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Research Article

TWO TEMPERATURE ELECTRO-MAGNETO THERMO-VISCO-ELASTIC RESPONSE WITH RHEOLOGICAL PROPERTIES AND TEMPERATURE DEPENDENT ELASTIC MODULI

[*]Manoj K. Mondal[1] and B. Mukhopadhyay[2]
[1] *Department of Mathematics, Seacom Engineering College, Howrah, India*
[2]*Department of Mathematics, Bengal Engineering and Science University, Howrah, India*
[*]*Author for Correspondence*

ABSTRACT
The present paper is concerned with a thermo-visco-elastic problem of an isotropic material in cylindrical hole with two-temperature due to the presence of a uniform magnetic field. Rheological properties of volume and density of material are considered here. The problem is based on the concept of temperature dependent mechanical properties. Generalized heat conduction equation due to Lord-Shulman and Green-Lindsay are utilized. Eigen value approach is used to solve the problem.

Keywords: *Thermo-Visco-Elasticity, Rheological Property, Vector-Matrix-Differential Equation, Relaxation Function, Two-Temperature*

INTRODUCTION
In classical theory of thermo-elasticity, the heat conduction equation is a parabolic type differential equation which predicts that the effect of a thermal disturbance will instantaneously manifest itself at infinitely large distance from the source. This prediction is unrealistic from a physical point of view. During last four decades, non-classical theories which predict finite speed of thermal signal in elastic solids have been developed to remove this drawback. In the theory of thermoelastic diffusion the coupled thermoelastic model which is used implies infinite speeds of propagation of thermoelastic waves. Lord and Shulman (1967) obtained a wave-type heat equation by constructing a new law of heat conduction to replace the classical Fourier's law which ensure finite speeds of propagation for heat and elastic waves which is known as the first generalization of the coupled thermo-elasticity theory. Green and Lindsay (1972) incorporate temperature rate term into the constitutive equations of thermo-elasticity and attained an explicit version of the constitutive equations. This theory depends on two relaxation times.
Linear visco-elasticity including thermal stresses and classical thermo-visco-elasticity increase the field area of elastic theory to the research workers. Drozdov (1996) derived a constitutive model in thermo-visco-elasticity which accounts for changes in elastic moduli and relaxation times. Using generalized theory proposed by Lord-Shulman and Green-Lindsay, the problem on visco-elastic materials has been discussed by Giorgi and Naso (2006). Explanation of electro-magneto-thermo-visco-elastic plane waves in rotating media with thermal relaxation was discussed by Choudhuri and Chattapadhyay (2007). The thermo-elastic problem with the effect of magnetic field and thermal relaxation under diffusion was discussed by Othman et al. (2013). At high temperature the mechanical properties of the material are temperature-dependent. Most investigation in thermo-visco-elasticity was done by ignoring the temperature-dependent mechanical properties. Thermo-visco-elasticity including the temperature-dependent mechanical properties increases the field area of elastic theory to the research workers. Problem on temperature-dependent mechanical properties was analyzed by Aouadi and El-Karamany (2004) in their paper. Ezzat *et al.*, (2010) expend their valuable efforts to recognize the effects of modified Ohm's and Fourier's laws on generalized magneto-thermo-visco-elasticity with relaxation volume properties. Kundu and Mukhopadhyay (2005) investigated a thermo-visco-elastic problem of an infinite medium containing a spherical cavity considering the rheological properties of volume, using the generalized theory of thermo-elasticity. Mondal and Mukhopadhyay (2013) discussed the effects of rheological volume and density properties on their problem having temperature dependent mechanical properties. The study of thermo-visco-elasticity with two-temperature is of interest in some branches of

material science, metallurgy, applied mathematics etc. Now a day the effect of two-temperature has become an important area of research. According to Gurtin and Williams (1967) the second law of thermodynamics for continuous bodies may involve with twin temperatures. In the theory of thermodynamics the temperature caused by the thermal process is known as conductive temperature φ and the temperature due to mechanical process in the material is known as thermodynamic temperature T. The theory of heat conduction depending on the above two temperatures was originated by Chen and Gurtin (1968). The propagation of harmonic plane waves in the theory of two-temperature thermo-elasticity ware investigated by Puri and Jordan (2006). Quintanilla (2004) analyzed the existence, structural stability, convergence and spatial behavior for the theory two-temperature thermo-elasticity. By means of two-temperature generalized thermo-elasticity Youssef and Al-Harby (2007) explained the state-space approach on an infinite body with spherical cavity. Ailawilia et al. (2009) investigated the deformation of a rotating generalized thermoelastic medium with two temperatures under the influence of gravity subjected to different type of sources. Banik and Kanoria (2011) investigated the effects of two-temperature on generalized thermo-elasticity for infinite medium with spherical cavity. The analysis of effects of two-temperature in the material having temperature dependent mechanical properties on generalized thermo-visco-elastic problem was discussed by Mondal and Mukhopadhyay (2013). Shaw and Mukhopadhyay (2013) discussed the moving heat source response in micropolar half-space with two-temperature theory. Ezzat and El-Karamany (2011) established the model of one-dimensional equations of the two-temperature generalized magneto-thermo-elasticity theory with two relaxation times. Youssef (2010) analyzed the effect of two-temperature in infinite medium under generalized thermoelasticity with cylindrical cavity.

Various problems related to visco-elasticity, done without taking the rheological density property and also the mechanical properties which are taken are not temperature dependent. The rheological properties of volume as well as density having temperature dependent mechanical properties are being considered here for an infinite visco-elastic solid with a cylindrical hole in the context of the theory of generalized two-temperature magneto-thermo-elasticity. The inverse of Laplace transform for the different expression is done numerically using the method adopted by Honig and Hirdes (1984). Solution for stresses, displacement and temperature are presented graphically with respect to space and time variables separately.

