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NATURE OF GENERALIZED ALGEBRAIC STRUCTURE

$$A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5 / a_i \in F \& G_i \in C(P)\}$$

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ABSTRACT

This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri SriRamakrishana. In the Present work first I proved that $(A, +, \cdot)$ is a non abelian ring. Second I proved that $(A, +, *)$ is a Commutative ring with unity, where $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5 / a_i \in F \& G_i \in C(P)\}$ and $C(P)$ = Class of Algebraic Structure.

Keywords: Binary Operation, Abelian Group, Semigroup, Ring, Field

INTRODUCTION

Herstein cotes in 1992

Definition: A nonempty set of elements G is said to form a group if in G there is defined a binary operation, called the product and defined by $*$, such that

1 $a, b \in G$ implies that $a*b \in G$

2 $a, b, c \in G$ implies that $(a*b)*c = a*(b*c)$

3 There exist an element $e \in G$ such that $a*e = e*a = a$ for all $a \in G$

4 For every $a \in G$ there exist an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Definition: A group G is said to be abelian (or Commutative) if for every $a, b \in G$,

$$a * b = b * a.$$

Definition: A nonempty set R is said to be an associative ring if in R there are defined two operations, defined by $+$ and $*$ respectively, such that for all a, b, c in R :

1 $a+b$ is in R .

2 $a+b = b+a$.

3 $(a+b)+c = a+(b+c)$.

4 There is an element 0 in R such that $a+0 = a, \forall a \in R$

5 There exist an element $-a$ in R such that $a + (-a) = 0$.

6 $a*b$ is in R

7 $a*(b*c) = (a*b)*c$.

8 $a * (b+c) = a * b + a * c$ and $(b+c) * a = b*a + c*a$.

It may very well happen, or not happen, that there is an element 1 in R such that $a*1 = 1*a = a$ for every a in R ; if there is such we shall describe R as a ring with unit element.

If the multiplication of R is such that $a*b = b*a$ for every a, b in R , then we call R a commutative ring.

Discussion

Let $A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5 / a_i \in F \& G_i \in C(P)\}$
 and $C(P)$ = Class of Algebraic Structure.

Let $x = a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5, a_i \in F$

$y = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4 + b_5G_5, b_i \in F$

$z = c_0G_0 + c_1G_1 + c_2G_2 + c_3G_3 + c_4G_4 + c_5G_5, c_i \in F$

$$-x = (-a_0)G_0 + (-a_1)G_1 + (-a_2)G_2 + (-a_3)G_3 + (-a_4)G_4 + (-a_5)G_5$$

$$0 = 0G_0 + 0G_1 + 0G_2 + 0G_3 + 0G_4 + 0G_5$$

$$G_0 = 1G_0 + 0G_1 + 0G_2 + 0G_3 + 0G_4 + 0G_5$$

$$cx = (ca_0)G_0 + (ca_1)G_1 + (ca_2)G_2 + (ca_3)G_3 + (ca_4)G_4 + (ca_5)G_5, c \in F$$

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Now we define first binary operation $+$ on A as

$$\begin{aligned} x + y &= (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5) \\ &\quad + (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4 + b_5G_5) \\ &= (a_0 + b_0)G_0 + (a_1 + b_1)G_1 + (a_2 + b_2)G_2 + (a_3 + b_3)G_3 + (a_4 + b_4)G_4 + (a_5 + b_5)G_5 \\ &\dots\dots\dots (1) \end{aligned}$$

$$\begin{aligned} \Rightarrow x + y &= y + x, \forall x, y \in A \\ x + (y + z) &= (x + y) + z, \forall x, y, z \in A \\ 0 + x &= x + 0, \forall x \in A \end{aligned}$$

$$\begin{aligned} x + (-x) &= (-x) + x = 0, \forall x \in A \\ \Rightarrow (A, +) &\text{ is an abelian group. } \dots\dots\dots (2) \end{aligned}$$

Case 1:

Now we define Second binary operation on A as

$$\begin{aligned} x \cdot y &= (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5) \\ &\cdot (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4 + b_5G_5) \\ x \cdot y &= (a_0 + a_1 + a_2 + a_3 + a_4 + a_5)y, \forall x, y \in A \\ &\dots\dots\dots (3) \end{aligned}$$

$$\Rightarrow x \cdot (y \cdot z) = (x \cdot y) \cdot z, \forall x, y, z \in A$$

& $x \cdot y \neq y \cdot x, \forall x, y \in A$
 Also

$$\begin{aligned} (x + y) \cdot z &= x \cdot z + y \cdot z, \forall x, y, z \in A \\ x \cdot (y + z) &= x \cdot y + x \cdot z, \forall x, y, z \in A \end{aligned}$$

Hence (A, \cdot) is a non-commutative semi group.

