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Research Article

NATURE OF ALGEBRAIC STRUCTURE

$A = \{a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m \,/\, ai \in F\}$

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ABSTRACT
This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri Sri Ramakrishana. In the Present work first I proved that (*A*, +,.) is a non commutative ring. Second I proved that $(A,+,*)$ is a Commutative ring with unity, where $A = \{a_0 + a_1 i + a_2 j + a_3 k +$ *a4l+a5m / ai ∈ F*

Keywords: *Binary Operation, Abelian Group, Semigroup, Ring, Field*

INTRODUCTION:
Herstein cotes in 1992
***Definition**: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by *, such that
1 a, b ∈ G implies that a*b ∈ G
2 a, b, c ∈ G implies that (a*b)*c = a * (b*c)
3 There exist an element e ∈ G such that a*e = e*a = a for all a ∈ G
4 For every a ∈ G there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$
***Definition**: A group G is said to be abelian (or Commutative) if for every a, b ∈ G,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by + and * respectively, such that for all a,b,c in R:
1 a+b is in R.
2 a+b = b+a.
3 (a+b)+c = a+(b+c).
4 There is an element 0 in R such that a+0 = a,∀ a ∈ R
5 There exist an element –a in R such that a + (-a) = 0.
6 a*b is in R
7 a*(b*c) = (a*b)*c.
8 a * (b+c) = a * b + a * c and (b+c) * a = b*a + c*a.
It may very well happen, or not happen, that there is an element 1 in R such that a*1 = 1*a = a for every a in R; if there is such we shall describe R as a ring with unit element.
If the multiplication of R is such that a*b = b*a for every a, b in R, then we call R a commutative ring.

DISCUSSION
Let $A = \{a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m \,/\, ai \in F\}$
And
Let $x = a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m$, $ai \in F$
$y = b_0 + b_1 i + b_2 j + b_3 k + b_4 l + b_5 m$, b$i \in F$
$z = c_0 + c_1 i + c_2 j + c_3 k + c_4 l + c_5 m$, c$i \in F$

$$-x = (-a_0) + (-a_1)i + (-a_2)j + (-a_3)k + (-a_4)l + (-a_5)m$$
$$O = 0 + 0i + 0j + 0k + 0l + 0m$$
$$1 = 1 + 0i + 0j + 0k + 0l + 0m$$

c$x = ca_0 + ca_1 i + ca_2 j + ca_3 k + ca_4 l + ca_5 m$, c ∈ F

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Now we define first binary operation + on A as

$$x + y = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m) + (b_0 + b_1 i + b_2 j + b_3 k + b_4 l + b_5 m)$$
$$= (a_0 + b_0) + (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k + (a_4 + b_4)l + (a_5 + b_5)m$$
………… (1)

$$=> \ x + y = y + x, \forall\, x, y, \in A$$
$$x + (y + z) = (x + y) + z, \forall\, x, y, z \in A$$
$$0 + x = x + 0, \forall\, x \in A$$
$$x + (-x) = (-x) + x = 0, \forall\, x \in A$$

⇨ (A,+) is an abelian group. ………… (2)

Case: 1

Second binary operation on A as defined as

$$x.y = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m).(b_0 + b_1 i + b_2 j + b_3 k + b_4 l + b_5 m)$$
$$= (a_0 + a_1 + a_2 + a_3 + a_4 + a_5)y, \forall\, x, y \in A$$
$$=> \ x.(y.z) = (x.y).z, \forall\, x, y, z \in A$$
$$\& x.y \neq y.x, \forall\, x, y, \in A$$
$$(x + y).z = x.z + y.z, \forall\, x, y, z \in A$$
$$x.(y + z) = x.y + x.z, \forall\, x, y, z \in A$$

Hence (A,.) is a non-commutative semi group.
& (A,+,.) is a non-abelian ring. ……….. (4)

Case: 2

$$x * y = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m) * (b_0 + b_1 i + b_2 j + b_3 k + b_4 l + b_5 m)$$
……….. (5)

⇨ $x * y = y * x, \forall\, x, y \in A$
⇨ $x * (y * z) = (x * y) * z, \forall\, x, y, z \in A$
⇨ $1 * x = x * 1 = x, \forall\, x \in A$

Let $x^{-1} = (a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m)^{-1}$ be a inverse of x in A
$= b_0 + b_1 i + b_2 j + b_3 k + b_4 l + b_5 m$ *(say)*

∴By definition

$$(a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m) * (b_0 + b_1 i + b_2 j + b_3 k + b_4 l + b_5 m)$$
$$= (b_0 + b_1 i + b_2 j + b_3 k + b_4 l + b_5 m) * (a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m)$$
$$= 1 + 0i + 0j + 0k + 0l + 0m$$

