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Research Article

NATURE OF GENERALIZED ALGEBRAIC STRUCTURE $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 \;/\; a_i \in F \;\&G_i \in C(P)\}$

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ABSTRACT
This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri SriRamakrishana. In the Present work first I proved that (A, +,.) is a non abelian ring. Second I proved that $(A,+,*)$ is a Commutative ring with unity,
Where A = $\{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 \;/\; a_i \in F \;\&G_i \in C(P)\}$
and $C(P)$= Class of Algebraic Structure.

Keywords: *Binary Operation, Abelian Group, Semigroup, Ring, Field*

INTRODUCTION

Herstein cotes in 1992

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that

1 a, b ∈ G implies that a*b ∈ G
2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Definition: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively, such that for all a,b,c in R:

1 a+b is in R.
2 a+b = b+a.
3 (a+b)+c = a+(b+c).
4 There is an element 0 in R such that a+0 = a,∀ a ∈ R
5 There exist an element –a in R such that a + (-a) = 0.
6 a*b is in R
7 a*(b*c) = (a*b)*c.
8 a * (b+c) = a * b + a * c and (b+c) * a = b*a + c*a.

It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R; if there is such we shall describe R as a ring with unit element.

If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

DISCUSSION

Let $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 \;/\; a_i \in F \;\&G_i \in C(P)\}$
Where $C(P)$= Class of Algebraic Structure, and
Let $x = a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4\,, a_i \in F$
$y = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4, b_i \in F$
$z = c_0G_0 + c_1G_1 + c_2G_2 + c_3G_3 + c_4G_4,\; c_i \in F$

$$-x = (-a_0)G_0 + (-a_1)G_1 + (-a_2)G_2 + (-a_3)G_3 + (-a_4)G_4$$
$$O = 0G_0 + 0G_1 + 0G_2 + 0G_3 + 0G_4$$
$$G_0 = 1G_0 + 0G_1 + 0G_2 + 0G_3 + 0G_4$$

$$cx = (ca_0)G_0 + (ca_1)G_1 + (ca_2)G_2 + (ca_3)G_3 + (ca_4)G_4 \,, c \in F$$

Now we define first binary operation $+$ on A as

$$x + y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4) + (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4)$$

$= (a_0 + b_0)G_0 + (a_1 + b_1)G_1 + (a_2 + b_2)G_2 + (a_3 + b_3)G_3 + (a_4 + b_4)G_4$

………… (1)

$$=> x + y = y + x \,, \forall x, y, \in A$$

$$x + (y + z) = (x + y) + z, \quad \forall x, y, z \in A$$

$$0 + x = x + 0, \quad \forall x \in A$$

$x + (-x) = (-x) + x = 0, \forall x \in A$

⇨ $(A,+)$ is an abelian group. ………… (2)

Case 1:

Now we define Second binary operation.onA as

$$x.y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4).(b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4)$$

$x.y = (a_0 + a_1 + a_2 + a_3 + a_4)y, \forall x, y \in A$ ………. (3)

$$=> x.(y.z) = (x.y).z, \forall x, y, z \in A$$

& $x.y \neq y.x, \ \forall x, y, \in A$

Hence (A,.) is a non-commutative semi group.

Also

$$(x + y).z = x.z + y.z \ \forall x, y, z \in A$$

$$x.(y + z) = x.y + x.z \ \forall x, y, z \in A$$

And (A, +, .) is a non-commutative ring. ……….. (4)

Case 2:

Now we define Second binary operation $*$ on A as

$$x * y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4) * (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4)$$

$= (a_0 b_0 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1) G_0$
$+ (a_0 b_1 + a_1 b_0 + a_2 b_4 + a_3 b_3 + a_4 b_2) G_1$
$+ (a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_4 + a_4 b_3) G_2$
$+ (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0 + a_4 b_4) G_3$
$+ (a_0 b_4 + a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0) G_4$ ……… (5)