And $(A, +, \cdot)$ is a non-commutative ring. (4)

Case: 2

Now we define Second binary operation $*$ on A as

$$\begin{aligned} x * y &= (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5) \\ &\quad * (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4 + b_5G_5) \\ &= (a_0b_0 + a_1b_5 + a_2b_4 + a_3b_3 + a_4b_2 + a_5b_1)G_0 \\ &\quad + (a_0b_1 + a_1b_0 + a_2b_5 + a_3b_4 + a_4b_3 + a_5b_2)G_1 \\ &\quad + (a_0b_2 + a_1b_1 + a_2b_0 + a_3b_5 + a_4b_4 + a_5b_3)G_2 \\ &\quad + (a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0 + a_4b_5 + a_5b_4)G_3 \\ &\quad + (a_0b_4 + a_1b_3 + a_2b_2 + a_3b_1 + a_4b_0 + a_5b_5)G_4 \\ &\quad + (a_0b_5 + a_1b_4 + a_2b_3 + a_3b_2 + a_4b_1 + a_5b_0)G_5 \\ &\dots\dots\dots (5) \end{aligned}$$

$$\begin{aligned} \Rightarrow x * y &= y * x, \forall x, y \in A \\ \Rightarrow x * (y * z) &= (x * y) * z, \forall x, y, z \in A \\ G_0 * x &= x * G_0 = x, \forall x \in A \end{aligned}$$

Let $x^{-1} = (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3 + b_4G_4 + b_5G_5)$ be the inverse of any x in A
 Where $x = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 + a_4G_4 + a_5G_5)$

\therefore By definition one obtains

$$(a_0b_0 + a_1b_5 + a_2b_4 + a_3b_3 + a_4b_2 + a_5b_1) = 1 \dots\dots\dots (6)$$

$$(a_0b_1 + a_1b_0 + a_2b_5 + a_3b_4 + a_4b_3 + a_5b_2) = 0 \dots\dots\dots (7)$$

$$(a_0b_2 + a_1b_1 + a_2b_0 + a_3b_5 + a_4b_4 + a_5b_3) = 0 \dots\dots\dots (8)$$

$$(a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0 + a_4b_5 + a_5b_4) = 0 \dots\dots\dots (9)$$

$$(a_0b_4 + a_1b_3 + a_2b_2 + a_3b_1 + a_4b_0 + a_5b_5) = 0 \dots\dots\dots (10)$$

$$(a_0b_5 + a_1b_4 + a_2b_3 + a_3b_2 + a_4b_1 + a_5b_0) = 0 \dots\dots\dots (11)$$

Rewriting eqⁿ (7), (8), (9), (10)& (11)

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$$(a_1 b_0 + a_0 b_1 + a_5 b_2 + a_4 b_3 + a_3 b_4 + a_2 b_5) = 0 \dots\dots\dots (12)$$

$$(a_2 b_0 + a_1 b_1 + a_0 b_2 + a_5 b_3 + a_4 b_4 + a_3 b_5) = 0 \dots\dots\dots (13)$$

$$(a_3 b_0 + a_2 b_1 + a_1 b_2 + a_0 b_3 + a_5 b_4 + a_4 b_5) = 0 \dots\dots\dots (14)$$

$$(a_4 b_0 + a_3 b_1 + a_2 b_2 + a_1 b_3 + a_0 b_4 + a_5 b_5) = 0 \dots\dots\dots (15)$$

$$(a_5 b_0 + a_4 b_1 + a_3 b_2 + a_2 b_3 + a_1 b_4 + a_0 b_5) = 0 \dots\dots\dots (16)$$

solving eqⁿ (12), (13), (14), (15) & (16) one obtains

$$\frac{b_0}{F_0} = \frac{-b_1}{F_1} = \frac{b_2}{F_2} = \frac{-b_3}{F_3} = \frac{b_4}{F_4} = \frac{-b_5}{F_5} = k$$

