G implies that a*b ∈ G
2. a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3. There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4. For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set V is said to be vector space over a field F if V is an abelian group under an operation which we denote by +, and if for every a ∈ F, v ∈ V; there is defined an element, written as av, in V subject to

1. a (v+w) = av + aw;
2. (a + b)v = av + bv;
3. a (bv) = (ab)v;
4. 1v = v;

For all a, b ∈ F; v, w ∈ V Where the 1 represent the unit element of F under multiplication.

Definition: The Vector Space V over F is said to be an inner product space if there is defined for any two vectors x, y ∈ V an element (x, y) in F such that

1. $(x, y) = \overline{(y, x)}, \forall x, y \in V$
2. $(x, x) \geq 0 \text{ and } (x, x) = 0 \text{ iff } x = 0$
3. $(c_1x + c_2y, z) = c_1(x, z) + c_2(y, z), \forall c_1, c_2 \in F \,\&\, x, y, z \in V$

DISCUSSION

Let $A = \{ a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \,/\, a_i \in F \text{ and } n \in N \}$ and

Let $x = a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n, a_i \in F \text{ and } n \in N,$

$y = b_0 + b_1x_1 + \cdots + b_{n-1}x_{n-1} + b_nx_n, b_i \in F \text{ and } n \in N,$

$z = c_0 + c_1x_1 + \cdots + c_{n-1}x_{n-1} + c_nx_n, c_i \in F \text{ and } n \in N,$

$-x = (-a_0) + (-a_1)x_1 + \cdots + (-a_{n-1})x_{n-1} + (-a_n)x_n, a_i \in F \text{ and } n \in N$

$O = 0 + 0x_1 + \cdots + 0x_{n-1} + 0x_n$

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$1 = 1 + 0x_1 + \cdots + 0x_{n-1} + 0x_n$

$cx = (ca_0) + (ca_1)x_1 + \cdots + (ca_{n-1})x_{n-1} + (ca_n)x_n, c \in F$

x = y iff $a_i = b_i, \forall\, i$

Now we define first binary operation $\oplus$ on A as

$x \oplus y = (a_0 + a_1x_1 + \cdots + a_{n-1}x + a_nx_n) \oplus (b_0 + b_1x_1 + \cdots + b_{n-1}x_{n-1} + b_nx_n)$

$= (a_0 + b_0) + (a_1 + b_1)x_1 + \ldots + (a_{n-1} + b_{n-1})x_{n-1} + (a_n + b_n)x_n$

………… (1)

$=> x \oplus y = y \oplus x, \forall\, x, y, \in A$

$x \oplus (y \oplus z) = (x \oplus y) \oplus z, \forall\, x, y, z \in A$

$0 \oplus x = x \oplus 0, \forall\, x \in A$

$x \oplus (-x) = (-x) \oplus x = 0, \forall\, x \in A$

(A, $\oplus$) is an abelian group. ………… (2)

Now we define second binary operation $\circledast$ on A as

$c \circledast x = cx, \forall\, c \in F \,\&\, x \in A$ ………… (3)

$=> c \circledast (x \oplus y) = (c \circledast x) \oplus (c \circledast y), \forall\, c \in F \,\&\, x, y \in A$

$(c_1 \oplus c_2) \circledast x = (c_1 \circledast x) \oplus (c_2 \circledast x), \forall\, c_1, c_2 \in F \,\&\, x \in A$

$c_1 \circledast (c_2 \circledast x) = (c_1 \circledast c_2) \circledast x, \forall\, c_1, c_2 \in F \,\&\, x \in A$

$1 \circledast x = x, \forall\, x \in A$

$=> (A, \oplus, \circledast)$ is a vector space. ………… (4)

Now we define inner product on A as
$(x, y) = (a_0 + a_1 + \cdots + a_{n-1} + a_n)\,(b_0 + b_1 + \cdots + b_{n-1} + b_n) \in F$

$=> (x, y) = \overline{(y, x)}, \forall\, x, y \in A$
$(x, x) \geq 0$ *and* $(x, x) = 0$ *iff* $x = 0$ *or* $\sum a_i = 0$
$(c_1x \oplus c_2y, z) = c_1(x, z) + c_2(y, z), \forall c_1, c_2 \in F \,\&\, x, y, z \in A$

…………. (5)
From (1) to (5) we come to the Conclusion that
$A = \{\, a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \,/\, a_i \in F$ *and* $n \in N \,\}$, Is a modified inner product space.

Conclusion

From the above discussion, I come to the following conclusions

$\left(A, \oplus\right)$ is an abelian group. $(A, \oplus, \circledast)$ is a vector space.

$A = \{\, a_0 + a_1x_1 + \cdots + a_{n-1}x_{n-1} + a_nx_n \;/\; a_i \in F \text{ and } n \in N \,\}$, Is a modified inner product space.

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