FORMULATION OF PROBLEM AND SOLUTION

We consider a homogeneous, isotropic, perfectly conducting long hollow visco-elastic cylinder with z-axis as the axis of the cylinder. We assume that the whole body is situated at a constant applied magnetic field $\boldsymbol{H}_0 = (0, 0, H_o)$ acted along z-axis. It produce an induce magnetic field $\boldsymbol{h} = (0, 0, h)$ and a perturbed electric field . For perfectly conducting slowly moving visco-elastic medium, the variation of magnetic field and electric field are given by Maxwell equations:

$$\text{curl}\,\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}, \quad (1)$$

$$\text{curl}\,\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad (2)$$

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}, \quad (3)$$

$$\boldsymbol{B} = \mu_1 \boldsymbol{H}, \quad (4)$$

$$\text{div}\,\boldsymbol{D} = 0, \quad (5)$$

$$\text{div}\,\boldsymbol{B} = 0 \quad (6)$$

together with the generalized Ohm's law

$$\boldsymbol{J} = \sigma_{\mathbf{0}}\left[\boldsymbol{E} + \mu_{\mathbf{1}}\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}\right] \quad (7)$$

where $\boldsymbol{H} = \boldsymbol{h} + \boldsymbol{H}_0$ is magnetic intensity vector, $\boldsymbol{J}$ is current density vector. ε_0, μ_1 and σ_0 are the electric permittivity, the magnetic permeability and the electric conductivity respectively. $\boldsymbol{u}$ is the displacement vector and $\boldsymbol{B}, \boldsymbol{D}$ respectively are the magnetic induction and electric displacement vector.

The cylindrical polar co-ordinates suppose to be taken as (r, θ, z). Due to radial symmetry the displacement can be taken as $\boldsymbol{u} = (u(r,t), 0,0)$ and the temperature be functions of r and t. Therefore the strain components take the form as $\varepsilon_{rr} = \frac{\partial u}{\partial r}, \varepsilon_{\theta\theta} = \frac{u}{r}, \varepsilon_{zz} = 0$ and dilatation is $= \frac{\partial u}{\partial r} + \frac{u}{r}$. The basic equations for two-temperature electro-magneto-thermo-visco-elastic solid in the context of generalized theory, where rheological properties of volume as well as density are considered, may be taken as:

The equation of motion

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + (\boldsymbol{J} \times \boldsymbol{B})_r = \rho \frac{\partial^2 u}{\partial t^2}\,. \quad (8)$$

The generalized equation of heat conduction

$$k\nabla^2\varphi = C_E \int_0^t R_3(t-\tau)\frac{\partial}{\partial \tau}\left(\frac{\partial T}{\partial \tau} + \tau_2 \frac{\partial^2 T}{\partial \tau^2}\right) d\tau + 3T_0\alpha_T \int_0^t R_2(t-\tau)\frac{\partial}{\partial \tau}\left(\frac{\partial e}{\partial \tau} + \tau_3 \frac{\partial^2 e}{\partial \tau^2}\right) d\tau. \quad (9)$$

The constitutive equations

$$S_{ij} = \int_0^t R_1(t-\tau)\frac{\partial e_{ij}}{\partial \tau} d\tau\,, \quad (10)$$

$$\sigma = \int_0^t R_2(t-\tau)\frac{\partial}{\partial \tau}\{e - 3\alpha_T(T - T_0 + \tau_1 \dot{T})\} d\tau\,. \quad (11)$$

Relation between conductive temperature φ and thermodynamic temperature T

$$\varphi - T = a\,\nabla^2\varphi \quad (12)$$

where $a \geq 0$ is two-temperature parameter (Youssf 2006).

Here e_{ij}, S_{ij} are the deviatoric parts of strain tensor and stress tensor respectively; T_0 is the reference temperature; e, k, C_E are dilatation, thermal conductivity, specific heat at constant strain respectively; ρ, α_T are the density and co-efficient of linear thermal expansion at absolute temperature respectively; τ_1, τ_2, τ_3 are thermal relaxation times; $R_1(t), R_2(t), R_3(t)$ are non-negative relaxation function, relaxation function characterized by rheological properties of volume and density respectively. ∇^2 is Laplacian operator.

Depending upon the values of thermal relaxation times τ_1, τ_2, τ_3 the above problem can be reduced according as bellow.

- When $\tau_1 = \tau_2 = \tau_3 = 0$ the above one corresponds the problem with classical theory.
- When $\tau_1 = 0, \tau_2 = \tau_3 \neq 0$ the above one corresponds the problem with generalized Lord-Shulman theory.
- When $\tau_1 \neq 0, \tau_2 \neq 0, \tau_3 = 0$ the above one corresponds the problem with generalized Green-Lindsay theory.

The relaxation functions are taken in the form

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$$\left.\begin{aligned} R_1(t) &= 2\mu\left(1 - M_1\int_0^t g(t)dt\right) \\ R_2(t) &= K\left(1 - M_2\int_0^t g(t)dt\right) \\ R_3(t) &= \rho\left(1 - M_3\int_0^t g(t)dt\right) \end{aligned}\right\}. (13)$$

The function $g(t)$ generally taken in the form $g(t) = e^{-\beta t}\ t^{\alpha-1}$ (Koltunov 1976) where

$$0 < \alpha < 1, \beta > 0, 0 \le M_2 \le M_3 \le M_1 < \Gamma(\alpha), 0 \le t < \infty.$$

Using equations (4), (7) and (10), (11) we have

$$(\boldsymbol{J} \times \boldsymbol{B})_r = \mu_1 H_0^2\ \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \mu_1^2 \varepsilon_0 H_0^2\ \frac{\partial^2 u}{\partial t^2},$$

$$\sigma_{ij} = \int_0^t R_1(t-\tau)\frac{\partial e_{ij}}{\partial \tau}\ d\tau + \delta_{ij}\int_0^t R_2(t-\tau)\frac{\partial}{\partial \tau}\{e - 3\alpha_T(T - T_0 + \tau_1 \dot{T})\}d\tau. (14)$$

After eliminating $\boldsymbol{J}$ with the help of equation (7), we have from equation (1) and (2)

$$\frac{\partial h}{\partial r} = -\sigma_0\left[E - \mu_1 H_0\frac{\partial u}{\partial t}\right] - \varepsilon_0\frac{\partial E}{\partial t}, (15)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE) = -\mu_1\frac{\partial h}{\partial t}. (16)$$

In the above we consider the material having temperature dependent mechanical properties which are in the form $\mu = \mu_0\psi_0(T), K = K_0\psi_0(T), \rho = \rho_0\psi_1(T), \alpha_T = \alpha_T^0\psi_2(T)$ (Lomakin 1976) where $\psi_i(T) = 1 - \alpha_i(T - T_r), i = 0,1,2$ and $\alpha_i > 0, i = 0,1$ and $\alpha_2 < 0$ in which μ_0, K_0, ρ_0 and α_T^0 are the Lame' constant, bulk modulus, density, Co-efficient of linear thermal expansion at room temperature T_r.

