International Journal of Physics and Mathematical Sciences ISSN: 2277-2111 (Online)
An Open Access, Online International Journal Available at http://www.cibtech.org/jpms.htm
2014 Vol. 4 (1) January-March, pp. 188-192/Solanke and Durge
Research Article

QUASI-STATIC TRANSIENT THERMAL STRESSES IN A DIRICHLET'S THIN HOLLOW CYLINDER WITH INTERNAL MOVING HEAT SOURCE

***D. T. Solanke[1] and M. H. Durge[2]**
[1]*Sudhakar Naik and Umashankar Khetan College Akola, Maharashtra State, India*
[2]*Anand Niketan College, Warora, Maharashtra State, India*
**Author for Correspondence*

ABSTRACT
This paper concerns with transient non-homogeneous thermoelastic problem with Dirichlet's boundary condition in thin hollow cylinder of isotropic material of inner radius a, outer radius b and height h, occupying the region R: $a \le r \le b,\ \ -\frac{h}{2} \le z \le \frac{h}{2},\ \ 0 \le \phi \le 2\pi$ having initial temperature $f(r,\phi,z)$ placed in an ambient temperature zero. The cylinder is subjected to the activity of moving heat source along circular trajectory of radius r_0, where $a < r_0 < b$, around the centre of the cylinder with constant angular velocity ω. The heat conduction equation containing heat generation term is solved by applying integral transform technique and Green's theorem is adopted in deducing the solution of heat conduction equation. The solution is obtained in a series form of Bessel function and trigonometric function and derived thermal stresses.

***Keywords**: Dirichlet's Thin Hollow Cylinder, Moving Heat Source, Thermal Stresses, Green's Theorem*

INTRODUCTION
During the second half of 20^{th} century, non-isothermal problems of the theory of elasticity became increasingly important. This is due to their wide application in diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of aircraft structure.

In this present paper we determine temperature, thermal stresses, in a Dirichlet's thin hollow cylinder, determined by R: $a \le r \le b, -\frac{h}{2} \le z \le \frac{h}{2}, 0 \le \phi \le 2\pi$ with internal moving heat source. Heat conduction equation with heat generation term is solved by applying integral transform technique and Green's theorem. Solution is obtained in series form of Bessel function and trigonometric function. The direct problems is very important in view of its relevance to various industrial mechanics subjected to heating such as main shaft of lathe, turbines and the role of rolling mill for base of furnace boiler of thermal power plant, gas power plant and measurement of aerodynamic heating.

Formulation of the Heat conduction problem

Consider a thin hollow cylinder of isotropic material of inner radius a, outer radius b and thickness h occupying the region R: $a \le r \le b, -\frac{h}{2} \le z \le \frac{h}{2}, 0 \le \phi \le 2\pi$ having initial temperature $f(r,\phi,z)$ placed in an ambient temperature zero. The cylinder is subjected to the activity of moving heat source which changes its place along circular trajectory of radius r_0, where $a < r_0 < b$, around the centre of the cylinder with constant angular velocity ω. The activity of moving heat source and initial temperature of the cylinder may cause the generation of heat due to nuclear interaction that may be a function of position and time in the form $g(r,\phi,z,t)$ w/s^3. The temperature distribution of the thin hollow cylinder is described by the differential equation of heat conduction with heat generation term as in [5] page no.8 is given by

$$\nabla^2 T + \frac{1}{k} g = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where $T = T(r,\phi,z,t)$ is temperature distribution, k is thermal conductivity of the material of the cylinder, $\alpha = \dfrac{k}{\rho C_p}$ is thermal diffusivity, ρ is density, C_p is specific heat of the material and g is the volumetric energy generation term in the cylinder. Where ∇^2 is Laplacian operator in cylindrical coordinates and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$

