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Research Article

INNER PRODUCT SPACE $A = \{a_0 + a_1 i + a_2 j \,/\, a_i \in F\}$

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ABSTRACT

This piece of work consist of $\left(A, \oplus\right)$ is an abelian group. $(A, \oplus, \circledast)$ is a vector space. $\left(A, \oplus, \circledast\right) = A = \{a_0 + a_1 i + a_2 j \,/\, a_i \in F\}$ is a modified inner product space.

Keywords: *Binary Operation, Abelian Group, Vector Space, Inner Product, Field.*

INTRODUCTION

Herstein Cotes in (1)

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that

1. a, b ∈ G implies that a*b ∈ G
2. a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3. There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4. For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set V is said to be vector space over a field F if V is an abelian group under an operation which we denote by +, and if for every a ∈ F, v ∈ V; there is defined an element, written as av, in V subject to

1. a (v+w) = av + aw;
2. (a + b)v = av + bv;
3. a (bv) = (ab)v;
4. 1v = v;

For all a, b ∈ F; v, w ∈ V Where the 1 represent the unit element of F under multiplication.

Definition: The Vector Space V over F is said to be an inner product space if there is defined for any two vectors x, y ∈ V an element (x, y) in F such that

1. $(x, y) = \overline{(y, x)}, \forall\, x, y \in V$
2. $(x, x) \geq 0$ and $(x, x) = 0$ iff $x = 0\ \forall\, x \in V$
3. $(c_1 x + c_2 y, z) = c_1(x, z) + c_2(y, z), \forall c_1, c_2 \in F \;\&\; x, y, z \in V$

DISCUSSION

Let $A = \{a_0 + a_1 i + a_2 j \,/\, a_0, a_1, a_2 \in F(field)\}$

And $x = a_0 + a_1 i + a_2 j,\ a_i \in F$;

$y = b_0 + b_1 i + b_2 j,\ b_i \in F$

$z = c_0 + c_1 i + c_2 j,\ c_i \in F$

$-x = (-a_0) + (-a_1)i + (-a_2)j,\ -a_i \in F$

$O = 0 + 0i + 0j = zero\ element\ of\ F$

$1 = 1 + 0i + 0j = multiplicative\ identity\ of\ F$

$c = c + 0i + 0j,\ c \in F$

$cx = (ca_0) + (ca_1)i + (ca_2)j, c \in F$

x = y iff $a_i = b_i, \forall\, i$

Now we define first binary operation $\oplus$ on A defined as

$x \oplus y = (a_0 + a_1 i + a_2 j) \oplus (b_0 + b_1 i + b_2 j)$

$= (a_0 + b_0) + (a_1 + b_1)i + (a_2 + b_2)j$ ………… (1)

$=> x \oplus y = y \oplus x, \forall\, x, y, \in A$

$x \oplus \left(y \oplus z\right) = \left(x \oplus y\right) + z, \forall\, x, y, z \in A$

$0 \oplus x = x \oplus 0 = x, \forall\, x \in A$

$x \oplus (-x) = (-x) \oplus x = 0, \forall\, x \in A$

$=> \left(A, \oplus\right)$ is an abelian group. ………… (2)

Now we define second binary operation $\circledast$ on A as

$c \circledast x = cx, \forall\, c \in F\ \&\ x \in A$ ………… (3)

$c \circledast (x \oplus y) = \left(c \circledast x\right) \oplus (c \circledast y), \forall\, c \in F\ \&\ x, y \in A$

$(c_1 \oplus c_2) \circledast x = (c_1 \circledast x) \oplus (c_2 \circledast x), \forall\, c_1, c_2 \in F\ \&\ x \in A$

$c_1 \circledast (c_2 \circledast x) = (c_1 \circledast c_2) \circledast x, \forall\, c_1, c_2 \in F\ \&\ x \in A$

$1 \circledast x = x, \forall\, x \in A$

$=> (A, \oplus, \circledast)$ is a vector space. ………… (4)

Now we define inner product on A as
$(x, y) = (a_0 + a_1 + a_2)\,(b_0 + b_1 + b_2) \in F$

$=> (x, y) = \overline{(y, x)}, \forall\, x, y \in A$

$(x, x) \geq 0\ and\ (x, x) = 0\ iff\ x = 0\ or\ \sum a_i = 0$

$\left(c_1 x \oplus c_2 y, z\right) = c_1(x, z) + c_2(y, z), \forall c_1, c_2 \in F\ \&\ x, y, z \in A$

$=> A = \{a_0 + a_1 i + a_2 j \,/\, a_i \in F\}$ is a modified inner product space.

Conclusion

From the above discussion, I come to the following conclusions

$\left(A, \oplus\right)$ is an abelian group.

$(A, \oplus, \circledast)$ is a vector space.

$\left(A, \oplus, \circledast\right) = A = \{a_0 + a_1 i + a_2 j \;/\; a_i \in F\}$ is a modified inner product space.

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