For more convenient form we use the following non-dimensional variables

$$r' = \frac{r}{a_1},\ u' = \frac{u}{a_1},\ t' = \frac{c_0}{a_1}t,\ \tau_1' = \frac{c_0}{a_1}\tau_1,\ \tau_2' = \frac{c_0}{a_1}\tau_2,\ \tau_3' = \frac{c_0}{a_1}\tau_3,\ h' = \frac{h}{a_1\sigma_0 H_0\mu_1 c_0},$$

$$E' = \frac{E}{a_1\sigma_0 H_0\mu_1^2 c_0^2},\ R_1' = \frac{2R_1}{3K_0}, R_2' = \frac{R_2}{K_0}, R_3' = \frac{R_3}{\rho_0},\ T' = \frac{\gamma(T - T_0)}{\rho_0 c_0^2},\ \varphi' = \frac{\gamma(\varphi - T_0)}{\rho_0 c_0^2},$$

$$\sigma_{ij}' = \frac{\sigma_{ij}}{K_0}$$

where

$$c_0^2 = \frac{\lambda_0 + 2\mu_0}{\rho_0}, a_1 = \frac{k}{c_0 c_E \rho_0}, \gamma = 3K_0\alpha_T^0$$

Using non-dimensional terms and taking Laplace transform with respect to t the equations (8), (9) and (12)-(16), dropping the primes, take the following forms as

$$p(\bar{R}_1 + \bar{R}_2 + \varepsilon_1)L(\bar{u}) - r^2 p\ m\ \psi_2\bar{R}_2(1 + \tau_1 p)\left\{\frac{\partial\bar{\varphi}}{\partial r} - \frac{\omega}{r^2}\ L\left(\frac{\partial\bar{\varphi}}{\partial r}\right)\right\}$$

$$= r^2p^2(m\,\psi_1 + c_0^2\mu_1\varepsilon_1\varepsilon_0)\bar{u}\,, (17)$$

$$L\left(\frac{\partial\bar{\varphi}}{\partial r}\right) = r^2a_2\frac{\partial\bar{\varphi}}{\partial r} + a_3L(\bar{u})\,, (18)$$

$$\bar{\varphi} - \bar{T} = \omega\,\nabla^2\bar{\varphi}\,, (19)$$

$$\left.\begin{aligned} \bar{R}_1 &= \tfrac{\beta_0\psi_0}{p}\left(1 - M_1\sqrt{\tfrac{\pi}{p+\beta}}\right) \\ \bar{R}_2 &= \tfrac{\psi_0}{p}\left(1 - M_2\sqrt{\tfrac{\pi}{p+\beta}}\right) \\ \bar{R}_3 &= \tfrac{\psi_1}{p}\left(1 - M_3\sqrt{\tfrac{\pi}{p+\beta}}\right) \end{aligned}\right\}\,, (20)$$

$$\bar{\sigma}_{rr} = p\bar{R}_1\left(\frac{\partial\bar{u}}{\partial r} - \frac{1}{2}\frac{\bar{u}}{r}\right) + p\bar{R}_2\left\{\left(\frac{\partial\bar{u}}{\partial r} + \frac{\bar{u}}{r}\right) - m\,\psi_2(1+\tau_1 p)(\bar{\varphi} - \omega\,\nabla^2\bar{\varphi})\right\}\,, (21)$$

$$\bar{\sigma}_{\theta\theta} = p\bar{R}_1\left(\frac{\bar{u}}{r} - \frac{1}{2}\frac{\partial\bar{u}}{\partial r}\right) + p\bar{R}_2\left\{\left(\frac{\partial\bar{u}}{\partial r} + \frac{\bar{u}}{r}\right) - m\,\psi_2(1+\tau_1 p)(\bar{\varphi} - \omega\,\nabla^2\bar{\varphi})\right\}\,, (22)$$

$$\bar{\sigma}_{zz} = -\frac{1}{2}p\bar{R}_1\left(\frac{\partial\bar{u}}{\partial r} + \frac{\bar{u}}{r}\right) + p\bar{R}_2\left\{\left(\frac{\partial\bar{u}}{\partial r} + \frac{\bar{u}}{r}\right) - m\,\psi_2(1+\tau_1 p)(\bar{\varphi} - \omega\,\nabla^2\bar{\varphi})\right\}\,, (23)$$

$$\frac{\partial\bar{h}}{\partial r} = p\bar{u} - (\delta_0 + v^2p)\bar{E}\,, (24)$$

$$L(\bar{E}) = -p\,r^2\frac{\partial\bar{h}}{\partial r}\,. (25)$$

Where bar denotes the Laplace transform with respect to t, p is the parameter of Laplace transform and

$$\omega = \frac{a}{a_1^2},\ \beta_0 = \frac{4\mu_0}{3K_0}, m = \frac{\rho_0 c_0^2}{K_0}, \varepsilon = \frac{T_0\gamma^2 a_1}{k\rho_0 c_0},\ \varepsilon_1 = \frac{\mu_1 H_0^2}{K_0},\ \delta_0 = \sigma_0\mu_1 c_0 a_1,\ v^2 = \varepsilon_0\mu_1 c_0^2,$$

$$a_2 = \frac{p^2\bar{R}_3(1+\tau_2 p)}{1 + p^2\omega\bar{R}_3(1+\tau_2 p)}, a_3 = \frac{\varepsilon\,\psi_2 p^2\bar{R}_2(1+\tau_3 p)}{1 + p^2\omega\bar{R}_3(1+\tau_2 p)}, L \equiv r^2\frac{\partial^2}{\partial r^2} + r\frac{\partial}{\partial r} - 1.$$

