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Research Article

BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A STRETCHING SURFACE IN THE PRESENCE OF MAGNETIC FIELD

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ABSTRACT
A steady two-dimensional laminar boundary layer flow of an incompressible viscous electrically conducting fluid near a stagnation point of stretching surface in the presence of magnetic field is analyzed. The governing partial differential equations are non-dimensionalzed and transformed into a system of nonlinear ordinary differential similarity equations, in a single independent variable. The resulting nonlinear ordinary differential equations are solved under appropriate transformed boundary conditions using Runge-Kutta-Fehlberg Forth-Fifth order method. The influence of various parameters are presented and discussed.

Keywords: *Boundary Layer Flow, Heat Transfer, Stretching Surface, Magnetic Field, Numerical Study*

NOMENCLATURE

a	proportionality constant of the free stream velocity
c	proportionality constant of the stretching surface velocity
C_p	specific heat of the fluid at constant pressure
Ec	Eckert number
f	dimensionless stream function
H_0	applied magnetic field
Ha	Hartmann number
Pr	Prandtl number
T	temperature of the fluid
T_w	temperature at the surface
T_∞	free stream temperature
u, v	velocity component of the fluid along the x and y directions, respectively
u_w	velocity at the surface
$u_e(x)$	free stream velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

ρ	density of the fluid
μ	viscosity of the fluid
μ_e	magnetic permeability
σ_e	electrical conductivity
η	dimensionless similarity variable
κ	thermal conductivity
υ	kinematic viscosity
Ψ	stream function
θ	dimensionless temperature

Superscript

$'$	derivative with respect to η

Subscripts

w	properties at the surface
∞	free stream condition

INTRODUCTION

The hydromagnetic flow and heat transfer in a viscous incompressible fluid over a moving continuous stretching surface is a significant type of flow has considerable practical applications in industries and engineering. For example, materials manufactured by extrusion processes, heat-treated materials raveling between a feed roll and a wind-up roll or on a conveyor belt possess the characteristics of a moving continuous surface. Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. The classical problem was introduced by Blasius (1908) where he considered the boundary layer flow on a fixed flat plate. Different from Blasius (1908), the boundary layer flow over a stretching sheet was first studied by Sakiadis (1961). Later, Crane (1970) extended this idea for the two-dimensional problem where the velocity is proportional to the distance from the plate. The heat and mass transfer over a stretching sheet subject to suction or blowing (injection) was investigated by Gupta and Gupta (1977) and Magyari and Keller (1999, 2000). Mahapatra and Gupta (2002, 2004) studied the heat transfer in the steady two-dimensional stagnation-point flow of a viscous, and incompressible Newtonian and viscoelastic fluids over a horizontal stretching sheet considering the case of constant surface temperature. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of magnetic field on the laminar flow over a stretching surface was studied by a number of researchers Jhankal and Kumar (2013), Pavlov (1974), Chakrabarthi and Gupta (1979), Chima (1993), Noor *et al.*, (2010) etc.

Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the problem of steady two-dimensional laminar boundary layer flow of an incompressible viscous electrically conducting fluid near a stagnation point of stretching surface in the presence of magnetic field.

Formulation of the Problem

Let us consider two-dimensional steady boundary layer flow of a viscous incompressible electrically conducting fluid near a stagnation point over a flat surface such that surface is stretched in its own plane with velocity proportional to the distance from the stagnation point in the presence of an externally applied normal magnetic field of constant strength H_0. The stretching surface has uniform temperature T_w and a linear velocity u_w while the velocity of the flow external to the boundary layer is $u_e(x)$. Under the usual boundary layer approximations, the governing equations of continuity, momentum and energy under the influence of externally imposed normal magnetic field are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_e \mu_e^2 H_0^2}{\rho}(u_e - u) \qquad \ldots(2)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma_e\mu_e^2 H_0^2 u^2 \qquad \ldots(3)$$