Where;

$$F_0 = \begin{vmatrix} a_0 & a_5 & a_4 & a_3 & a_2 \\ a_1 & a_0 & a_5 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_5 & a_4 \\ a_3 & a_2 & a_1 & a_0 & a_5 \\ a_4 & a_3 & a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_1 = \begin{vmatrix} a_1 & a_5 & a_4 & a_3 & a_2 \\ a_2 & a_0 & a_5 & a_4 & a_3 \\ a_3 & a_1 & a_0 & a_5 & a_4 \\ a_4 & a_2 & a_1 & a_0 & a_5 \\ a_5 & a_3 & a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_2 = \begin{vmatrix} a_1 & a_0 & a_4 & a_3 & a_2 \\ a_2 & a_1 & a_5 & a_4 & a_3 \\ a_3 & a_2 & a_0 & a_5 & a_4 \\ a_4 & a_3 & a_1 & a_0 & a_5 \\ a_5 & a_4 & a_2 & a_1 & a_0 \end{vmatrix}$$

$$F_3 = \begin{vmatrix} a_1 & a_0 & a_5 & a_3 & a_2 \\ a_2 & a_1 & a_0 & a_4 & a_3 \\ a_3 & a_2 & a_1 & a_5 & a_4 \\ a_4 & a_3 & a_2 & a_0 & a_5 \\ a_5 & a_4 & a_3 & a_1 & a_0 \end{vmatrix}$$

$$F_4 = \begin{vmatrix} a_1 & a_0 & a_5 & a_4 & a_2 \\ a_2 & a_1 & a_0 & a_5 & a_3 \\ a_3 & a_2 & a_1 & a_0 & a_4 \\ a_4 & a_3 & a_2 & a_1 & a_5 \\ a_5 & a_4 & a_3 & a_2 & a_0 \end{vmatrix}$$

$$F_5 = \begin{vmatrix} a_1 & a_0 & a_5 & a_4 & a_3 \\ a_2 & a_1 & a_0 & a_5 & a_4 \\ a_3 & a_2 & a_1 & a_0 & a_5 \\ a_4 & a_3 & a_2 & a_1 & a_0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \end{vmatrix}$$

$$\Rightarrow b_0 = kF_0 \dots\dots\dots (17)$$

$$b_1 = -kF_1 \dots\dots\dots (18)$$

$$b_2 = kF_2 \dots\dots\dots (19)$$

$$b_3 = -kF_3 \dots\dots\dots (20)$$

$$b_4 = kF_4 \dots\dots\dots (21)$$

$$b_5 = -kF_5 \dots\dots\dots (22)$$

From eqⁿ (6), (17), (18), (19), (20), (21) & (22) one obtains

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$$k = 1 / \{a_0 F_0 - a_1 F_1 + a_2 F_2 - a_3 F_3 + a_4 F_4 + a_5 F_5\} = \infty$$

if $(a_0 = a_1 = a_2 = a_3 = a_4 = 0 \text{ \& } a_5 \neq 0)$

Hence the inverse of each element in A not exist

From the above discussion one obtains

$(A, +, *)$ Is a Commutative ring with unity,

Where $A = \{a_0 G_0 + a_1 G_1 + a_2 G_2 + a_3 G_3 + a_4 G_4 + a_5 G_5 / a_i \in F \text{ \& } G_i \in C(P)\}$

and $C(P)$ = Class of Algebraic Structure.

Conclusion

From the above discussion, I come to the following conclusions

first I proved that $(A, +, .)$ is a non commutative ring. Second I proved that $(A, +, *)$ Is a Commutative ring with unity.

Where $A = \{a_0 G_0 + a_1 G_1 + a_2 G_2 + a_3 G_3 + a_4 G_4 / a_i \in F \text{ \& } G_i \in C(P)\}$

and $C(P)$ = Class of Algebraic Structure,

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