Solving equations (17) and (18) and combining equations (24), (25) we have

$$L(\bar{u}) = r^2a_{11}\bar{u} + r^2a_{12}\frac{\partial\bar{\varphi}}{\partial r}\,, (26)$$

$$L\left(\frac{\partial\bar{\varphi}}{\partial r}\right) = r^2a_{21}\bar{u} + r^2a_{22}\frac{\partial\bar{\varphi}}{\partial r}\,, (27)$$

$$L\left(\frac{\partial\bar{h}}{\partial r}\right) = r^2a_{31}\bar{u} + r^2a_{32}\frac{\partial\bar{\varphi}}{\partial r} + r^2a_{33}\frac{\partial\bar{h}}{\partial r}\ (28)$$

where

$$a_{11} = \frac{(\psi_1 m + v^2\varepsilon_1)p^2}{q}, a_{12} = \frac{\bar{R}_2\psi_2 m\,p(1+\tau_1 p)(1-\omega a_2)}{q}, a_{21} = a_3a_{11},$$

$$a_{22} = a_2 + a_3 a_{12}, a_{31} = p a_{11}, a_{32} = p\, a_{12}\,, a_{33} = p\,(\delta_0 + v^2 p),$$

$$q = p\,(\bar{R}_1 + \bar{R}_2) + \varepsilon_1 + m\omega p \bar{R}_2 \psi_2 a_3 (1 + \tau_1 p).$$

Now the equations (26), (27) and (28) in the form of vector-matrix differential equation may be written as
$L\,\tilde{V} = r^2 \mathbb{A}\,\tilde{V}$ (29)
where

$$\tilde{V} = \begin{bmatrix} \bar{u} \\ \dfrac{\partial \bar{\varphi}}{\partial r} \\ \dfrac{\partial \bar{h}}{\partial r} \end{bmatrix};\ \mathbb{A} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

Solution of equation (29) may be written as

$$\tilde{V} = \sum_{i=1}^{3} A_i\, \tilde{X}_i(\lambda_i^2)\, w_i(r, \lambda_i) \quad (30)$$

where A_i are the arbitrary constants, $\tilde{X}_i(\lambda_i^2)$ are the eigenvectors of the matrix $\mathbb{A}$ corresponding to the eigenvalue λ_i^2 of the matrix $\mathbb{A}$ and $w_i(r, \lambda_i)$ satisfies the modified Bessel's differential equation

$$r^2 \frac{d^2 w}{dr^2} + r\frac{dw}{dr} - (r^2 \lambda^2 + 1) w = 0\,. (31)$$

Solution of equation (31) is

$$w_i(r, \lambda_i) = \frac{1}{\lambda_i^2} K_1(\lambda_i r)\ (32)$$

where $K_1(\lambda_i r)$ is the modified Bessel's function of second kind of order 1 , so that

$$\tilde{X}_i(\lambda_i^2) = \begin{bmatrix} -a_{12} \\ (a_{11} - \lambda_i^2) \\ \dfrac{a_{31} a_{12} - a_{32}(a_{11} - \lambda_i^2)}{(a_{33} - \lambda_i^2)} \end{bmatrix}; \lambda_i^2 \neq a_{33}\,, i = 1{,}2 \text{ and } \tilde{X}_3(\lambda_3^2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{for } \lambda_3^2 = a_{33}. (33)$$

Therefore from (30) we have

$$\bar{u} = -\sum_{i=1}^{2} A_i\ \ a_{12} \frac{1}{\lambda_i^2} K_1(\lambda_i r)\,, (34)$$

$$\bar{\varphi} = -\sum_{i=1}^{2} A_i\, (a_{11} - \lambda_i^2)\, \frac{1}{\lambda_i^3} K_0(\lambda_i r)\,, (35)$$

$$\bar{h} = -\sum_{i=1}^{2} A_i\, \frac{a_{31} a_{12} - a_{32}(a_{11} - \lambda_i^2)}{(a_{33} - \lambda_i^2)}\, \frac{1}{\lambda_i^3} K_0(\lambda_i r) - A_3 \frac{1}{\lambda_3^3} K_0(\lambda_3 r)\ (36)$$

where $K_0(\lambda_i r)$ is the modified Bessel's function of second kind of order 0.
Using (34)-(36) we have from equations (19) and (21)-(24)

$$\bar{T} = \sum_{i=1}^{2} A_i \left(a_{11} - \lambda_i^2\right) \left[\frac{\omega}{\lambda_i} - \frac{1}{\lambda_i^3}\right] K_0(\lambda_i r), (37)$$

$$\bar{\sigma}_{rr} = \sum_{i=1}^{2} A_i\, a_{12} \left[\frac{p(\bar{R}_1 + \bar{R}_2)}{\lambda_i} K_0(\lambda_i r) + \frac{3p\bar{R}_1}{2r\lambda_i^2} K_1(\lambda_i r)\right] - \bar{Q}\,, (38)$$

$$\bar{\sigma}_{\theta\theta} = \sum_{i=1}^{2} A_i\, a_{12} \left[\frac{p\left(\bar{R}_2 - \frac{1}{2}\bar{R}_1\right)}{\lambda_i} K_0(\lambda_i r) - \frac{3p\bar{R}_1}{2r\lambda_i^2} K_1(\lambda_i r)\right] - \bar{Q}\,, (39)$$

$$\bar{\sigma}_{zz} = \sum_{i=1}^{2} A_i\, a_{12} \left[\frac{p\left(\bar{R}_2 - \frac{1}{2}\bar{R}_1\right)}{\lambda_i} K_0(\lambda_i r)\right] - \bar{Q}\,, (40)$$

$$\bar{E} = \frac{-1}{(\delta_0 + v^2 p)} \left[\sum_{i=1}^{2} A_i \left\{p a_{12} + \frac{a_{31}a_{12} - a_{32}\left(a_{11} - \lambda_i^2\right)}{\left(a_{33} - \lambda_i^2\right)}\right\} \frac{1}{\lambda_i^2} K_1(\lambda_i r) + \frac{A_3}{\lambda_3^2} K_1(\lambda_3 r)\right] (41)$$

where

$$\bar{Q} = \bar{R}_2 \psi_2 m\, p(1 + \tau_1 p) \sum_{i=1}^{2} A_i \left(a_{11} - \lambda_i^2\right) \left[\frac{\omega}{\lambda_i} - \frac{1}{\lambda_i^3}\right] K_0(\lambda_i r)\,.$$

In free space, where $\boldsymbol{J} \to \mathbf{0}$, if E_0 and h_0 represent the component of electric and induced magnetic field intensities in the direction of θ and z respectively, they satisfy the following non-dimensional equations in transform domain

$$\frac{1}{r}\frac{\partial}{\partial r}(r\bar{E}_0) = -p\bar{h}_0\,, (42)$$

$$\frac{\partial \bar{h}_0}{\partial r} = -v^2\, p\, \bar{E}_0\,. (43)$$

With the help of solution of (42), the solution of (43) becomes

$$\bar{h}_0 = -\frac{1}{p^2\, v}\, I_0(pvr)\ (44)$$

where I_0 is the modified Bessel function of first kind of order 0.