Along with the boundary conditions are:

$$y = 0: u = u_w = cx, v = 0, T = T_w$$
$$y \to \infty: u \to u_e(x) = ax, T \to T_\infty \qquad \ldots(4)$$

The continuity equation (1) is satisfied by introducing a stream function Ψ such that $u = \frac{\partial \Psi}{\partial y}$ and $= -\frac{\partial \Psi}{\partial x}$. …(5)

The momentum and energy equations can be transformed into the corresponding ordinary nonlinear differential equations by using the following transformations:

$$\eta = y\left(\frac{c}{\nu}\right)^{1/2}, \; f(\eta) = \frac{\Psi}{x(c\nu)^{1/2}} \text{ and } \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad \ldots(6)$$

Then, the transformed non-linear differential equations are:

$$f''' + ff'' - f'^2 - H_a^2 f' + H_a^2\lambda + \lambda^2 = 0 \qquad \ldots(7)$$

$$\frac{1}{Pr}\theta'' + f\theta' + Ecf''^2 + H_a^2 Ecf'^2 = 0 \qquad \ldots(8)$$

The transformed boundary conditions are:

$\eta = 0: f = 0, f' = 1, \theta = 1$
$\eta \to \infty: f' = \lambda, \theta = 0.$ …(9)

Where prime denotes differentiation with respect to η, $H_a = \mu_e H_0 \left(\frac{\sigma_e}{\rho c}\right)^{1/2}$ is the Hartmann number, $Pr = \frac{\mu c_p}{\kappa}$ is the Prandtl number, $\lambda = \frac{a}{c}$ is the velocity parameter and $Ec = \frac{u_w^2}{C_p\,(T_w - T_\infty)}$ is the Eckert number.

NUMERICAL SOLUTION AND DISCUSSION

The non-linear differential equations (7) and (8) subject to the boundary conditions (9) is solved numerically using Runge-Kutta-Fehlberg Forth-Fifth order method. To solve this equation we adopted symbolic algebra software Maple. Maple uses the well known Runge-Kutta-Feulberg Forth-Fifth (RKF45) order method to generate the numerical solution of boundary value problem.

The numerical results are obtained for velocity parameter (λ) 0.1 and 2.0, fixed values of Prandtl number (Pr) 0.71 and Eckert number (Ec) 0.01. The effect of Hartmann number (H_a) on the velocity and temperature are presented in Figures 1 to 4.

