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Research Article

GAUSSIAN DIOPHANTINE QUADRUPLE WITH PROPERTY D(4)

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ABSTRACT
A set of m Gaussian integer is called a complex Diophantine m-tuple with the property D(z) if the product of its any two distinct elements increased by z is a square of a Gaussian integer. In this paper, we present five sets of Gaussian Diophantine quadruples with the property D(4).

***Keywords**: Diophantine Quadruples, Gaussian Integer.*

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INTRODUCTION
A set of positive integers $\{a_1, a_2......a_m\}$ is said to have the property D(n), $n \in Z - \{0\}$, if $a_i a_j + n$, a perfect square for all $1 \le i < j \le m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any arbitrary integer n [1] and also for any linear polynomials in n. Further, various authors considered the connections of the problem of Diophantus, Davenport and Fibonacci numbers in (Hoggatt and Bergum, 1977; Horadam, 1987; Jones, 1978; Long and Bergum, 1988; Morgado, 1983-1984, 1991, 1995; Gupta and Singh, 1985; Beardon and Deshpande, 2002; Brown, 1985; Deshpande, 2003; Bugeaud *et al.*, 2007; Liqun, 2007; Fujita, 2008; Srividhya, 2009; Gopalan and Pandichelvi, 2011; Yasutsugu and Togbe, 2011; Gopalan and Srividhya, 2012; Gopalan and Srividhya, 2012; Gopalan and Srividhya, 2012).
In this paper we consider the analogous problem for Gaussian integer. Let z be any Gaussian integer and let $m \ge 2$ be an integer. A set $\{a_1, a_2......a_m\} \subset z(i) \setminus \{0\}$ is said to have this property D(z) if the product of its any two distinct elements increased by z is a square of a Gaussian integer. If the set $\{a_1, a_2......a_m\}$ is a complex Diophantine quadruple then the same is true for the set $\{-a_1, -a_2......, -a_m\}$. Particularly in (Dujella *et al.*, 1997; Vidhyalakshmi *et al.*, 2014), the authors have analyzed the problem of the existence of the complex Diophantine quadruples. In this paper, we present five sets of Gaussian Diophantine quadruples with the property D(4).

METHOD OF ANALYSIS
Let $a = p + 1 + iq$ and $b = 9p - 3 + 9iq$ be two Gaussian integers.
Observe that $ab + 4 = \alpha^2$.
Thus (a, b) is a Gaussian Diophantine double with property D(4).
Let c be any non-zero Gaussian integer such that

$$ac + 1 = \beta^2 \quad (1)$$

$$bc + 1 = \gamma^2 \quad (2)$$

Case (i):
Setting $\beta = a + \alpha, \gamma = b + \alpha$ and subtracting (1) from (2), we obtain

$$c == a + b + 2\alpha = 16p + i16q$$

Case (ii):

Similarly by choosing $\beta = a - \alpha, \gamma = b - \alpha$, we obtain

$c = a + b - 2\alpha = 4p - 4 + i4q$

We have a well known result that the fourth tuple for the property D(4) is given by

$$d = a + b + c + \frac{1}{2}[abc + \alpha\beta\gamma]$$

Thus, the Gaussian Diophantine quadruples are given by

Case (i):

$$\{p+1+iq,\ 9p-3+i9q,\ 16p+i16q,$$
$$144p^3 - 432pq^2 + 96p^2 - 96q^2 + 4p - 4 + i(-144q^3 + 432p^2q + 192pq + 4q)\}$$

Case (ii):

$$\{p+1+iq,\ 9p-3+i9q,\ 4p-4+i4q,$$
$$36p^3 - 108pq^2 - 12p^2 + 12q^2 - 8p + i(-36q^3 + 108p^2q - 24pq - 8q)\}$$

For simplicity, a few examples are exhibited below:

Ex:1

$$a = 3p - 3 + i3q,\ b = 12p - 4 + i12q,\ c = 27p - 15 + i27q,$$
$$d = 972p^3 - 2916pq^2 + 1836p^2 - 1836q^2 + 1128p - 224 + i(-972q^3 + 2916p^2q + 3672pq + 1128q)$$

Ex:2

$$a = 2p + 1 + i2q,\ b = 2p - 3 + i2q,\ c = 8p - 4 + i8q,$$
$$d = 32p^3 - 96pq^2 - 48p^2 + 48q^2 + 16p + i(-36q^3 + 96p^2q - 96pq + 16q)$$

Ex:3

$$a = p - 2 + iq,\ b = p + 2 + iq,\ c = 4p + i4q,$$
$$d = 4p^3 - 12pq^2 - 4p + i(-4q^3 + 12p^2q - 4q)$$

CONCLUSION

In this paper, we have exhibited five Gaussian quadruples with property D(4) starting with linear Gaussian polynomials. One may search for Gaussian Diophantine quadruples consisting of polygonal numbers and centered polygonal numbers with suitable property.