Boundary Conditions

The transverse components of the electric and induced magnetic field intensities are continuous across the surface of the cylinder and hence

$E(r,t) = E_0(r,t)\,;\ h(r,t) = h_0(r,t)$ for $t > 0$ at the surface of the hole. (45)

For mechanical and thermal boundary conditions we consider the following two cases:

Case I) $\sigma_{rr} = H(t), \frac{\partial\varphi}{\partial r} = 0$ at the surface of the hole;

Case II) $\sigma_{rr} = \delta(t), \frac{\partial \varphi}{\partial r} = 0$ at the surface of the hole.

When the boundary conditions are used, a system of three linear equations in A_1, A_2, A_3 can be obtained from the equations (35), (38) and combination of (36) with (44). Solving these equations, we achieve the values of A_i for two distinct cases.

Case I)

$$A_1 = \frac{X_2}{p(Y_1X_2 - X_1Y_2)}, A_2 = \frac{-X_1}{p(Y_1X_2 - X_1Y_2)}, A_3 = \frac{I_0}{vp^2Z_3} - \frac{(Z_1X_2 - X_1Z_2)}{pZ_3(Y_1X_2 - X_1Y_2)}$$

Case II)

$$A_1 = \frac{X_2}{(Y_1X_2 - X_1Y_2)}, A_2 = \frac{-X_1}{(Y_1X_2 - X_1Y_2)}, A_3 = \frac{I_0}{vp^2Z_3} - \frac{(Z_1X_2 - X_1Z_2)}{Z_3(Y_1X_2 - X_1Y_2)}$$

where

$$X_i = \frac{N_{1i}\,K_{1i}}{\lambda_i^2},\ Y_i = a_{12}\left[\frac{p\bar{R}}{\lambda_i}K_{0i} + \frac{3p\bar{R}_1}{2\lambda_i^2}K_{1i}\right] - N\,N_{1i}\left[\frac{\omega}{\lambda_i} - \frac{1}{\lambda_i^3}\right]K_{0i}\ ,$$

$$Z_i = \frac{a_{31}a_{12} - a_{32}N_{1i}}{N_{3i}}\frac{1}{\lambda_i^3}K_{0i}\ ,\ K_{0i} = K_0(\lambda_i),\ K_{1i} = K_1(\lambda_i),$$

$$N_{1i} = \left(a_{11} - \lambda_i^2\right),\ N_{3i} = \left(a_{33} - \lambda_i^2\right);\ i = 1,2$$

and $Z_3 = \frac{1}{\lambda_3^3}K_0(\lambda_3), N = \bar{R}_2\psi_2 m\, p(1 + \tau_1 p), \bar{R} = (\bar{R}_1 + \bar{R}_2),\ I_0 = \ I_0(pv).$

Equations (34)-(41) together with the above derived values of A_1, A_2 and A_3 for distinct cases provide the eventual solutions in transform domain. Now for graphical representation it required the numerical information's.

RESULTS AND DISCUSSION

The expressions for displacement, stress, temperature and the fields can found out numerically with the help of above consequent values of A_i. For infinitesimal temperature deviations from reference temperature we can take $\psi_i(T_0) = 1 - \alpha_i(T_0 - T_r); i = 0,1,2$ such that $\alpha_0 > 0, \alpha_1 > 0\ \&\ \alpha_2 < 0$ (Nowacki 1959). To study the behavior of the quantities in details and with the intention of demonstrating the outcomes obtain in the above we attempt to achieve the numerical values of the different characteristic parameters of the material. For execution of the graphical representation we take for granted the numerical values for a magnesium crystal-like material as (Ezzat *et al.*, 2010)

$\rho_0 = 1.74 \times 10^3 kg/m^3\ , C_E = 1020\, J/K\, kg, k = 156 W/K\, m, \lambda_0 = 3543 \times 10^7 N/m^2,$
$\mu_0 = 1518 \times 10^7 N/m^2\ , \lambda_0 + 2\mu_0 = 6579 \times 10^7 N/m^2\ , \alpha_T^o = 25.2 \times 10^{-6}\ 1/K,$
$T_o = 298K,\ K_0 = 4555 \times 10^7 N/m^2\ , \gamma = (3\lambda_0 + 2\mu_0)\alpha_T^o = 3.444 \times 10^5 N/m^2\, K.$

The above considered numerical values imply $m = 1.4443, \varepsilon = 0.3027 \times 10^{-2}, \beta_0 = 0.444346$. For the graphical evaluation the other constants in this paper may be taken as $\beta = 0.05, \alpha = \frac{1}{2},\ M_1 = 0.106,\ M_2 = M_3 = 0.08,\ \psi_0 = 0.82\ ,\ \psi_1 = 0.90\ ,\ \psi_2 = 1.25,$
$\varepsilon_1 = .0005,\ \delta_0 = .008, v = .373 \times 10^{-5}.$

To get the solution for thermal displacement, temperature, stress and the fields we apply inverse Laplace transform numerically using the method based on Honig and Hirdes (1984) to the equations (34)-(41). The results of the present investigations are given in form of figures for different values of time, radial distance and two-temperature parameter (temperature discrepancy). Moreover we draw the graphs considering temperature independent mechanical properties (TIMP) and considering temperature

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dependent mechanical properties (TDMP) and in addition comparison are also made considering with rheological density property (WDP) and considering without rheological density property (WODP).
In particular cases when we choose,