Hence we get

$(a_0 b_0 + a_1 b_5 + a_2 b_4 + a_3 b_3 + a_4 b_2 + a_5 b_1) = 1$ ………. (6)
$(a_0 b_1 + a_1 b_0 + a_2 b_5 + a_3 b_4 + a_4 b_3 + a_5 b_2) = 0$ ………. (7)
$(a_0 b_2 + a_1 b_1 + a_2 b_0 + a_3 b_5 + a_4 b_4 + a_5 b_3) = 0$ ………. (8)
$(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0 + a_4 b_5 + a_5 b_4) = 0$ ………. (9)
$(a_0 b_4 + a_1 b_3 + a_2 b_2 + a_3 b_1 + a_4 b_0 + a_5 b_5) = 0$ ………. (10)
$(a_0 b_5 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1 + a_5 b_0) = 0$ ………. (11)

Rewriting eq^n (7), (8), (9), (10) & (11)

$(a_1 b_0 + a_0 b_1 + a_5 b_2 + a_4 b_3 + a_3 b_4 + a_2 b_5) = 0$ ……… (12)
$(a_2 b_0 + a_1 b_1 + a_0 b_2 + a_5 b_3 + a_4 b_4 + a_3 b_5) = 0$ ………… (13)
$(a_3 b_0 + a_2 b_1 + a_1 b_2 + a_0 b_3 + a_5 b_4 + a_4 b_5) = 0$ ………… (14)
$(a_4 b_0 + a_3 b_1 + a_2 b_2 + a_1 b_3 + a_0 b_4 + a_5 b_5) = 0$ ………… (15)
$(a_5 b_0 + a_4 b_1 + a_3 b_2 + a_2 b_3 + a_1 b_4 + a_0 b_5) = 0$...………. (16)

$$\frac{b_0}{F_0} = \frac{-b_1}{F_1} = \frac{b_2}{F_2} = \frac{-b_3}{F_3} = \frac{b_4}{F_4} = \frac{-b_5}{F_5} = k$$

$$F_0=\begin{vmatrix} a_0 & a_5 & a_4 & a_3 & a_2 \\ a_1 & a_0 & a_5 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_5 & a_4 \\ a_3 & a_2 & a_1 & a_0 & a_5 \\ a_4 & a_3 & a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_1=\begin{vmatrix} a_1 & a_5 & a_4 & a_3 & a_2 \\ a_2 & a_0 & a_5 & a_4 & a_3 \\ a_3 & a_1 & a_0 & a_5 & a_4 \\ a_4 & a_2 & a_1 & a_0 & a_5 \\ a_5 & a_3 & a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_2=\begin{vmatrix} a_1 & a_0 & a_4 & a_3 & a_2 \\ a_2 & a_1 & a_5 & a_4 & a_3 \\ a_3 & a_2 & a_0 & a_5 & a_4 \\ a_4 & a_3 & a_1 & a_0 & a_5 \\ a_5 & a_4 & a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_3=\begin{vmatrix} a_1 & a_0 & a_5 & a_3 & a_2 \\ a_2 & a_1 & a_0 & a_4 & a_3 \\ a_3 & a_2 & a_1 & a_5 & a_4 \\ a_4 & a_3 & a_2 & a_0 & a_5 \\ a_5 & a_4 & a_3 & a_1 & a_0 \end{vmatrix}$$

$$F_4=\begin{vmatrix} a_1 & a_0 & a_5 & a_4 & a_2 \\ a_2 & a_1 & a_0 & a_5 & a_3 \\ a_3 & a_2 & a_1 & a_0 & a_4 \\ a_4 & a_3 & a_2 & a_1 & a_5 \\ a_5 & a_4 & a_3 & a_2 & a_0 \end{vmatrix}$$

$$F_5=\begin{vmatrix} a_1 & a_0 & a_5 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_5 & a_4 \\ a_3 & a_2 & a_1 & a_0 & a_5 \\ a_4 & a_3 & a_2 & a_1 & a_0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \end{vmatrix}$$

$=> b_0 = kF_0$………… (17)
$b_1 = -kF_1$ ………… (18)
$b_2 = kF_2$………… (19)
$b_3 = -kF_3$ ………… (18)
$b_4 = kF_4$ ………… (19)
$b_5 = -kF_5$ ………… (20)

From eq^n (6), (17), (18), (18), (19)& (20) one obtains
$k\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3 + a_4F_4 - a_5F_5\} = 1$
Hence
$k = {}^1/_{\{a_0F_0 - a_1F_1 + a_2F_2 - a_3F_3 + a_4F_4 - a_5F_5\}} = \infty$
If $a_0 = a_1 = a_2 = a_3 = a_4 = 0$ &$a_5 \neq 0$
From the above discussion we get

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$x^{-1} = \frac{F_0 - F_1 i + F_2 j - F_3 k + F_4 l - F_5 m}{\{a_0 F_0 - a_1 F_1 + a_2 F_2 - a_3 F_3 + a_4 F_4 - a_5 F_5\}} = \infty$
$=> x^{-1}$ is exist for each non zero element in A.
Hence
$(A, +, *)$ is a Commutative ring with unity.
Where $A = \{a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m \ / \ ai \ \in \ F \}$

Conclusion
From the above discussion, I come to the following conclusions first I proved that (A, +,.) is a non abelian ring.
Second I proved that $(A, +, *)$ Is a Commutative ring with unity.
Where $A = \{a_0 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 m \ / \ ai \ \in \ F \}$

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