$=> x * y = y * x, \ \forall \ x, y \in A$
$=> x * (y * z) = (x * y) * z, \forall x, y, z \in A$
$G_0 * x = x * G_0 = x, \forall x \in A$

Let $x^{-1} = (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4)$ be the inverse of any x in A
Where $x = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4)$

∴By definition one obtains

$a_0 b_0 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1 = 1$ …………(6)
$a_0 b_1 + a_1 b_0 + a_2 b_4 + a_3 b_3 + a_4 b_2 = 0$ ………… (7)
$a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_4 + a_4 b_3 = 0$ ………… (8)
$a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0 + a_4 b_4 = 0$ ………… (9)
$a_0 b_4 + a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0 = 0$ ………… (10)

Rewriting eq^n (7), (8), (9), (10)

$a_1 b_0 + a_0 b_1 + a_4 b_2 + a_3 b_3 + a_2 b_4 = 0$ ………… (11)
$a_2 b_0 + a_1 b_1 + a_0 b_2 + a_4 b_3 + a_3 b_4 = 0$ ………… (12)
$a_3 b_0 + a_2 b_1 + a_1 b_2 + a_0 b_3 + a_4 b_4 = 0$ ………… (13)
$a_4 b_0 + a_3 b_1 + a_2 b_2 + a_1 b_3 + a_0 b_4 = 0$ ………… (14)

solving eq^n (11), (12), (13) &(14) one obtains

$$\frac{b_0}{F_0} = \frac{-b_1}{F_1} = \frac{b_2}{F_2} = \frac{-b_3}{F_3} = \frac{b_4}{F_4} = k$$

Where; $F_0 = \begin{vmatrix} a_0 & a_4 & a_3 & a_2 \\ a_1 & a_0 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_4 \\ a_3 & a_2 & a_1 & a_0 \end{vmatrix}$

$$F_1 = \begin{vmatrix} a_1 & a_4 & a_3 & a_2 \\ a_2 & a_0 & a_4 & a_3 \\ a_3 & a_1 & a_0 & a_4 \\ a_4 & a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_2 = \begin{vmatrix} a_1 & a_0 & a_3 & a_2 \\ a_2 & a_1 & a_4 & a_3 \\ a_3 & a_2 & a_0 & a_4 \\ a_4 & a_3 & a_1 & a_0 \end{vmatrix}$$

$$F_3 = \begin{vmatrix} a_1 & a_0 & a_4 & a_2 \\ a_2 & a_1 & a_0 & a_3 \\ a_3 & a_2 & a_1 & a_4 \\ a_4 & a_3 & a_2 & a_0 \end{vmatrix}$$

$$F_4 = \begin{vmatrix} a_1 & a_0 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_4 \\ a_3 & a_2 & a_1 & a_0 \\ a_4 & a_3 & a_2 & a_1 \end{vmatrix}$$

$=> b_0 = kF_0$ ………… (15)
$b_1 = -kF_1$ ………… (16)
$b_2 = kF_2$ ………… (17)
$b_3 = -kF_3$………… (18)
$b_4 = kF_4$………… (19)
From eq^n (6), (15), (16), (17), (18), (19) one obtains
$k = {}^{1}/_{\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3 + a_4F_4\}} = \infty$ if $a_0 = a_1 = a_2 = a_3 = 0$ &$a_4 \neq 0$

⇨ Inverse of each of the element in A is not exist
⇨ $(A, +,*)$ Is a Commutative ring with unity,
Where A = $\{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 \ / \ a_i \in F \ \&G_i \in C(P)\}$
Where $C(P)$= Class of Algebraic Structure, and

Conclusion

From the above discussion,I come to the following conclusions
first I proved that (A, +,.) is a non commutative ring. Second I proved that $(A, +,*)$ Is a Commutative ring with unity.
Where A = $\{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 \ / \ a_i \in F \ \&G_i \in C(P)\}$
and $C(P)$= Class of Algebraic Structure,

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