- $a = 0$ that is $\omega = 0$, our considered problem reduced to one, related to unique-temperature.
- $M_3 = 0$, our considered problem reduced to one, related to null rheological density property.
- $M_2 = 0$, our considered problem reduced to one, related to null rheological volume property.
- $M_3 = 0$ and $M_2 = 0$, our considered problem reduced to one, related to null rheological density property as well as null rheological volume property.
- $\psi_0 = \psi_1 = \psi_2 = 1$, our considered problem reduced to one, related to temperature independent mechanical properties.
- $\psi_0 = \psi_1 = \psi_2 = 1, \omega = 0$, our considered problem reduced to one, related to temperature independent mechanical properties and unique-temperature.
- $\psi_0 = \psi_1 = \psi_2 = 1, M_3 = 0$, our considered problem reduced to one, related to temperature independent mechanical properties and null rheological density property.
- $\psi_0 = \psi_1 = \psi_2 = 1, M_3 = 0, M_2 = 0$, our considered problem reduced to one, related to temperature independent mechanical properties and null rheological density property as well as null rheological volume property.
- $\psi_0 = \psi_1 = \psi_2 = 1, M_3 = 0, M_2 = 0, \omega = 0$, our considered problem reduced to one, related to temperature independent mechanical properties and null rheological density property as well as null rheological volume property with unique-temperature.