Now consider an instantaneously moving heat source $g_s{}^i$ located at a point (r_0, ϕ', ξ) and releasing its energy spontaneously at time τ. Such volumetric heat source in cylindrical coordinates is given by

$$g(r,\phi,z,t) = g_s{}^i \frac{1}{2\pi r}\delta(r-r_0)\delta(\phi-\phi')\delta(z-\xi)\delta(t-\tau)$$

Hence above equation reduces to

$$\nabla^2 T + g_s{}^i \frac{1}{2\pi k r}\delta(r-r_0)\delta(\phi-\phi')\delta(z-\xi)\delta(t-\tau) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (3.1)$$

$$\phi' = \omega t \quad (3.2)$$

With initial and homogeneous boundary conditions,

$$T = 0 \text{ at } r = a \text{ and } r = b \quad (3.3)$$

$$T = 0 \text{ at } z = -\tfrac{h}{2} \quad (3.4)$$

$$T = 0 \text{ at } z = \tfrac{h}{2} \quad (3.5)$$

$$T = f(r,\phi,z) \text{ at } t = 0, \; \tau = -\infty \quad (3.6)$$

Formulation of thermoelastic Problem

Let us introduce a thermal stress function χ related to component of stress in the cylindrical coordinates system as in [3]

$$\sigma_{rr} = \frac{1}{r}\frac{\partial \chi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \chi}{\partial r^2} \quad (4.1)$$

$$\sigma_{\phi\phi} = \frac{\partial^2 \chi}{\partial r^2} \quad (4.2)$$

$$\sigma_{r\phi} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \chi}{\partial \phi}\right) \quad (4.3)$$

The boundary condition for a traction free body are

$$\sigma_{rr} = 0, \sigma_{r\phi} = 0 \text{ at } r = a \text{ or } r = b \quad (4.4)$$

Where $\chi = \chi_c + \chi_p$ (4.5)

Where χ_c is complementary solution and χ_p is particular solution and

χ_c satisfies the equation $\nabla^4 \chi_c = 0$ (4.6)

χ_p satisfies the equation $\nabla^4 \chi_p = -\lambda E \nabla^2 \Gamma$ (4.7)

Where Γ is temperature change $\Gamma = T - T_i$, T_i is initial temperature

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$$ since cylinder is thin, z component is negligible

Solution

We define integral transform of $T(r,\phi,z,t)$ by

$$\overline{T}(\beta_m, v, \alpha_n, t) = \int_R T(r,\phi,z,t) R_v(\beta_m r)\cos v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2})dv \quad (5.1)$$

And its inverse by

$$T(r,\phi,z,t) = \frac{2}{\pi h}\sum_{m,n,v=0}^{\infty} \frac{\overline{T}(\beta_m, v, \alpha_n, t) R_v(\beta_m r)\cos v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2})}{N(\beta_m)} \quad (5.2)$$

Where $R_v(\beta_m r) = J_v(\beta_m r)Y_v(\beta_m b) - J_v(\beta_m b)Y_v(\beta_m r)$ (5.3)

$$N(\beta_m) = \int_a^b r R^2{}_v(\beta_m r)dr = \frac{2}{\pi^2}\frac{J^2{}_v(\beta_m a) - J^2{}_v(\beta_m b)}{\beta_m{}^2 J^2{}_v(\beta_m a)} \quad (5.4)$$

β_m is root of the transcendental equation $J_v(\beta_m a)Y_v(\beta_m b) - J_v(\beta_m b)Y_v(\beta_m a) = 0$ (5.5)

$v = 0,1,2,3,............$ (5.6)

$$\alpha_n = \frac{p\pi}{h} \quad p = 0,1,2,3,....... \quad (5.7)$$

By taking integral transform of equation (3.1) and using following Green's theorem

$$\int_R \nabla^2 T\psi_k dv = \int_R T\nabla^2\psi_k dv + \sum_{i=1}^{N}\int_{s_i}\left[\psi_k\frac{\partial T}{\partial n_i} - T\frac{\partial\psi_k}{\partial n_i}\right]ds_i \quad (5.8)$$