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REFERENCES

Andrej Dujella, Zagreb, Croatia (1997). The problem of Diophantus and Davenport for Gaussian Integers. *Glasnik Matematicki, Ser III* **32** 1-10.

Baker A and Davenport H (1969). The equations $3x^2 - 2 = y^2$ and $8x^2 - 7 = z^2$, *Quarterly Journal of Mathematics: Oxford ser.***20**(2) 129-137.

Beardon AF and Deshpande MN (2002). Diophantine triples. *The Mathematical Gazette* **86** 253-260.

Brown E (1985). Sets in which xy+k is always a square. *Mathematics of Computation* **45** 613-620.

Bugeaud Y, Dujella A and Mignotte (2007). On the family of Diophantine triples $(k-1, k+1, 16k^3 - 4k)$. *Glasgow Mathematical Journal* **49** 333-344.

Deshpande MN (2003). Families of Diophantine Triplets. *Bulletin of the Marathwada Mathematical Society* **4** 19-21.

Fujita Y (2008). The extensibility of Diophantine pairs (k-1,k+1). *Journal of Number Theory* **128** 322-353.

Gopalan MA and Pandichelvi V (2011). The Non Extendibility of the Diophantine Triple $(4(2m-1)^2n^2, 4(2m-1)n+1, 4(2m-1)^4n^4-8(2m-1)^3n^3)$. *Impact Journal of Science and Technology* **5**(1) 25-28.

Gopalan MA and Srividhya G (2012). Diophantine Quadruple for Fionacci and Lucas Numbers with property D(4). *Diophantus Journal of Mathematics* **1**(1) 15-18.

Gopalan MA and Srividhya G (2012). Some non extendable P_{-5} sets. *Diophantus Journal of Mathematics* **1**(1) 19-22.

Gopalan MA and Srividhya G (2012). Two Special Diophantine Triples. *Diophantus Journal of Mathematics* **1**(1) 23-27.

Gupta H and Singh K (1985). On k-triad Sequences. *International Journal of Mathematics and Mathematical Sciences* **5** 799-804.

Hoggatt VE and Bergum GE (1977). A Problem of Fermat and the Fibonacci sequence, *Fibonacci Quarterly* **15** 323-330.

Horadam AF (1987). Generalization of a result of Morgudo. *Portugaliae Mathematica* **44** 131-136.

Jones BW (1978). A second variation on a problem of diophantus and davenport. *Fibonacci Quarterly* **16** 155-165.

Long C and Bergum GE (1988). On a problem of Diophantus. In: *Application of Fibonacci Numbers*. Edited by Philippou AN, Horadam AF and Bergum GE (Kluwer, Dordrecht) **2** 183-191.

Morgado J (1983-1984). Generalization of a result of Hoggatt and bergum on Fibonacci numbers. *Portugaliae Mathematica* **42** 441-445.

Morgado J (1991). Note on a Shannon's theorem concerning the Fibonacci numbers. *Portugaliae Mathematica* **48** 429-439.

Morgado J (1995). Note on the Cheyshev polynomials of the second kind. *Portugaliae Mathematica* **52** 363-378.

Srividhya G (2009). Diophantine Quadruples for Fibbonacci numbers with property D(1). *Indian Journal of Mathematics and Mathematical Science* **5**(2) 57-59.

Tao Liqun (2007). On the property P_{-1}. *Electronic Journal of Combinatorial Number Theory* **7** #A47.

Vidhyalakshmi S, Gopalan MA and Lakshmi K (2014). Gaussian Diophantine quadruples with property D(1). *IOSR Journal of Mathematics* **10**(3) Ver.II 12-14.

Yasutsugu Fujita and Alain Togbe (2011). Uniqueness of the extension of the $D(4k^2)$-triple $(k^2-4, k^2, 4k^2-4)$. *Notes on Number Theory and Discrete Mathematics* **17**(4) 42-49.