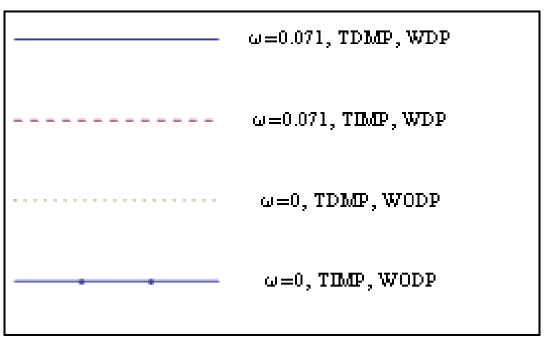

Figure1: Curves representation

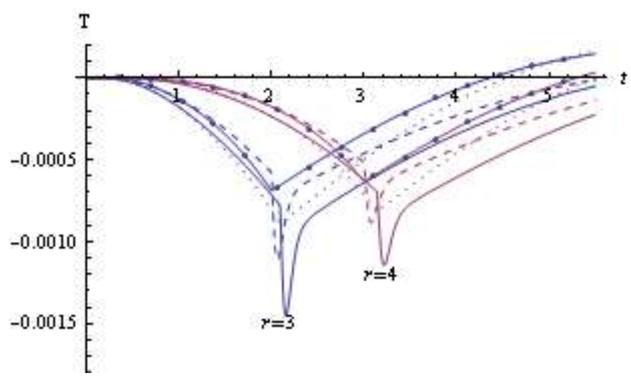

Figure2: Plot thermodynamic temperature vs time

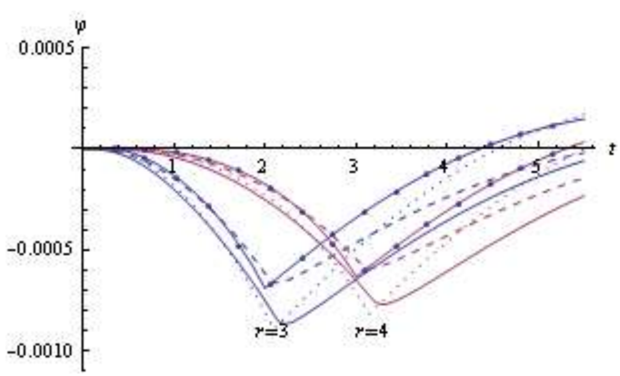

Figure 3: Plot conductive temperature vs time

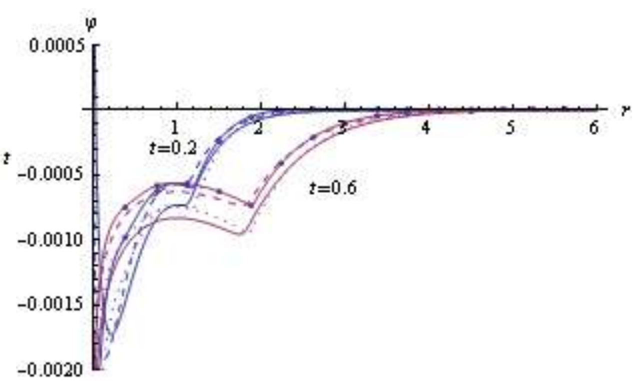

Figure 4: Plot conductive temperature vs radius

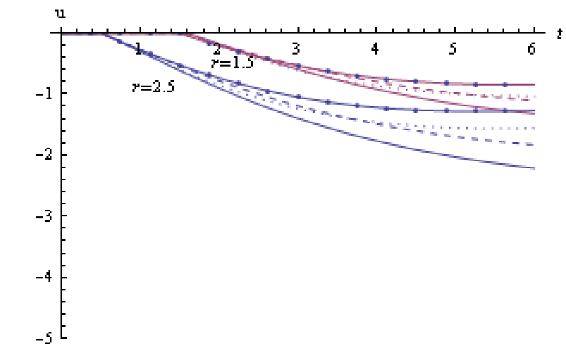

Figure 5: Plot displacement vs time

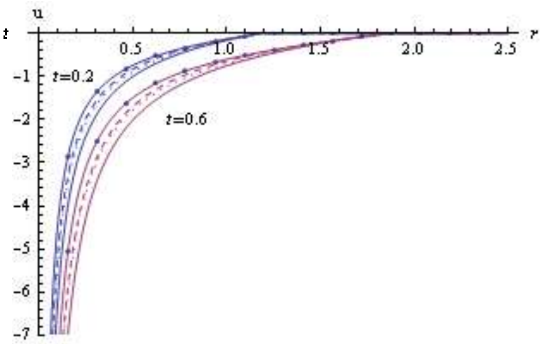

Figure 6: Plot displacement vs radius

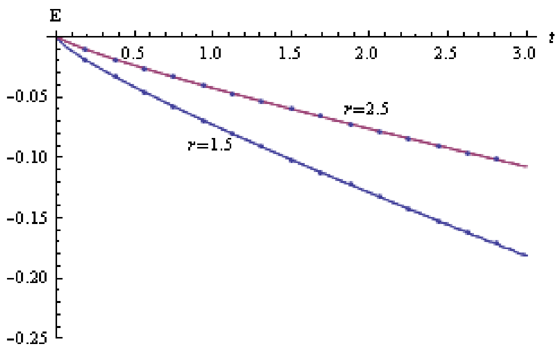

Figure 7: Plot electric field vs time

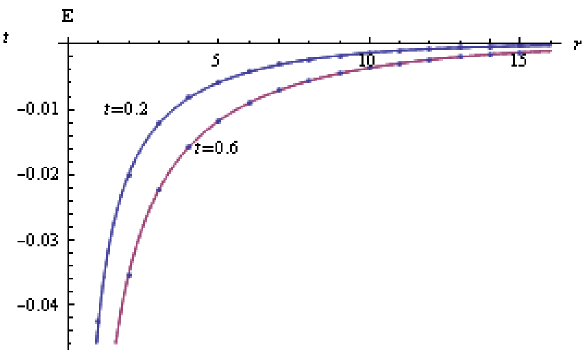

Figure 8: Plot electric field vs radius

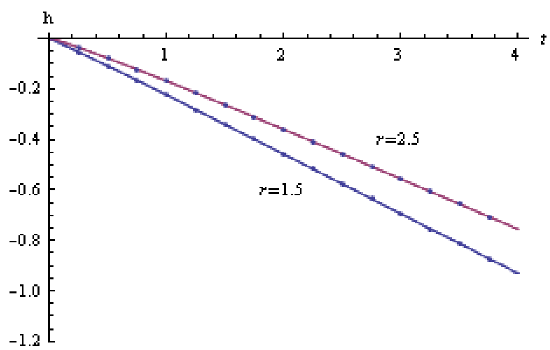

Figure 9: Plot perturbed magnetic field vs time

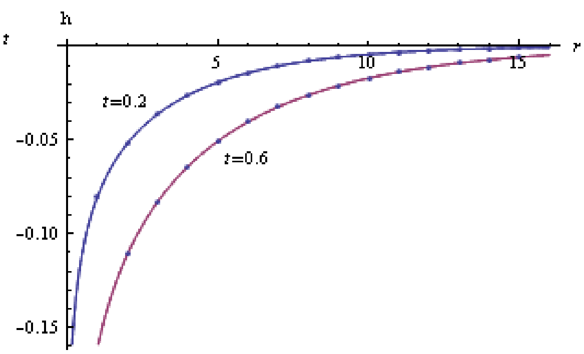

Figure 10: Plot perturbed magnetic field vs radius

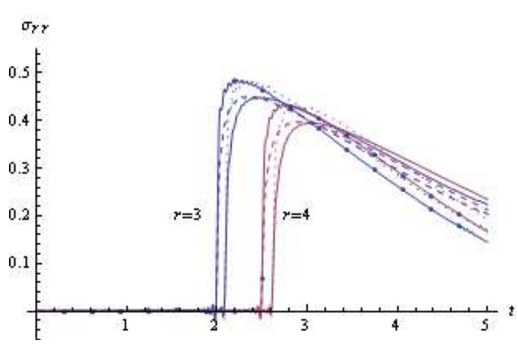

Figure 11: Plot stress component vs time

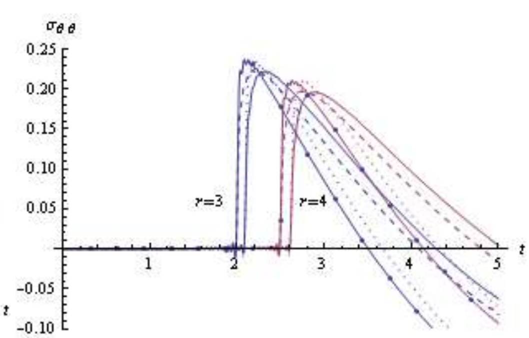

Figure 12: Plot stress component vs time

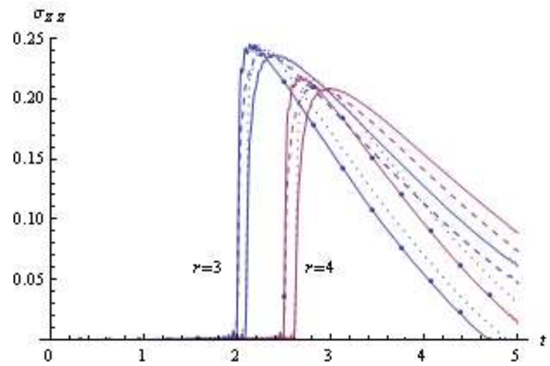

Figure 13: Plot stress component vs time

Here the figures represent the variation of thermodynamic temperature, conductive temperature, displacement, electric field, perturbed magnetic field and stresses components when continuous radial stress is applied on the thermally insulated boundary and all the graphs are shown in view of two relaxation times. Figure 1 stands for representation of curves for remaining all graphs. The variations of thermodynamic temperature verses time in the presence of rheological density property, as well as without it, are plotted in the figure 2 and the assessment is made between TDMP and TIMP. The effect of two-temperature parameter is clearly seen from this figure. At a distance from the vertical axis all type of curves produce the notable variations. It is observed from this figure that for increasingly fixed values of r, the deviation gradually decreases. Figure 3 and figure 4 describe the variations of conductive temperature with respect of time variable and radial distance respectively. It is observed from both these figures that the conductive temperature takes different values in the presence of density property and two-temperature parameter and without them. From figure 4 it is found that the curves for TDMP always provide lower values than that of the curves for TIMP and steadily converge to zero. Figure 5 depicts the variation of displacement versus time t. For fixed value of r, a little change in displacement is observed at the initial stage and after a certain time-pass it gradually decreases and then converges to a certain value. From this figure it is seen that when time increases the curves for TDMP and TIMP converge nearly parallel to each other and the nature of the curves just change for WDP and WODP. It is also observed that the initial time-pass increases with the increasing value of r. Figure 6 shows the variation of displacement with respect to radial distance. It is found from the figure that for fixed t the magnitude of displacement decreases with the increasing value of the radial distance and that it approaches zero at a distance far from the boundary of the hole. Figures 7-8 represent the distributions of electric field and figures 9-10 represent the distributions of perturbed magnetic field when continues radial stress is applied at the boundary of the hole. It is seen from both the figures 7 and 9 that as time increase both the fields' decreases and the magnitude of the fields' are inversely proportional with radial distance. Figures 8 and 10 show that for fixed values of t, electric field and perturbed magnetic field increase with the radial distance and after the traverse of certain distance the steady state condition arrives and gradually converges to zero. Figures 11-13 illustrate the stress distributions with respect to time for WDP and WODP as well as taking TDMP and TIMP. After a certain span of time it is found from the figures that the stress components expose a sudden increments and then decrease with time. It is noticed that the magnitude of the sudden increments decreases with the increasing value of the radial distance.