Which yield as

$$\frac{d\overline{T}}{dt} + \alpha(\beta_m{}^2 + \alpha_n{}^2)\overline{T} = \frac{\alpha}{2\pi k} g_s{}^i R_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})\delta(t-\tau)$$

This is linear differential equation of first order whose solution by applying initial condition (3.6) is

$$T = \frac{2}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{R_v(\beta_m r)\cos v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2})}{N(\beta_m)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s{}^i R_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right].$$
$$e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t} \quad (5.9)$$

$$\Gamma = \frac{2}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{R_v(\beta_m r)\cos v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2})}{N(\beta_m)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s{}^i R_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right].$$
$$\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t} - 1\right] \quad (5.10)$$

Solution of Thermoelastic Problem:

Let suitable form of χ_c satisfying equation (4.6) be

$$\chi_c = \sum_{v=0}^{\infty}\left(Ar^{v+2} + Br^{-v+2}\right)cos v(\phi-\phi') + \left(Cr^{v+2} + Dr^{-v+2}\right)\sin v(\phi-\phi') \quad (6.1)$$

Let suitable form of χ_p satisfying equation (4.7) be

$$\chi_p = \frac{2\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{R_v(\beta_m r)\cos v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2})}{\beta_m{}^2 N(\beta_m)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s{}^i R_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]$$

$$\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.2)$$

From (4.5) (6.1) and (6.2) we obtain

$$\chi=\sum_{v=0}^{\infty}\left(Ar^{v+2}+Br^{-v+2}\right)cosv(\phi-\phi')+\left(Cr^{v+2}+Dr^{-v+2}\right)\sin v(\phi-\phi')+$$
$$\frac{2\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{R_v(\beta_m r)\cos v(\phi-\phi')\sin\alpha_n(z+\frac{h}{2})}{\beta_m{}^2N(\beta_m)}\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^iR_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]$$
$$\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.3)$$

From (4.1) and (6.3) we obtain

$$\sigma_{rr}=\sum_{v=0}^{\infty}\left\{\left[A(2+v-v^2)r^v+B(2-v-v^2)r^{-v}\right]\cos v(\phi-\phi')+\left[C(2+v-v^2)r^v+D(2-v-v^2)r^{-v}\right]\sin(\phi-\phi')\right\}+$$
$$\frac{2\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{1}{N(\beta_m)}\frac{1}{\beta_m{}^2r^2}\left[\beta_m rR_{v-1}(\beta_m r)-(v+v^2)R_v(\beta_m r)\right]\cos v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2}).$$
$$\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^iR_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.4)$$

From (4.2) and (6.3) we obtain

$$\sigma_{\phi\phi}=\sum_{v=0}^{\infty}\left\{\left[A(v^2+3v+2)r^v+B(v^2-3v+2)r^{-v}\right]\cos v(\phi-\phi')+\left[C(v^2+3v+2)r^v+D(v^2-3v+2)r^{-v}\right]\sin(\phi-\phi')\right\}+$$
$$\frac{2\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{1}{N(\beta_m)\beta_m{}^2}\left[\frac{\beta_m}{r}R_{v+1}(\beta_m r)-\beta_m{}^2R_v(\beta_m r)\right]\cos v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2}).$$
$$\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^iR_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.5)$$

From (4.3) and (6.3) we obtain

$$\sigma_{r\phi}=\sum_{v=0}^{\infty}\left\{\left[A(-v^2-v)r^v+B(v^2-v)r^{-v}\right]\sin v(\phi-\phi')+\left[C(v^2+v)r^v+D(-v^2+v)r^{-v}\right]\cos(\phi-\phi')\right\}-$$
$$\frac{2\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{1}{N(\beta_m)\beta_m{}^2r^2}\left[\beta_m rR_{v-1}(\beta_m r)-(1+v)R_v(\beta_m r)\right]\sin v(\phi-\phi')\sin\alpha_n(z+\tfrac{h}{2}).$$
$$\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^iR_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.6)$$