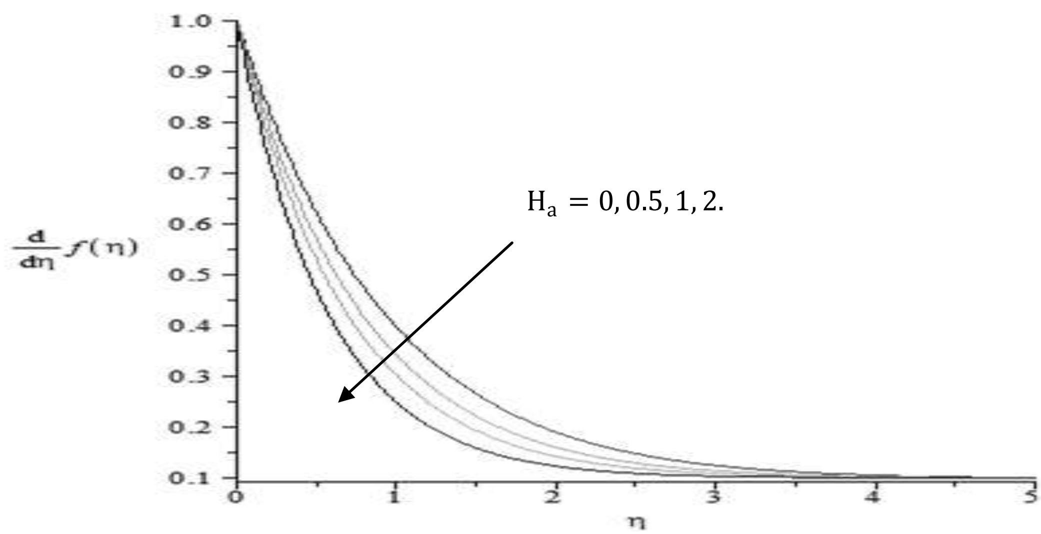

Figure 1: Velocity profile for various value of Hartmann number H_a when λ=0.1

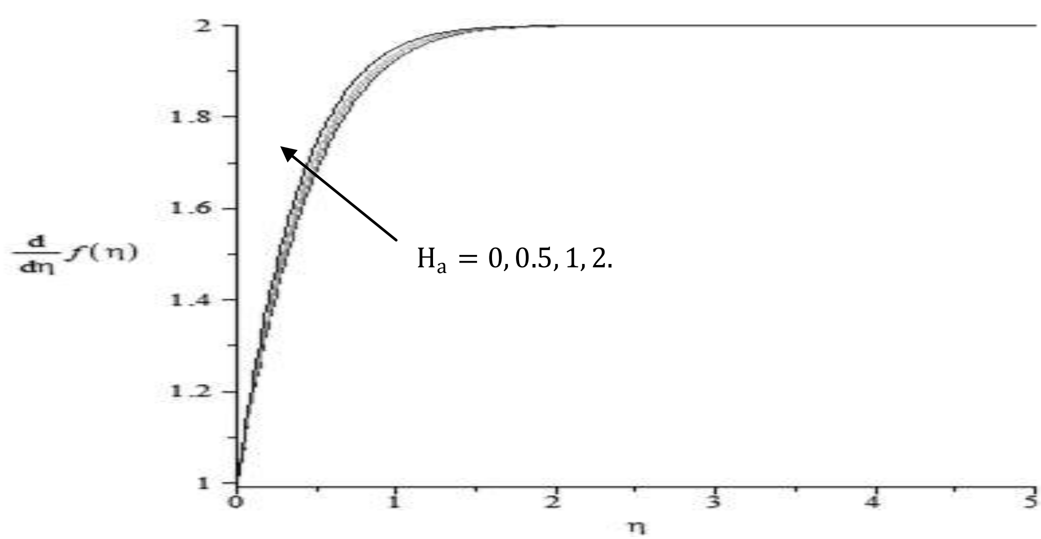

Figure 2: Velocity profile for various value of Hartmann number H_a when λ=2.0

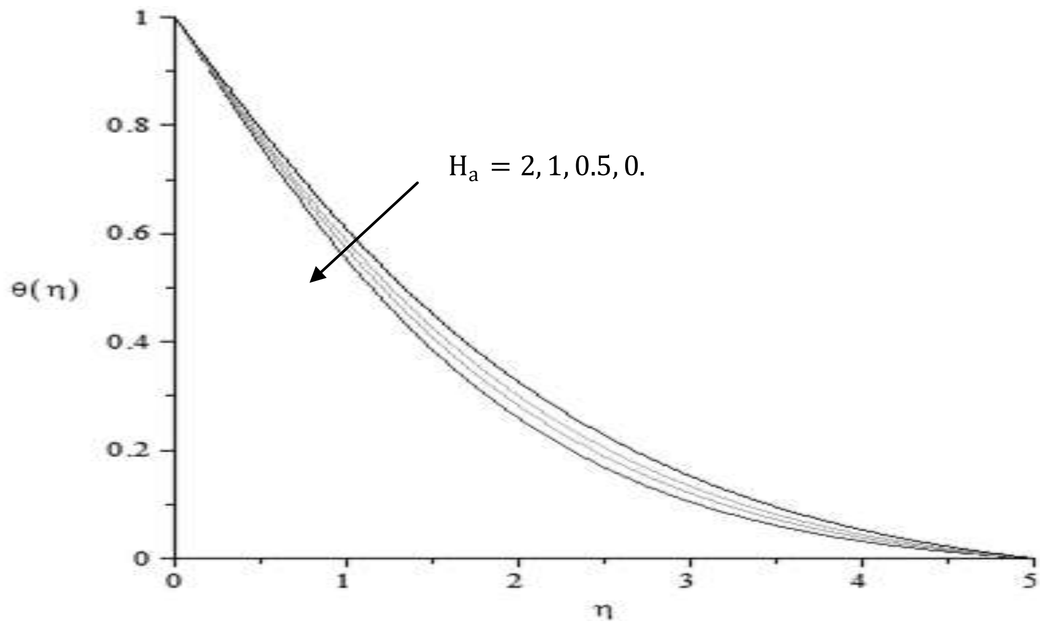

Figure 3: Temperature profile for various value of Hartmann number H_a when λ=0.1, Pr=0.71 and Ec=0.01

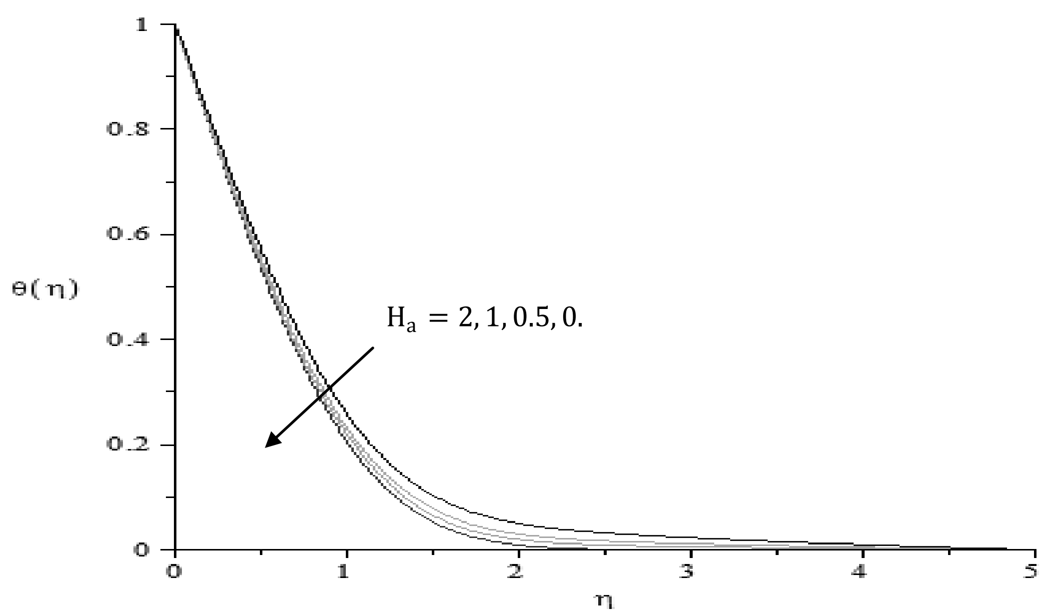

Figure 4: Temperature profile for various value of Hartmann number H_a when λ=2.0, Pr=0.71 and Ec=0.01

Conclusion

In this study, a mathematical model has been presented for the boundary layer flow and heat transfer over a stretching surface in the presence of magnetic field.

We notice from the figure 1 (when λ=0.1<1.0), velocity boundary layer thickness decreases with increases in Hartmann number H_a, whereas opposite phenomenon occurs in figure 2, when λ=2.0>1.0. Thus we conclude that we can control the velocity field by introducing magnetic field.

On the other hand, the temperature profiles for various values of Hartmann number (H_a) for velocity parameter (λ) 0.1 and 2.0, fixed values of Prandtl number (Pr) and Eckert number (Ec) are plotted against the similarity variable in figures 3 and 4. It is observed from the figures that the thermal boundary layer thickness increases with increases in Hartmann number H_a.

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