Conclusions

In the presence of uniform magnetic field and in the context of classical theory and generalized theory, the governing equations of thermo-visco-elasticity with two-temperature have been investigated with the effect of temperature dependent mechanical properties and rheological density property. The leading equations have been derived for the cases when continuous radial stress and instantaneous radial stress applied on the thermally insulated boundary. Eigen value approach has been used to solve the vector-matrix-differential equation. We conclude that the magnitude of the displacement for the curve TIMP is lesser then that of the curve for TDMP. In the presence of temperature discrepancy ($\omega = 0.071$) and rheological density property the magnitude of the displacement increases. The conductive temperature and the thermodynamic temperature exhibit the remarkable changes on behalf of various situations. The electric field and perturbed magnetic field take the values with very small disparity for different conditions. Stress components provide the similar nature as it was for null temperature discrepancy and for void rheological density property.

REFERENCES

Ailawalia P, Khurana G and Kumar S (2009). Effect of rotation in a generalized thermoelastic medium with two temperature under the influence of gravity. *International Journal of Applied Mathematics and Mechanics* **5**(5) 99-116.

Aouadi M and El-Karamany AS (2004). The relaxation effects of volume properties in two-dimensional generalized thermoviscoelastic problem. *Applied Mathematics and Computation* **151** 689-711.

Banik S and Kanoria M (2011). Two-Temperature Generalized Thermoelastic Interactions in an Infinite Body with a Spherical Cavity. *International Journal of Thermophysics* **32** 1247-1270.

Chen PJ and Gurtin ME (1968). On a theory of heat conduction involving two temperatures. *Zeitschrift für Angewandte Mathematik und Physik* **19**(4) 614-627.

Drozdov AD (1996). A constitutive model in thermo-viscoelasticity. *Mechanics Research Communications* **23** 543-548.

Ezzat MA and El-Karamany AS (2011). Two-temperature theory in generalized magneto-thermoelasticity with two relaxation times. *Meccanica* **46** 785-794.

Ezzat MA, Zakaria M and El-Karamany AS (2010). Effects of modified Ohm's and Fourier's laws on generalized magnetoviscoelastic thermoelasticity with relaxation volume properties. *International Journal of Engineering Science* **48** 460-472.

Giorgi C and Naso MG (2006). Mathematical models of Reissner–Mindlin thermoviscoelastic plates. *Journal of Thermal Stresses* **29** 699-716.

Green AE and Lindsay KA (1972). Thermoelasticity. *Journal of Elasticity* **2** 1-7.

Gurtin ME and Williams WO (1967). An axiom foundation for continuum thermodynamics. *Archive for Rational Mechanics and Analysis* **26** 83-117.

Honig G and Hirdes U (1984). A method for the numerical inversion of Laplace transforms. *Journal of Computational and Applied Mathematics* **10** 113-132.

Koltunov MA (1976). *Creeping and Relaxation* (Izd. Vishaya, Shkola, Moscow).

Kundu MR and Mukhopadhyay B (2005). A Thermoviscoelastic Problem of an Infinite Medium with a Spherical Cavity Using Generalized Theory of Thermoelasticity. *Mathematical and Computer Modelling* **41** 25-32.

Lomakin VA (1976). *The Theory of Elasticity of Non-Homogeneous Bodies* (Moscow State University Press, Moscow).

Lord H and Shulman Y (1967). A generalized dynamical theory of thermoelasticity. *Journal of the Mechanics and Physics of Solids* **15** 299-309.

Mondal MK and Mukhopadhyay B (2013). A cylindrical problem with rheological volume, density property on thermo-visco-elastic medium in magnetic field. *International Journal of Applied Mathematics and Mechanics* **9**(2) 22-38.

Mondal MK and Mukhopadhyay B (2013). Effect of two-temperature on thermoviscoelasticity problem with rheological properties. *International Journal of Physics and Mathematical Sciences* **3**(2) 36-47.

Nowacki W (1959). Some Dynamic Problems of Thermoviscoelasticity. *Archiwum Mechaniki Stosowane* **11** 259.

Othman MIA, Farouk RM and El Hamied HA (2013). The effect of magnetic field and thermal relaxation on 2-D problem of generalized thermoelastic diffusion. *International Applied Mechanics* **49**(2) 245-255.

Puri P and Jordan P (2006). On the propagation of harmonic plane waves under the two-temperature theory. *International Journal of Engineering Science* **44** 1113-1126.

Quintanilla R (2004). On existence, structural stability, convergence and spatial behavior in thermoelasticity with two temperatures. *Acta Mechanica* **168** 61-73.

Roy Choudhuri SK and Chattopadhyay M (2007). Electro-magneto-thermo-visco-elstic plane waves in rotating media with thermal relaxation. *International Journal of Thermophysics* **28** 1401-1419.

Shaw S and Mukhopadhyay B (2013). Moving heat source response in micropolar half-space with two-temperature theory. *Continuum Mechanics and Thermodynamics* **25** 523-535.

Youssef HM (2006). Theory of two-temperature generalized thermoelasticity. *IMA Journal of Applied Mathematics* **71** 383-390.

Youssef HM (2010). Two-temperature generalized thermoelastic infinite medium with cylindrical cavity subjected to moving heat source. *Archive of Applied Mechanics* **80** 1213-1224.

Youssef HM and Al-Harby AH (2007). State-space approach of two-temperature generalized thermoelasticity of infinite body with a spherical cavity subjected to different type thermal loading. *Archives of Applied Mechanics* **77** 675-687.