Applying condition (4.4) to (6.4) and (6.6) we obtain

$$A=-\frac{\lambda E}{\pi hv}\frac{b^{-v}R_{v-1}(\beta_m b)\sin\alpha_n(z+\frac{h}{2})}{\beta_m bN(\beta_m)}\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^iR_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.5)$$

$$B=\frac{\lambda E}{\pi h}\frac{b^{v}R_{v-1}(\beta_m b)\sin\alpha_n(z+\frac{h}{2})}{\beta_m bN(\beta_m)}\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^iR_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.6)$$

$$C=D=0 \quad (6.7)$$

Substituting the value of constants in above equations we obtain

$$\sigma_{rr}=\frac{\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\left\{\left[\left(\frac{2-v-v^2}{v}\right)b^v r^{-v}-\left(\frac{2+v-v^2}{v}\right)b^{-v}r^v\right]\frac{R_{v-1}(\beta_m b)}{\beta_m b}+2\left[\frac{\beta_m rR_{v-1}(\beta_m r)-(v+v^2)R_v(\beta_m r)}{\beta_m{}^2r^2}\right]\right\}.$$

Research Article

$$\frac{\cos v(\phi-\phi')\sin\alpha_n(z+\frac{h}{2})}{N(\beta_m)}\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^{i}R_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.8)$$

$$\sigma_{\phi\phi}=\frac{\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\left\{\left[\left(\frac{v^2-3v+2}{v}\right)b^v r^{-v}-\left(\frac{v^2+3v+2}{v}\right)b^{-v}r^v\right]\frac{R_{v-1}(\beta_m b)}{\beta_m b}+\frac{2}{\beta_m{}^2}\left[\frac{\beta_m}{r}R_{v+1}(\beta_m r)-\beta_m{}^2R_v(\beta_m r)\right]\right\}.$$

$$\frac{\cos v(\phi-\phi')\sin\alpha_n(z+\frac{h}{2})}{N(\beta_m)}\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^{i}R_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.9)$$

$$\sigma_{r\phi}=\frac{\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\left\{\left[(1-v)b^v r^{-v}-(1+v)b^{-v}r^v\right]\frac{R_{v-1}(\beta_m b)}{\beta_m b}+2v\left[\frac{\beta_m rR_{v-1}(\beta_m r)-(1+v)R_v(\beta_m r)}{\beta_m{}^2r^2}\right]\right\}.$$

$$\frac{\sin v(\phi-\phi')\sin\alpha_n(z+\frac{h}{2})}{N(\beta_m)}\left[\overline{f}(\beta_m,v,\alpha_n)+\frac{\alpha}{2\pi k}g_s{}^{i}R_v(\beta_m r_0)\sin\alpha_n(\xi+\tfrac{h}{2})e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right]\left[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1\right] \quad (6.10)$$

Conclusion

In this paper we determined the temperature distribution in three dimensions and thermal stresses in a Dirichlet's thin hollow cylinder with moving heat source with analytical approach on the surface is established. By giving particular values to the parameter one can obtain their desired results by putting values of the parameters in the equations (5.9), (6.8), (6.9), (6.10). From these equation we observe that initially ($t=0$) all stresses vanishes.

REFERENCES

Solanke DT and Durge MH (2014). Quasi-Static transient Thermal Stresses in a Dirichlet's thin Solid cylinder with internal moving heat source *(IOSR-JM)* **10(**2)1 51-55.

Solanke DT and Durge MH. Quasi-Static transient Thermal Stresses in a Neumann's thin Solid cylinder with internal moving heat source. *AJER* **3**(3) 75-79

Solanke DT and Durge MH (2014). Quasi-Static transient Thermal Stresses in a Robin's thin Solid cylinder with internal moving heat source *AJCEM* **3**(2) 8-11.

Ozisik M Necati (No date). `Heat conduction, Second Edition, A Wiley-Interscience Publication John Wiley and Sons, inc. New-York.

Noda N, Hetnarski RB, Tanigawa Y (2002). Thermal Stresses, Second edition.